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Keywords: *s-geodetic cover, s-geodetic basis, s-geodetic number, pseudo s-geodetic set, pseudo s-geodetic number.*

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I. INTRODUCTION

Zadeh in 1965 [27] brought the concept of fuzzy sets into existence which gave a platform for describing the uncertainties prevailing in day-today life situations. Later on, the theory of fuzzy graphs was developed by Rosenfeld in the year 1975 [21] along with Yeh and Bang [26]. Rosenfeld also obtained the fuzzy analogue of several graph theoretic concepts like paths, cycles, trees and connectedness along with some of their properties [21] and the concept of fuzzy trees [17], automorphism of fuzzy graphs [2], fuzzy interval graphs [14], cycles and co-cycles of fuzzy graphs [15] etc has been established by several authors during the course of time. Fuzzy groups and the notion of a metric in fuzzy graphs was introduced by Bhattacharya [1]. The concept of strong arcs [5] and geodesic distance in fuzzy graphs [4] were introduced by Bhutani and Rosenfeld in the year 2003. The definition of fuzzy end nodes and some of their properties were established by the same authors in [3]. Several other important works on fuzzy graphs can be found in [18, 12, 24]. Studies in fuzzy graphs using μ-distance was carried out by Rosenfeld [22] in 1975 and was further studied by Sunitha and Vijayakumar in [24]. In crisp graph, the concept of geodetic iteration number was first introduced by Harary and Nieminen in 1981 [10]. This concept along with that of geodetic numbers in graphs was again discussed by several authors in [6], [8] and [7]. Later on, these concepts were extended to fuzzy graphs using geodesic distance by Suvarna and Sunitha in [25] and the same based on μ-distance was introduced by Linda and Sunitha in [11].The concept of sum distance and some of its metric aspects was introduced by Mini Tom and Sunitha in [13]. *s*-Geodetic iteration number and *s*-geodetic number of a fuzzy graph based on sum distance was introduced by Sameeha and Sunitha in [19].

In this paper, the concept of Pseudo *s*-geodetic number of fuzzy graphs is introduced. The Pseudo *s*-geodetic number of paths, fuzzy trees and of complete bipartite fuzzy graphs are obtained. The limiting values of Pseudo *s*-geodetic numbers are established.

II. PRELIMINARIES

A fuzzy graph [16] is a triplet $G : (V, \sigma, \mu)$ where V is vertex set, σ a fuzzy subset of V and μ a fuzzy relation on σ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v), \forall u, v \in V$.

We assume that V is finite and non-empty, μ is reflexive (*i.e.*, $\mu(x, x) = \sigma(x), \forall x$) and symmetric (*i.e.*, $\mu(x, y) = \mu(y, x), \forall (x, y)$). Also we denote the underlying crisp graph [9] by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V / \mu(u, v) > 0\}$. Here we assume $\sigma^* = V$.

A fuzzy graph $G : (V, \sigma, \mu)$ is called trivial if $\sigma^* = 1$. Otherwise it is called non-trivial.

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A fuzzy graph $G : (V, \sigma, \mu)$ is a complete fuzzy graph [16] if $\mu(u, v) = \sigma(u) \wedge \sigma(v)\ \forall u, v \in \sigma^*$.

A weakest arc of $G : (V, \sigma, \mu)$ is an arc with least non zero membership value. A path P of length n is a sequence of distinct nodes $u_0, u_1, ..., u_n$ such that $\mu(u_{i-1}, u_i) > 0$, $i = 1, 2, 3, ..., n$ and the degree of membership of a weakest arc in the path is defined as its strength. The strength of connectedness between two nodes u and v is defined as the maximum of the strengths of all paths between u and v, and is denoted by $CONN_G(u, v)$. A $u - v$ path P is called a strongest $u - v$ path if its strength equals $CONN_G(u, v)$. A fuzzy graph $G : (V, \sigma, \mu)$ is connected if for every u, v in σ^*, $CONN_G(u, v) > 0$.

An arc (u, v) of a fuzzy graph is called strong if its weight is at least as great as the strength of connectedness of its end nodes u, v when the arc (u, v) is deleted and a $u - v$ path P is called a strong path if P contains only strong arcs [5].

