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Shallow Water Formulae for Toe Scour, Requisite Armour Mass and Wave Overtopping for the Concept Design of Rock Armoured Coastal Revetments

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ABSTRACT

This article has investigated concept design formulae for revetment toe scour, rock armour mass stability and wave overtopping discharge for rock armoured coastal revetments. The investigation has utilised data from published scale model studies, normalising the laboratory data using factors based on wave energy. As wave characteristics change significantly in shallow water due to nonlinearities and wave breaking, the depth at half the nearshore wavelength fronting the structures has been adopted as a surrogate for a design wave height. Along with incorporating wave period, this has resulted in developing the formulae with predictions that were more coherent and accurate than those in current use. The Iribarren number has distinguished formulae for toe scour and armour mass stability between spilling and plunging breaking waves. Some model and scale effects have been discussed and recommendations for further investigations have been made.

Index Terms: coastal revetment • depth-limited breaking waves • toe scour • rock armour stability • wave overtopping discharge • wave energy • Iribarren number • physical model • scale effects

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RESEARCH ARTICLE

Shallow Water Formulae for Toe Scour, Requisite Armour Mass and Wave Overtopping for the Concept Design of Rock Armoured Coastal Revetments

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Abstract

This article has investigated concept design formulae for revetment toe scour, rock armour mass stability and wave overtopping discharge for rock armoured coastal revetments. The investigation has utilised data from published scale model studies, normalising the laboratory data using factors based on wave energy. As wave characteristics change significantly in shallow water due to nonlinearities and wave breaking, the depth at half the nearshore wavelength fronting the structures has been adopted as a surrogate for a design wave height. Along with incorporating wave period, this has resulted in developing the formulae with predictions that were more coherent and accurate than those in current use. The Iribarren number has distinguished formulae for toe scour and armour mass stability between spilling and plunging breaking waves. Some model and scale effects have been discussed and recommendations for further investigations have been made.

Keywords: *coastal revetment, depth-limited breaking waves, toe scour, rock armour stability, wave overtopping discharge, wave energy, Iribarren number, physical model, scale effects*

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1 Introduction

Coastal revetments protect shoreline assets from wave erosion and inundation (Figure 1). During storm surges they are in shallow water, subjected to breaking waves.

Hydrodynamic considerations for the design of rock-armoured coastal revetments include:

- Seabed toe scour, which informs geotechnical global stability of the structure and armour stability (Figure 2), being the primary cause of failure of many coastal structures (Sutherland *et al.* 2006)
- Armour stability, the primary protection from wave erosion
- Wave overtopping, which informs revetment design regarding armour stability, flooding, asset safety, vehicle and personnel safety (CIRIA 2007 p31-32). Critical values for overtopping discharge are given in CIRIA (2007). The additional hydraulic loading on the crest may contribute to slope instability (van Gent and Pozueta 2004).

Current practice for the assessment of toe scour, requisite armour mass and overtopping discharge is based empirically on data obtained from flume studies. Wave characteristics change significantly in shallow water due to non-linearities and wave breaking, which results in a significant change to the wave height and period, especially when severe breaking occurs and infragravity waves become significant (van der Meer *et al.* 2024). Hence, formulae for toe scour, the requisite rock armour mass and wave overtopping discharge of coastal revetments subjected to breaking waves in shallow water must be based on flume studies of shallow water wave breaking.

When designing for breaking waves, it is necessary to determine the maximum breaker height to which the revetment structure might reasonably be subjected (USACE 1984, p. 7–8). The design breaker height depends on the depth some distance seaward from the structure toe

Figure 1. Rock-armoured revetment, Cabbage Tree Harbour, NSW, Australia (photo: A Nielsen)

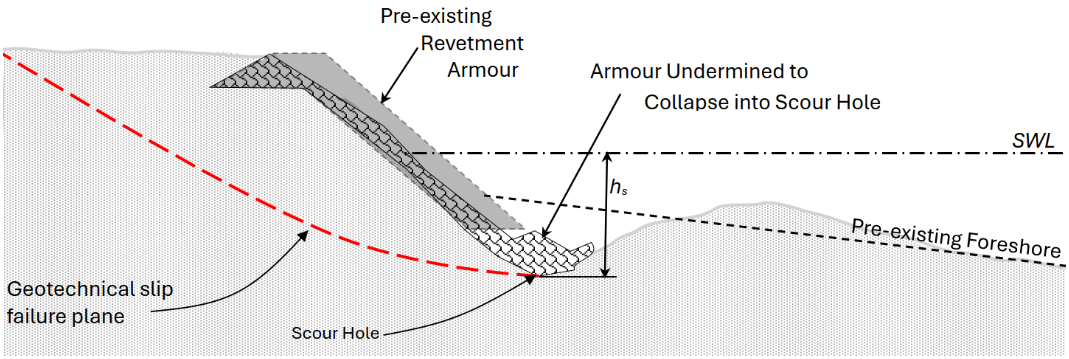

Figure 2. Possible failure mechanisms resulting from revetment toe scour

where the wave first begins to break (USACE 1984, p. 7–8). The breaking process extends over a distance equal to around half the shallow-water wavelength, which is based on the depth at this seaward position (USACE 1984, p. 7-4). Other factors determining the breaker height include the seabed slope over which the waves are shoaling, and wave period (Weggel 1972). The depth at half the wavelength in front of the structure has been adopted herein for determining nearshore wavelength and as a surrogate for the design wave height for determining toe scour, requisite armour mass and average overtopping discharge.

In current practice, data obtained from flume studies has been normalised, that is, made non-dimensional, with wave height factors. This approach synthesises data to a notionally common scale, thereby allowing for the development of design formulae based on large databases. However, it is the energy of breaking waves that is dissipated, *inter alia*, in seabed scour (Steetzel 1993), the rocking and displacement of revetment armour, and in wave runup and overtopping (Hudson 1958; Ahrens *et al*., 1986). Hence, normalising data with wave energy factors has been adopted herein to investigate improving the coherence of the database and the predictability of toe scour formulae (Nielsen 2023), stability formulae (Nielsen and Gordon 2025) and formulae for overtopping discharge (Nielsen 2025).