Two nodes u and v in a fuzzy graph $G : (V, \sigma, \mu)$ are neighbors if $\mu(u, v) > 0$ and v is called a strong neighbor of u if the arc (u, v) is strong. Also $N(u)$ denotes the set of neighbors of u other than u and degree of u is $deg(u) = N(u)$. A node u with $deg(u) = 1$ is an end node and a node u with $deg(u) > 1$ is an internal node. A node v is called a fuzzy end node of G if it has exactly one strong neighbor in G [3].

A connected fuzzy graph $G : (V, \sigma, \mu)$ is called a fuzzy tree [21] if it has a spanning fuzzy subgraph $F : (V, \sigma, \nu)$ which is a tree such that for all arcs (u, v) not in F, $CONN_F(u, v) > \mu(u, v)$.

A fuzzy graph G is said to be bipartite [23] if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that $\mu(v_1, v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$. Further if $\mu(u, v) = \sigma(u)\ \ \sigma(v)\ \forall u \in V_1$ and $v \in V_2$, then G is called a complete bipartite fuzzy graph and is denoted by $K_{\sigma 1, \sigma 2}$, where σ_1 and σ_2 are respectively the restrictions of σ to V_1 and V_2.

For any path $P : u_0 - u_1 - u_2 - ... - u_n$, length of P, $L(P)$, is defined as the sum of the weights of the arcs in P. That is, $L(P) = \Sigma^n\ \mu(u_{i-1}, u_i)$.

If $n = 0$, define $L(P) = 0$ and for $n \geq 1$, $L(P) > 0$. For any two nodes u, v in $G : (V, \sigma, \mu)$, if $P = \{P_i : P_i$ is a u-v path, $i = 1, 2, 3, ...\}$, then the sum distance between u and v is defined as $d_s(u, v) = Min\{L(P_i) : P_i \in P, i = 1, 2, 3, ...\}$[13].

Any path P from x to y whose length is $d_s(x, y)$ is called s-geodesic from x to y [19].

Let $S \subseteq V$ be a set of nodes of a connected fuzzy graph $G : (V, \sigma, \mu)$. Then the s-geodetic closure of S, with respect to sum distance, is the set of all nodes of S as well as all nodes that lie on s-geodesics between nodes of S and is denoted by (S) [19].

A set $S \subseteq V(G)$ such that every node of G is contained in an s-geodesic joining some pair of nodes in S is called an s-geodetic cover(s-geodetic set) of G. In other words if $(S) = V(G)$, then S is an s-geodetic cover of G [19].

The s-geodetic number of G, denoted by s-$gn(G)$, is the minimum order of its s-geodetic covers and any cover of order s-$gn(G)$ is an s-geodetic basis [19].

A node v in a fuzzy graph G is called an extreme node if the fuzzy sub- graph induced by its neighbors is a complete fuzzy graph [19].

Throughout this paper we consider only connected fuzzy graphs.

III. PSEUDO S-GEODETIC NUMBER OF A FUZZY GRAPH

In fuzzy graph theory, the concept of Pseudo geodesic number, using geodesic distance, was introduced by Sameeha and Sunitha in [20]. In this section, using sum distance, we introduce the concept of Pseudo s-geodetic number of fuzzy graphs. We also show that a Pseudo s-geodetic set is not in general a complementary s-geodetic set of a fuzzy graph.

Definition 3.1: Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph and S be an s-geodetic basis of G. Then the set of nodes which do not belong to any s-geodetic basis of G is the Pseudo s-geodetic set S' of G.

The cardinality of Pseudo s-geodetic set S' is called Pseudo s-geodetic num- ber and is denoted by s-$gn'(G)$.

Example 3.2: Consider the fuzzy graph G given in Fig.1.

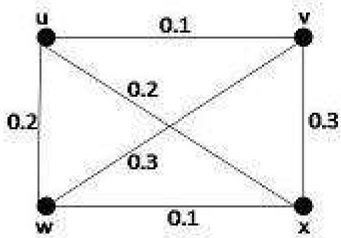

Fig. 1

Here, if $S=\{v, x, w\}$ then $(S)=\{u, v, x, w\}=V(G)$. Therefore S is an s-geodetic cover of G. Also S is the unique s-geodetic basis of G and so $s\text{-}gn(G) = 3$. Hence $S' = \{u\}$ and so $s\text{-}gn'(G) = 1$.

Remark 3.3: For the fuzzy graph G given in Fig.1, $S' = S^c$ and hence $s\text{-}gn'(G) = |V - S|$.
But in general S' is not the complement of the set S.

Example 3.4: Consider the fuzzy graph G given in Fig.2.

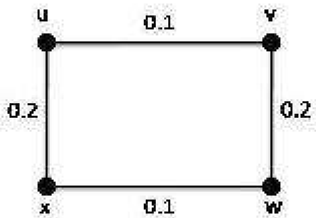

Fig. 2

Here $S_1 = \{u, w\}$ and $S_2 = \{v, x\}$ are both s-geodetic bases of G and so $S' = \varphi$. Hence $s\text{-}gn'(G) = 0$.

Proposition 3.5. [19] For any non-trivial connected fuzzy graph G on n nodes, $2 \le s\text{-}gn(G) \le n$.

Proposition 3.6. For a connected fuzzy graph G on n nodes, $0 \le gn'(G) \le n - 2$.
Proof. The proof follows directly from Proposition 3.5 that $2 \le s\text{-}gn(G) \le n$.

Since $s\text{-}gn(G) \ge 2$, we get $s\text{-}gn'(G) \le n - 2$. Also since $s\text{-}gn(G) \le n$, we get $s\text{-}gn'(G) \ge 0$*Proposition 3.7.* If $S_1, S_2, \ldots, S_n$ are the s-geodetic bases of a fuzzy graph
$G : (V, \sigma, \mu)$, then the pseudo s-geodetic number $s\text{-}gn'(G) = |\cap_{i=1}^{n} S_i^c|$.

Proof. Let S' be the pseudo s-geodetic set of G. To show that $s\text{-}gn'(G) = |\cap_{i=1}^{n} S_i^c|$ it is enough to show that $S' = |\cap_{i=1}^{n} S_i^c|$

Let v be a node of G such that $v \in S'$.

Then by definition 3.1, v does not belong to any s-geodetic basis of G.

i.e, $v \notin S_i\ \forall i = 1, 2, \ldots, n$.
$\Rightarrow v \in S_i^c\ \forall i = 1, 2, \ldots, n$.
$\Rightarrow v \in \cap_{i=1}^{n} S_i^c$.
$\Rightarrow S' \subseteq \cap_{i=1}^{n} S_i^c$ ....(1)

Conversely, let u be a node of G such that $u \in \cap_{i=1}^{n} S_i^c$.
Then $u \in S_i^c\ \forall i = 1, 2, \ldots, n$.
$\Rightarrow u \notin S_i\ \forall i = 1, 2, \ldots, n$.
Hence by definition 3.1, $u \in S'$ and so $\cap_{i=1}^{n} S_i^c \subseteq S'$....................(2)
From (1) and (2), $S' = \cap_{i=1}^{n} S_i^c$ and so $s\text{-}gn'(G) = |\cap_{i=1}^{n} S_i^c|$.

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Remark 3.8. The set of internal nodes (non end-nodes) of a path P_n $(n \geq 2)$ is its Pseudo s-geodetic set so that $s - gn'(P_n) = n - 2$. Thus the path P_n has the largest possible Pseudo s-geodetic number $n - 2$.

Proposition 3.9. No extreme node of a fuzzy graph $G : (V, \sigma, \mu)$ belongs to its Pseudo s-geodetic set.

Proof. Let S be an s-geodetic cover of G and v be an extreme node of G. Let $\{v_1, v_2, ..., v_n\}$ be the neighbors of v and (v, v_i), $1 \leq i \leq n$ be the edges incident on v. Since v is an extreme node, v_i and v_j are adjacent for $i \neq j$ $(1 \leq i, j \leq n)$. Hence any s-geodetic which contains v, is either (v_i, v), $(1 \leq i \leq n)$ or $u_1, u_2, ..., u_m$, v_i, v where each $'u_i, (1 \leq i \leq m)$ is different from vi. Hence it follows that $v \in S$ and so $v \notin S$.

Proposition 3.10. [19] Let $G : (V, \sigma, \mu)$ be a fuzzy tree such that G^* is a tree. Then the set of all fuzzy end nodes of G form an s-geodetic basis for G and s-$gn(G)$ is the number of fuzzy end nodes of G.