Using published data from comprehensive scale model studies of toe scour, armour stability and overtopping discharge for simple rock-armoured coastal revetments, with the aim to improve predictions, new concept design formulae were derived utilising the normalisation of flume data with a wave energy factor $f(H^2L)$ based on the depth at half a wavelength fronting the structures ($h_{L/2}$), being a surrogate for the design wave height (H) as well as the depth to determine the wavelength (L) along with the wave period (T). Bed slope (m) has been considered where data were available. Complex structures such as revetments with berms, wave deflectors, falling toes or environmentally friendly seawalls have not been examined.

2 Methodology

2.1 Incident Wave conditions

Depth-limited waves incident on coastal revetments may be spilling, plunging or surging (Figure 3), depending upon the steepness of the incident breaking waves and the bed slope upon which they are shoaling and breaking, that is, the breaking wave surf similarity parameter (or breaking wave Iribarren Number), ξ_b:

$$\xi_b = \frac{m}{\sqrt{\frac{H_b}{L_0}}} \tag{1}$$

where m is the tangent of the bed slope (-), H_b is the breaking wave height (m) and L_0 is the deepwater wavelength (m), given by $gT^2/2\pi$ (USACE 2006 pII-I-66) where g is gravitational acceleration (m/s^2) and T is the wave period (s). Spilling breakers occur when $\xi_b < 0.4$, plunging breakers when $0.4 \le \xi_b \le 2.0$ and surging breakers when $\xi_b > 2.0$ (Battjes 1974).

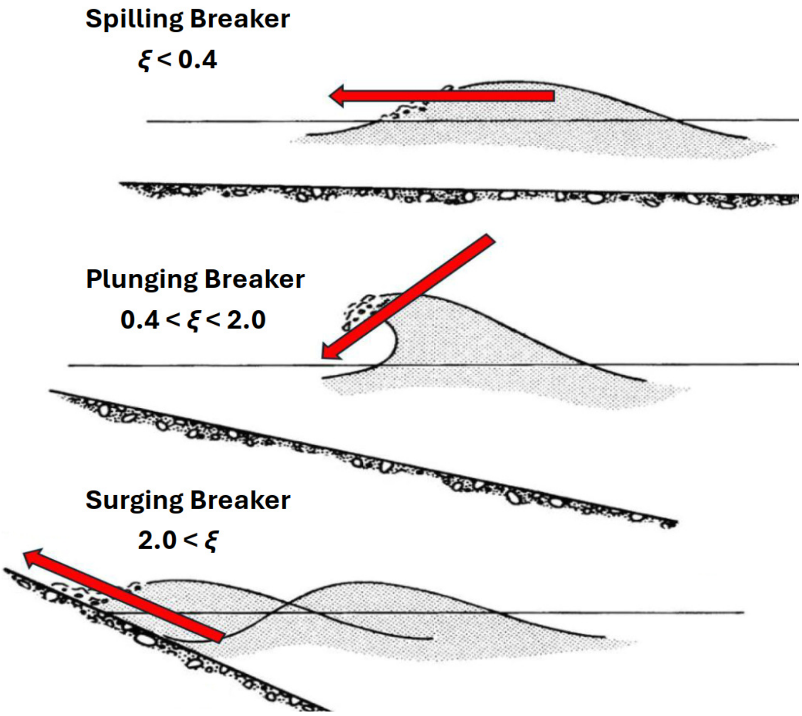

Figure 3. Types of breaking waves (modified from Coastal Wiki 2025 Figure 27)

The breaker type may influence structure performance, the damage varying by how the wave energy is dissipated. Plunging breakers can form a jet creating high velocities normal to the structure with intense turbulence over short periods, which can be effective in dislodging armour units and scouring seabed sediment. Surging breakers can create strong uplift and drag forces, albeit over longer periods, lifting and transporting armour rock, while spilling breakers may be less effective than plunging breakers, dissipating their energy across wider areas over longer periods.

Incident wave energy, E, is a function of the product of wave height squared and wavelength (Equation 2; USACE 1984 p2-26, eqn2-38) thus:

$$E = \frac{\rho_w g H^2 L}{8} \tag{2}$$

where ρ_w is the unit mass of seawater (kg/m^3), g is gravitational acceleration (m/s^2), H is the incident wave height (m), L is the nearshore wavelength (m), which in shallow water can be approximated by Equation 3 (Battjes 1974; USACE 1984 p2-25 eqn2-37):

$$L = T\sqrt{gh} \tag{3}$$

where h is the depth (m) and T is wave period (s). Herein, H and h have been substituted with $h_{L/2}$, the depth located at half the nearshore wavelength in front of the structure (Figure 4).

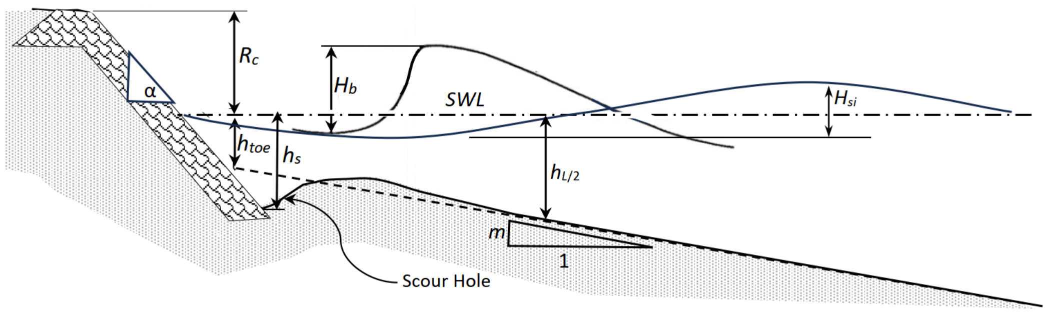

Figure 4. Definition schema for depth-limited waves breaking onto a sloping revetment

2.2 Toe Scour

2.2.1 Current practice. Previous research has related scour depth at dune revetments variously to the maximum height of an unbroken wave at the toe of the structure (USACE 1984, 2006), the volume of sand behind the seawall that would have been eroded in the absence of the seawall (Dean 1986), the minimum depth over the surf zone bar upon which the incident wave breaks (Steetzel 1988, Boers *et al.* 2011, van Rijn 2018), the depth at the toe of the structure (Silvester, in Herbich 1990) and the deep water offshore significant wave height (Sutherland *et al.* 2006). The Coastal Engineering Manual (CEM) presents a simple "rule-of-thumb" that serves as an engineering guideline for sloping revetments under breaking waves, a conservative estimate being that the seabed toe scour occurs to a depth of $H_{\max}$ below the pre-storm seabed level (USACE 2006 p.VI-5-236-237). The scour "depth" as defined in USACE (2006), illustrated in Figure 5, creates an application issue because pre-storm beach levels are not related to storm scouring conditions and, in most instances, vary considerably and are unknown at the time of the storm event. Further, being subaerial they are unlikely to be related to the subaqueous beach profile over which the scouring waves are shoaling and breaking. Hence, the pre-existing sand level cannot be considered relevant for toe scour assessment. Not one of the preceding references provides a method by which a scour *level* can be determined for concept design.

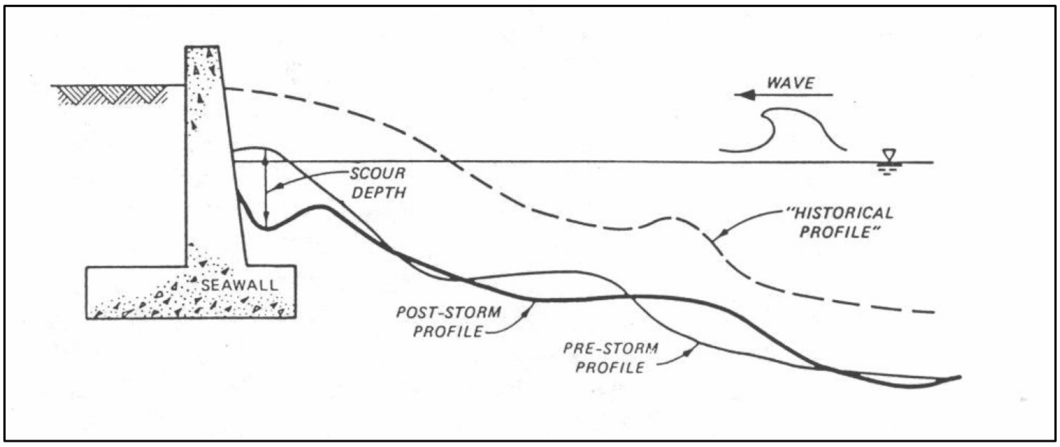

Figure 5. USACE (2006) definition of scour depth at shoreline structures (Figure VI-5-117)

2.2.2 Wave Energy Considerations. Data for toe scour at sloping revetments is presented in Table 1, derived from several studies covering a large range of scales (Nielsen 2023). In that research, toe scour depth was scaled carefully and referenced to a still water level datum, rather than to the pre-existing seabed level. From those data, Figure 6 shows the scour depth as a function of a wave energy factor. The level to which the incident wave has the energy required to scour the seabed sediment is determined from the still water level with Equation 4.

$$\text{Design scour level} = \text{SWL} - 0.53\left(h_{L/2}^2 L_{h_{L/2}}\right)^{0.33} \tag{4}$$

Table 1. Scour data for sloping revetments under breaking waves from movable bed flume model studies (modified from Nielsen 2023)

Researcher	Test No. (-)	h_{toe} (m)	m (-)	T (s)	$h_{L/2}$ (m)	$L_{hL/2}$ (m)	$\xi_b\ (h_{L/2})$ (-)	h_s (m)
Salauddin &	-	0.060	0.050	1.13	0.079	0.994	0.251	0.110
Pearson (2019)	-	0.075	0.050	1.13	0.095	1.089	0.229	0.113
	-	0.100	0.050	1.13	0.121	1.230	0.203	0.134
Sutherland	26	0.200	0.013	1.87	0.218	2.736	0.067	0.270
et al. (2006)	27	0.200	0.013	3.24	0.233	4.894	0.112	0.308
	28	0.200	0.013	1.55	0.215	2.251	0.056	0.274
Steetzel (1985)	T2, M2505-II	0.145	0.024	3.10	0.187	4.195	0.215	0.239
	T3, M2505-II	0.141	0.024	3.40	0.183	4.556	0.238	0.300
Steetzel (1988)	T1, H298-I	0.830	0.024	5.00	1.050	16.047	0.146	1.500
	T2, H298-I	0.830	0.024	5.00	1.050	16.047	0.146	1.500
Van der Meer &	-	1.300	0.033	4.40	1.590	17.377	0.145	2.000
Pilarczyk (1988)	-	0.260	0.033	2.00	0.318	3.532	0.148	0.380
Fowler (1992)	10	0.061	0.067	1.93	0.136	2.232	0.435	0.155
	11	0.061	0.067	1.97	0.138	2.292	0.442	0.143
	13	0.061	0.067	2.40	0.161	3.018	0.498	0.213
	14	0.061	0.067	2.45	0.165	3.116	0.502	0.186
Tsai et al. (2009)	5	0.090	0.200	1.40	0.349	2.590	0.592	0.381
	10	0.150	0.200	1.40	0.441	2.912	0.527	0.428
	15	0.210	0.200	1.40	0.529	3.189	0.481	0.474