Proposition 3.11. The Pseudo s-geodetic number of a fuzzy tree $G : (V, \sigma, \mu)$ such that G^* is a tree equals the number of non fuzzy end-nodes in G.

Proof. By Proposition 3.10 a fuzzy tree, whose underlying crisp gaph G^* is a tree, has a unique s-geodetic basis consisting of its fuzzy end nodes. Thus for such a fuzzy tree $G : (V, \sigma, \mu)$, the set of all non fuzzy end-nodes of G is the Pseudo s-geodetic set of G. Hence, the Pseudo s-geodetic number $s\ gn'(G)$ equals the number of non fuzzy end-nodes in G.

Remark 3.12. However, the above result is not true in general for every fuzzy tree.

Example 3.13. Consider the fuzzy tree G given in Fig.3.

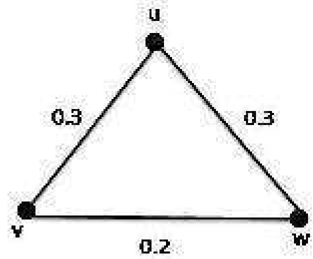

Fig. 3

Here v and w are the fuzzy end nodes of G but $\{v, w\}$ is not an s-geodetic cover since $(\{v, w\}) = \{v, w\} = V(G)$ and the s-geodetic basis is $\{u, v, w\}$.

Thus the Pseudo s-geodetic set $S' = \emptyset$. Hence, s-$gn'(G) = 0$. But the number of non fuzzy end nodes in G is 1.

Proposition 3.14. For a complete bipartite fuzzy graph $K_{\sigma_1,\sigma_2} = (V_1 \cup V_2, \sigma, \mu)$, the Pseudo s-geodetic number

1. $s - gn'(K_{\sigma_1,\sigma_2}) = 0$ if $|V_1| = |V_2| = 1$.
2. $s - gn'(K_{\sigma_1,\sigma_2}) = |V_1|$ if $|V_1| = 1$ and $|V_2| \geq 2$.
3. $s - gn'(K_{\sigma_1,\sigma_2}) = max\{r, s\}$ if $|V_1| = r$ and $|V_2| = s$ where $r, s \geq 2$, $r \neq s$.
4. $s - gn'(K_{\sigma_1,\sigma_2}) = 0$ if $r = s$.

Proof:

1. This follows directly from Remark 3.8.
2. This follows from Proposition 3.11.
3. Let $r, s \geq 2$. First assume that $r < s$.

Let $V_1 = \{u_1, u_2, ..., u_r\}$ and $V_2 = \{w_1, w_2, ..., w_s\}$ be a bipartition of K_{σ_1},σ_2. Let $S = V_1$. We prove that S is an s-geodetic basis of K_{σ_1},σ_2. That is, we prove that S is an s-geodetic cover of K_{σ_1},σ_2 having minimum cardinality.

Any node w_j, $(1 \leq j \leq s)$ lies on the s-geodesic $u_i w_j u_k$ for any i, so that S is an s-geodetic cover of K_{σ_1},σ_2. Let T be any set of nodes such that $|T| < |S|$. If $T \subset V_1$ then there exists a vertex $u_i \in V_1$ such that $u_i \notin T$. Then the only s-geodesics containing u_i are $u_i w_j u_k$, $(k \neq i)$, $(1 \leq j \leq s)$ and $w_j u_i w_l$, $(l \neq j)$ and so u_i cannot lie on an s-geodetic joining 2 nodes of T.

Thus T is not an s-geodetic cover of K_{σ_1,σ_2}.
If $T \subset V_2$, then by a similar argument, T is not an s-geodetic cover of $K\sigma_1,\sigma_2$.

Now if $T \subset S \cup V_2$ such that T contains at least one node from each of S and V_2, then since $|T| < |S|$, there exists nodes $u_i \in V_1$ and $w_j \in V_2$ such that $u_i \notin T$ and $w_j \notin T$. Then clearly at least one of the end nodes of the edge (u_i, w_j) does not lie on an s-geodesic connecting 2 nodes of T so that T is not an s-geodetic cover.

Thus in any case, T is not an s-geodetic cover of K_{σ_1,σ_2}. Hence S is the unique s-geodetic basis of K_{σ_1,σ_2}. so that $s\text{-}gn(K_{\sigma_1,\sigma_2}) = |S| = r = min\{r, s\}$.