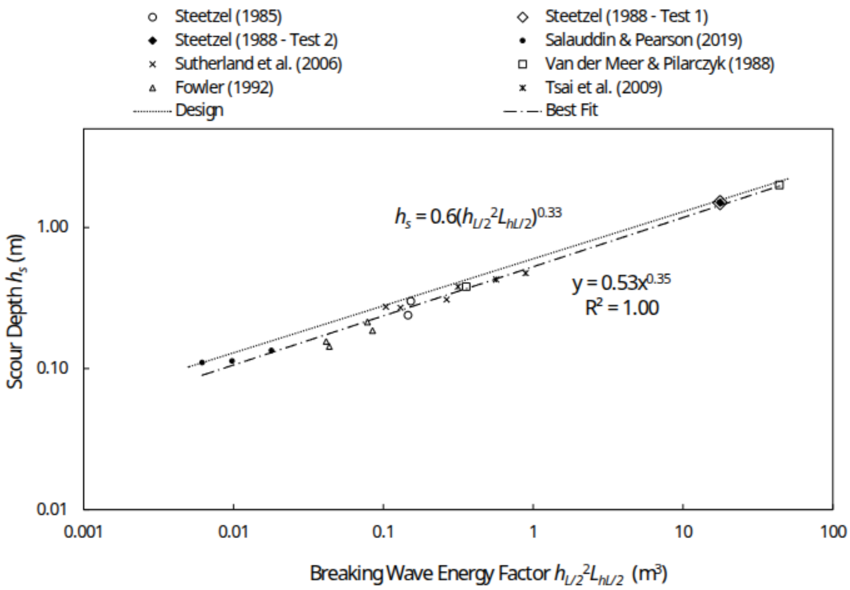

Figure 6. Results from flume studies of toe scour depth at sloping revetments under breaking waves in shallow water presented as a function of breaking wave energy

2.3 Requisite Armour Mass

2.3.1 Introduction. The stability of rock armour is a key design consideration for coastal revetments subjected to breaking waves. Armour mass must be sufficient to resist the cumulative hydrodynamic forces generated by wave breaking over the duration of the design storm, recognising that damage is progressive and develops over many wave cycles (Nielsen and Gordon 2025). Current practice relies largely on empirical relationships that relate armour size to wave height, slope geometry and material properties, with calibration coefficients, derived from flume test data, reflecting accepted levels of damage. These empirical relationships have not been developed from shallow water wave breaking flume tests, resulting in inaccuracies (van der Meer 2024). A concept design formula, presented herein, is normalised with incident wave energy, accounting for wave period explicitly, improving the coherence of the calibrating data, and is based on shallow water wave breaking flume data. Armour mass is the dependent variable.

In current practice, concept design formulae for revetment rock armour define the median requisite armour mass as a function of wave height, rock density, water density and revetment slope following the passage of a number of waves. The formulae are in the form:

$$M_{50} = \frac{\rho_a H_s^3}{K_D \Delta^3 \cot^n \alpha} \tag{5}$$

where M_{50} is the median requisite stable armour mass (kg), ρ_a is the armour density (kg/m^3), H_s is the incident significant wave height (m), Δ is the relative submerged density of the armour to water $(\rho_a - \rho_w)/\rho_w$ (-), $\cot\alpha$ is the revetment armour slope (-) and K_D is a damage coefficient (-) derived empirically (Hudson 1958). Subsequent developments of this formula have included additional factors in the calibration coefficient for the extent of damage (Gordon 1973; Foster & Gordon 1973), the number of waves, the porosity of the revetment core and wave period (van der Meer 1988), giving formulae variations for various spilling, plunging and surging waves, as summarised in Table 2, where D is damage (%), $S = D/1.25$ (-) (van der Meer 1988), N is the number of waves (-), ξ_m surf similarity parameter based on mean wave period (-) and, assuming in very shallow water, $H_{2\%} = 1.4H_s$ (van der Meer 1988; Goda 2000 p91).

Table 2. Damage coefficients (K_D) and $\cot\alpha$ exponents (n) applied to the Hudson equation in shallow water for standard rock armour placed pell mell over impermeable underlayers

Researcher Reference	n; K_D	Remarks
Hudson (1958)/Van Gent (2004)	1; 1	1000 regular waves
Foster & Gordon (1973)[1]	1; $0.44D^{0.51}$	2000 Plunging regular breakers
Van der Meer (2021)[2]	0; $28.7\left(\frac{S}{\sqrt{N}}\right)^{0.6} \varepsilon_m^{-1.5}$	N Plunging random breakers
Van der Meer (2021)[3]	1.5; $0.82\left(\frac{S}{\sqrt{N}}\right)^{0.6} \varepsilon_m^{0.3}$	N Surging random breakers
Van Gent *et al.* (2003)	1; $5.4\left(\frac{S}{\sqrt{N}}\right)^{0.6}$	N Spilling & Plunging random breakers

1. K_D factor adjusted for using $H_s = 0.7H_b$ with the Hudson formula
2. $C_{pl} = 6.49$
3. $C_s = 0.97$

2.3.2 Data.

2.3.2.1 UNSW Water Research Laboratory (1973). Testing of rough angular rock armour on slopes of cotα = 1.25, 1.5 and 2.0 with regular waves was undertaken at UNSW Water Research Laboratory in a flume 1 m wide, 1.5 m deep and 30 m long (Gordon 1973; Foster & Gordon 1973). The flume bed for 8 m fronting the structure had a slope $m = 0.033$, beyond which the bed was flat. The limestone rock armour had a specific gravity of 2.7, was graded with a ratio of maximum to minimum mass of approximately 2.0, a median mass of $M_{50} = 0.064$ kg, an aspect ratio not exceeding 2.0 with edges that were moderately sharp and was placed randomly in two layers, the core being impermeable.

For a given toe depth, the wave height and period were adjusted until the wave formed a plunging break immediately onto the structure. The toe depth was increased by small increments allowing larger waves to break onto the structure. Damage was assessed as the percentage of rock dislodged over the revetment area extending from one wave height below SWL to the limit of wave uprush.

The testing regime was set up to enable the effects of up to 10,000 regular waves to be observed for each water level and period combination. Progressive damage to the armour was observed up to failure, which occurred generally after around 2,500 waves (Foster & Gordon 1973), developing once damage had reached around 20%. For lower damage levels, the structures adopted a stable configuration once the initial damage had occurred. Data are in Table 3.

Table 3. Test data for rock armour damage under regular plunging waves (Gordon 1973)

$cot\alpha$ (-)	T (s)	$h_{L/2}$ (m)	L (m)	D (%)	N (-)
2	1.30	0.095	1.254	1.0	1800
2	1.37	0.120	1.486	3.0	2500
2	1.35	0.125	1.494	6.0	3000
2	1.35	0.141	1.587	18.0	3000
1.5	1.31	0.096	1.273	2.2	3000
1.5	1.37	0.107	1.403	3.4	2000
1.5	1.35	0.117	1.449	8.3	4000
1.25	1.36	0.080	1.203	1.3	1500
1.25	1.34	0.088	1.245	6.5	5000
1.25	1.35	0.099	1.331	11.6	5000

2.3.3 Delft Hydraulics (2003).

Tests were undertaken in the 1 m wide flume of Delft Hydraulics (van Gent *et al*. 2003). The armour comprised well-sorted "stantard rough angular rock" of specific gravity 2.75 with $D_{n50} = 0.026$ m (van Gent 2004). The revetments overlaid impermeable cores at slopes of $cot\alpha = 2$ and $cot\alpha = 4$. The seabed slope was $m = 0.033$. Only considered herein were results with damage $1.6 < S < 17$ ($2\% < D < 20\%$). Severe wave breaking was defined herein as when $h_{toe}/H_s < 1.5$. Data are in Table 4.

Table 4. Data derived from testing for rock armour damage under random spilling waves at Delft Hydraulics (van Gent et al. 2003)

Test No (-)	N (-)	h_{toe} (m)	$h_{L/2}$ (m)	T_p (s)	L (m)	$S_{measured}$ (-)	D (%)	ξ_b (-)
Structure 6 cotα = 2								
15	967	0.150	0.176	1.240	1.63	4.11	5.1	0.20
16	1,193	0.150	0.195	1.981	2.74	11.97	15.0	0.30
17	1,216	0.150	0.192	1.863	2.56	10.13	12.7	0.28
18	1,039	0.125	0.157	1.589	1.97	7.09	8.9	0.26
19	1,286	0.125	0.179	2.471	3.27	10.34	12.9	0.39
Structure 7 cotα = 4								
6	1,034	0.250	0.295	1.623	2.76	6.82	8.5	0.20
7	1,267	0.250	0.324	2.525	4.50	17.76	22.2	0.29
8	1,032	0.200	0.248	1.888	2.95	6.96	8.7	0.24
9	1,333	0.200	0.282	3.000	4.99	12.20	15.2	0.38
10	1,017	0.150	0.211	2.584	3.72	1.72	2.2	0.38
11	1,068	0.150	0.225	3.038	4.51	3.55	4.4	0.43

2.3.4 NSW Manly Hydraulics Laboratory (2023).

Scale modelling was undertaken in 2023 at the New South Wales Government Manly Hydraulics Laboratory for the rock armour protection of some 70 km of clay embankments for the Mardie Salt and Potash facility, Western Australia (Figure 7, Figure 8; Nielsen and Gordon 2025).

Two-dimensional physical model testing for the rock-armoured revetments of the Mardie clayey embankments was undertaken in a 1 m wide flume. Model armour comprised two layers of a well-graded igneous rock of specific gravity *s.g.* = 2.76. The rock was placed with a porosity of 40% on an impermeable slope of $cot\alpha = 2.5$. Testing was undertaken with random waves in fresh water. The seabed slope was flat ($m = 0$) and $M_{50} = 0.056$ kg. Most of the tests comprised some 3,000 waves of peak period $T_p = 1.2$ s.

The modelling comprised testing the armour rock stability over a range of water levels with depth-limited breaking waves. Damage was observed by comparing photographs taken before and after exposure to the test conditions. Damage was quantified by counting the number of units displaced by more than one nominal diameter as a percentage of the total number of units within the wave runup/rundown section of the revetment. All rocks in the two armour layers were considered to have been exposed. Only results with damage less than 20% for severe breaking were considered, defined as when $H_s/h_{toe} > 0.4$ for these very flat slopes on which the maximum wave height that can be sustained is around 55% of the depth (Nelson 1975; Riedel & Byrne 1976). Data are in Table 5.

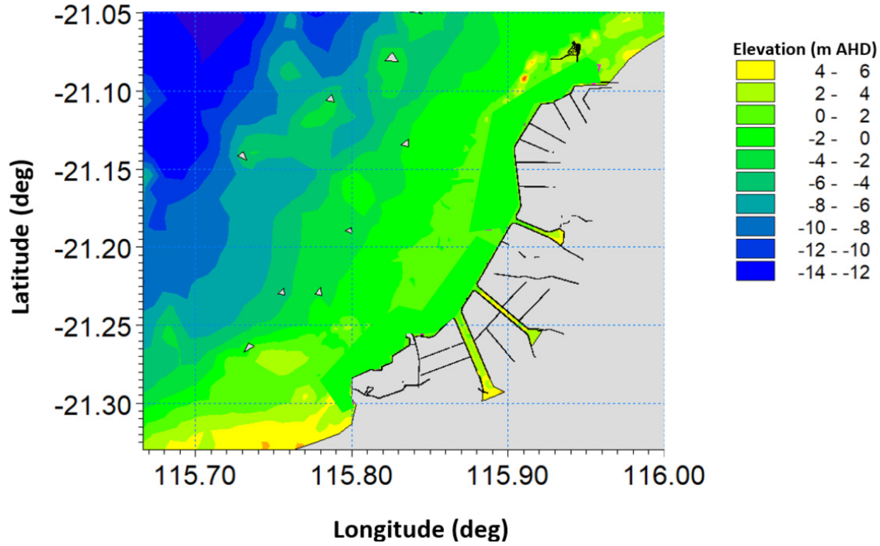

Figure 7. Pond layout and bathymetry for the Mardie Salt and Potash facility near Onslow, Western Australia.