Thus $S' = V_2$ and so $gn'(K_{\sigma_1,\sigma_2}) = |V_2| = s = max\{r, s\}$.

4. Now if $r = s$, then as in (3), V_1 and V_2 are both s-geodetic basis of K_{σ_1,σ_2}. Hence by defintion, $s\text{-}gn'(K_{\sigma_1,\sigma_2}) = 0$.

IV. AN APPLICATION OF PSEUDO S-GEODETIC SETS IN LOCATION THEORY

Location theory addresses questions of what economic activities are located where and why. It is generally assumed that agents act in their own self- interest. Firms thus choose locations that maximize their profits and indi- viduals choose locations that maximize their utility.

The topic of location theory generally observes patterns across geographic space typically associated with human settlement, industry siting, service competition, and, more generally, consumer behavior. Ultimately, location theory aims at gaining a sound grasp on the factors associated with loca-tional decision making and the question of what makes a good location.

In this section, the concept of pseudo s-geodetic sets is utilized in locating areas least suitable for placing Gas stations in a locality, the main agenda being to choose locations that maximize utility and minimize the number of Gas stations. For example, being the head of the city, we know that the Mayor of a city officially speaks for both the government and the community as a whole.

Suppose the Mayor issues an oder that the Gas stations in the city should be placed in such a way that they are minimum in number and of maximum utility to the consumers. The s-geodetic number of a fuzzy graph representation of a city, with each node representing important places in the city and each edge representing the roads connecting them, gives an idea about the minimum number of Gas stations to be placed so as to get maximum utility among consumers. Hence the s-geodetic bases give appro- priate locations for placing these Gas stations. A node that does not belong to any of the s-geodetic bases will eventually represent a location that is least suitable for placing a Gas station. In other words, the pseudo s-geodetic sets represent locations that are least suitable for placement of Gas stations in a certain city so as to satisfy the norms put forward by the Mayor of the city.

An illustrative example

Consider the fuzzy graph $G : (V, \sigma, \mu)$ given in Figure 4.

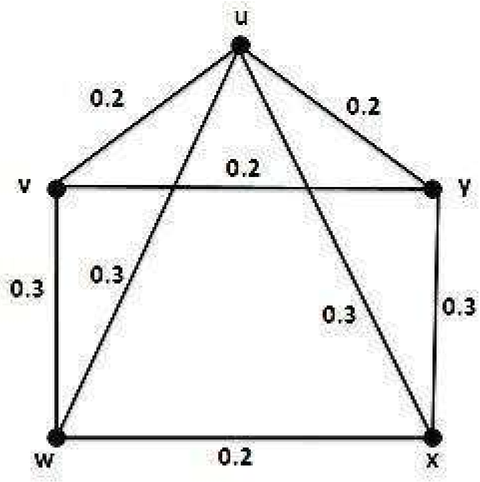

Fig. 4

Suppose that G is a fuzzy graph model of a city with nodes representing important places in the city and edges representing the roads connecting them. $S_1 = \{w, y\}$ and $S_2 = \{v, x\}$ are the s-geodetic bases of G. Therefore, $s\text{-}gn(G) = 2$. Thus the minimum number of Gas stations to be placed in such a way that utility is maximum among consumers is 2 and they can be placed either at the nodes w and y or at v and x as all other nodes lie on the s-geodetic paths joining w to y or on the s-geodetic path joining v to x. Here, the pseudo s-geodesic set $S' = u$. Thus the node u is least suitable for placing a Gas station.

V. CONCLUSION

In this paper, we introduced the idea of Pseudo *s*-geodetic number of fuzzy graphs and showed that a Pseudo *s*-geodetic set need not be the comple- ment of its geodesic basis always. An upper and a lower bound for Pseudo *s*-geodetic number of fuzzy graphs is exhibited. The Pseudo *s*-geodetic num- ber of paths, fuzzy trees and of complete bipartite fuzzy graphs are obtained. The concept of pseudo *s*-geodetic numbers have numerous applications in various real-life situations. One such application has been demonstrated in Location theory where the concept of pseudo *s*-geodetic sets is utilized in locating areas least suitable for placing Gas stations in a locality, the main agenda being to choose locations that maximize utility and minimize the number of Gas stations.

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