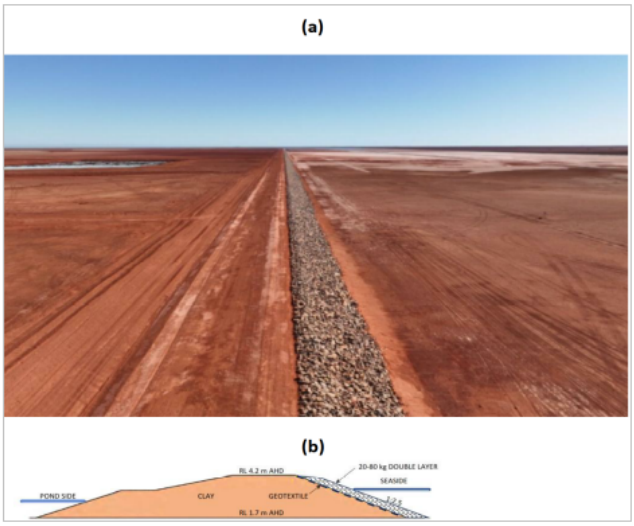

Figure 8. Typical clay embankment for the Mardie Salt and Potash facility: (a) aerial view courtesy BCI Minerals Ltd; (b) modelled cross-section – toe scour not tested.

Table 5. Data derived from testing for rock armour damage under random spilling waves at Manly Hydraulics Laboratory[1] (Nielsen and Gordon 2025)

Test No	T_p (s)	N (-)	$h_{L/2}$ (m)	$L_{hL/2}$ (m)	D (%)
27	1.2	3000	0.19	1.638	1.0
28	1.2	3000	0.19	1.638	1.5
29	1.2	3000	0.20	1.681	1.5
30	1.2	3000	0.20	1.681	3.5
31	1.2	3000	0.18	1.595	0.5
32	1.2	3000	0.18	1.595	0.7
33	1.2	3000	0.21	1.722	5.5
34	1.2	3000	0.21	1.726	3.0
35	1.2	3000	0.22	1.763	9.5
36	1.2	3000	0.22	1.763	9.5

2.4 Wave Overtopping

2.4.1 Introduction. Various methods for predicting wave runup and overtopping have been developed based on research utilising laboratory data, there being little prototype-scale data available. Measured discharges, q, have been related to the freeboard of revetment crests, R_c, generally with equations of the form (USACE 2006 p.VI-5-22; EurOtop 2018 p58):

$$q = ae^{-bR_c} \tag{6}$$

where q is the average overtopping discharge per meter width of revetment ($m^3/s/m$), R_c is the freeboard (m) above the still water level (SWL), with a and b being empirical constants.

2.4.2 Current practice. For various seawall and revetment configurations, the Overtopping Manual (EurOtop 2018) presents current practice for the relationships between measured discharges, q ($m^3/s/m$), normalised by the factor $\left(gH_{m0}^3\right)^{0.5}$ and the freeboards, R_c (m), normalised by the factor H_{m0} (m). For mild slope rubble-mound structures in shallow waters, $h_{\text{toe}}/H_s < 1.5$, current practice uses Equation 7 for mean overtopping discharge (EurOtop 2018 Eqn5.10 p115):

$$\frac{q}{\sqrt{gH_{si}^3}} = \frac{a}{\sqrt{\tan\alpha_{sf}}}\gamma_b\xi_{m-1,0}\exp\left[-\left(b\frac{R_c}{\xi_{m-1,0}H_{si}\gamma_b\gamma_f\gamma_\beta\gamma^*}\right)^{1.3}\right] \tag{7}$$

where $a = 0.023$, $b = 2.7$ (but for design, $a = 0.026$, $b = 2.5$), $\xi_{m-1,0}$ is the surf similarity parameter $\tan\alpha/(H_{si}/L_0)^{0.5}$ (-), $\tan\alpha_{sf}$ the equivalent shallow foreshore revetment slope calculated using Equation 8 with L_{slope} defined in Figure 9, and γ_b, γ_f, γ_β and γ^* are influence factors that account for revetment berms, armour slope roughness, angle of wave incidence and a range of revetment geometrical factors (respectively). Guidance for their quantification is given in the Overtopping Manual. For a two-layered rock-armoured revetment with an impermeable core, no berms and normal wave incidence factors, $\gamma_b = 1.0$, $\gamma_f = 0.55$ (EurOtop 2018 Table 6.2 p175), $\gamma_\beta = 1.0$ with $\gamma^* = 0.6$ derived herein for the specific configuration in Figure 10 and used to calibrate Equation 7. The revetment geometrical factor $\gamma^* = 0.6$ applies only to the revetment configuration in Figure 10. Equations developed for revetments with geometrical configurations different from those in Figure 10 will need to be calibrated for their influence factors with flume testing.

$$\tan\alpha_{sf} = \frac{R_{u2\%} + 1.5H_{si}}{L_{\text{slope}}} \tag{8}$$

$$R_{u2\%} = \gamma_f H_{si}\left(4 - \frac{1.5}{\sqrt{\xi_b}}\right) \tag{9}$$

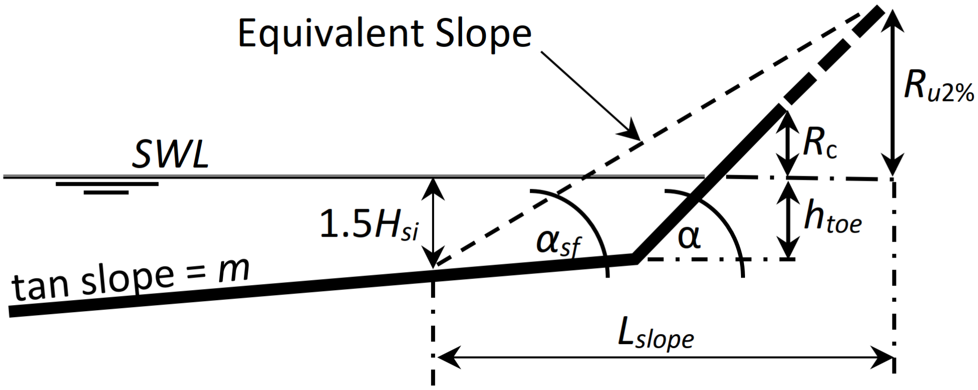

Figure 9. Schema to determine the equivalent runup slope for revetments in very shallow water (Altomare et al 2016)

2.4.3 Data. Data used for this research comprised results from a comprehensive set of flume experiments undertaken at the US Army Engineer Waterways Experiment Station's Coastal Engineering Research Centre (CERC) to investigate the amelioration of flooding at Roughans Point Massachusetts (Ahrens *et al.* 1986). The data were sourced from the scale modelling of a simple rock rubble revetment (Figure 10), conducted in a 1 m by 1 m by 45 m long wave flume. Model armour stone had a specific mass of 2650 kg/m^3 and grading ranged from 170 g to 320 g with a median mass of 250 g. The seabed fronting the revetment had a slope of $m = 0.01$ (1V:100H).

Test data are presented in Table 6. A wide range of wave conditions was represented in the tests, with the incident significant wave heights (H_{m0}) measured at the revetment toe. During a single test run, irregular waves were generated continuously for 33 minutes, each test comprising some 1,000 waves. The wave paddle was programmed to produce a modified Joint North Sea Wave Program (JONSWAP) wave spectrum for the water depth at the wave generation blade. Overtopping rates were determined by measuring the change in water level in the overtopping container behind the model during a test run using point gauges. The data published were presented in prototype imperial units, which have been reduced herein to scale model metric values.

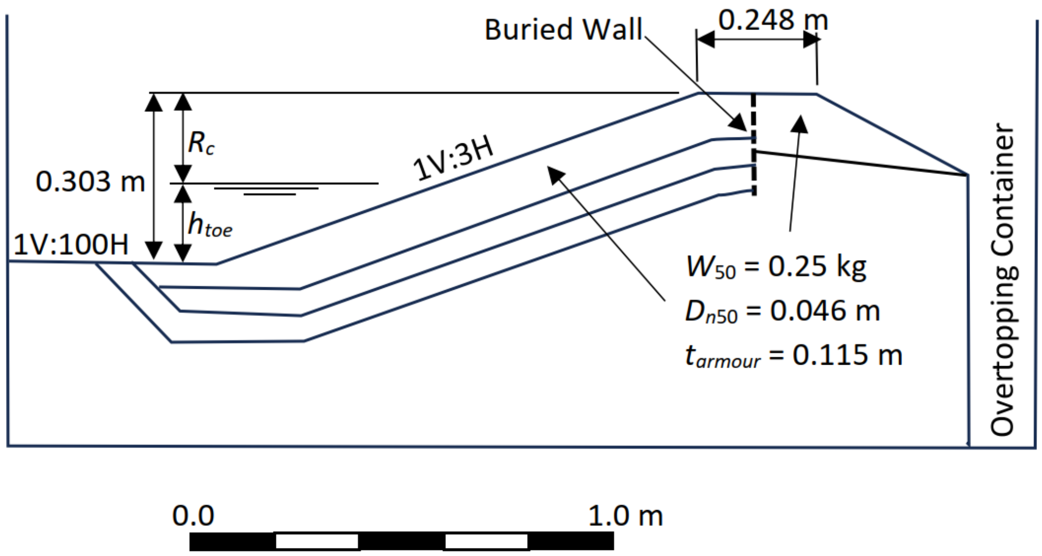

Figure 10. Schematic of Configuration 10 rock rubble revetment (Ahrens et al. 1986). Distinguishing geometrical factors comprise a buried wall, specific crest width, armour size, armour porosity, underlayer properties, seabed slope

Table 6. Test data from Configuration 10 (Figure 9) of Ahrens et al. (1986) converted to model metric units

Test No (-)	T_p (s)	h_{toe} (m)	R_c (m)	q ($m^3/s/m$)
149	1.75	0.181	0.156	2.50E-04
154	1.75	0.182	0.157	2.15E-04
158	1.75	0.167	0.161	8.86E-05
166	1.75	0.173	0.165	5.52E-05
170	1.75	0.150	0.182	1.89E-05
176	1.75	0.154	0.179	4.35E-05
182	1.75	0.153	0.186	2.18E-05
186	1.75	0.141	0.194	1.02E-05
192	1.75	0.141	0.211	7.26E-06
150	2.00	0.182	0.155	7.88E-04
155	2.00	0.180	0.157	3.25E-04
159	2.00	0.166	0.159	3.21E-04
163	2.00	0.166	0.168	1.60E-04
174	2.00	0.154	0.172	1.41E-04
177	2.00	0.153	0.177	1.18E-04
183	2.00	0.154	0.181	6.10E-05
187	2.00	0.139	0.183	3.05E-05
193	2.00	0.140	0.191	1.89E-05
151	2.25	0.180	0.150	7.61E-04
156	2.25	0.174	0.160	4.01E-04
160	2.25	0.167	0.154	3.96E-04
164	2.25	0.167	0.170	2.55E-04
172	2.25	0.152	0.173	1.28E-04
178	2.25	0.150	0.172	1.23E-04
188	2.25	0.138	0.180	4.35E-05
194	2.25	0.139	0.189	4.21E-05
152	2.50	0.180	0.157	6.72E-04
157	2.50	0.180	0.163	3.51E-04
161	2.50	0.166	0.163	2.28E-04
165	2.50	0.166	0.176	3.95E-04
169	2.50	0.140	0.179	1.95E-04
173	2.50	0.152	0.179	1.42E-04
179	2.50	0.143	0.183	2.19E-04
195	2.50	0.140	0.193	2.18E-05

3 Results

3.1 Toe Scour

Table 1 indicated that scour was tested mostly under spilling breakers ($\xi_b < 0.4$), with scour under plunging breakers ($0.4 < \xi_b < 2.0$) occurring with some tests from Fowler (1992) and Tsai *et al.* (2009). These data allowed examination of the relative impacts of wave height, represented by the surrogate depth ($h_{L/2}$), wave period (T) and bed slope (m) to scour depth.

Figure 11 presents the flume study data for the scour depth, normalised with the deepwater wavelength, *versus* the surf similarity parameter (the breaking wave Iribarren number ξ_b).

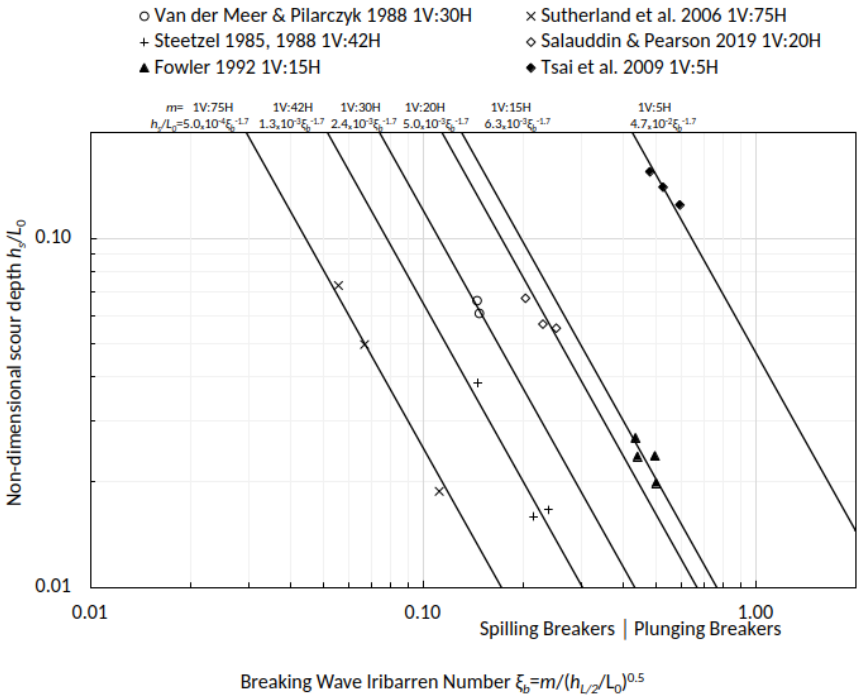

Figure 11. Non-dimensional scour depth versus the breaking wave Iribarren number

Figure 11 indicated that the scour depth, normalised with deepwater wavelength, was related to the breaking wave Iribarren number through the bed slope and wave height by a factor, b, as follows:

$$\frac{h_s}{L_0} = b\xi_b^{-1.7} \tag{10}$$

with b given in Table 7 for each of the various seabed slopes that were tested. This allowed for relationships being defined for the Iribarren number factor, b, with bed slope for spilling and plunging waves, as shown in Figure 12.

Substituting the relationships for b from Figure 12 into Equation 10 gave Equation 11 for spilling waves:

$$h_s = 0.97m^{0.04}T^{0.3}h_{L/2}^{0.85} \tag{11}$$

and Equation 12 for plunging waves:

$$h_s = 0.95m^{0.13}T^{0.3}h_{L/2}^{0.85} \tag{12}$$

Equations 11 and 12 indicated that bed toe scour was strongly dependent on water depth (*aka* wave height), moderately dependent upon wave period and weakly dependent on bed slope. However, it is noted that wave height is strongly dependent on bed slope, with steeper slopes generating larger breaking waves (Weggel 1972), which are likely to be plunging.

Table 7. Relationships between the scour depth, normalised with deepwater wavelength, to the breaking wave Iribarren number for various seabed slopes

m=1V:75H	m=1V:42H	m=1V:30H	m=1V:20H	m=1V:15H	m=1V:5H
$h_s/L_0 = 5.0\times10^{-4}\xi_b^{-1.7}$	$h_s/L_0 = 1.3\times10^{-3}\xi_b^{-1.7}$	$h_s/L_0 = 2.4\times10^{-3}\xi_b^{-1.7}$	$h_s/L_0 = 5.0\times10^{-3}\xi_b^{-1.7}$	$h_s/L_0 = 6.3\times10^{-3}\xi_b^{-1.7}$	$h_s/L_0 = 4.7\times10^{-2}\xi_b^{-1.7}$

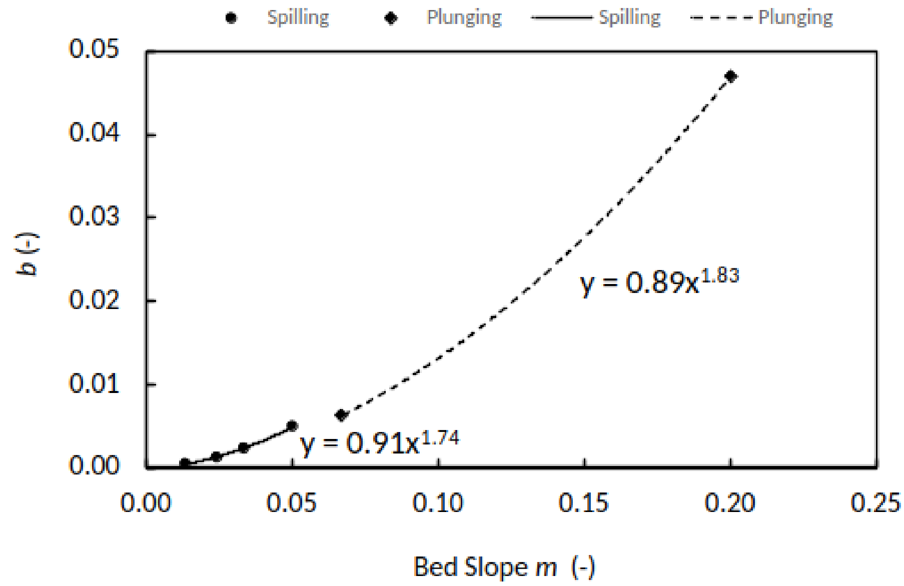

Figure 12. Relationships between factors (b) for the breaking wave Iribarren number with bed slope (m) for spilling waves and plunging waves

3.2 Requisite Armour Mass

For the wave energy method under various breaking wave conditions (Table 8), Figure 13 shows the relationship between armour mass M_{50} (kg), a wave energy factor $h_{L/2}^2 T_p \sqrt{g h_{L/2}}$ (m^3), revetment slope tanα (-), relative density $\Delta = \frac{\rho_a}{\rho_w} - 1$ (-) where ρ_a is armour mass density (kg/m^3), and the level of damage D (-) divided by the number of waves N (-), as given in Equation 13, which is recommended for concept design:

$$M_{50} = \frac{\rho_a h_{L/2}^2 T_p \sqrt{g h_{L/2}}}{K' \left(\frac{D}{\sqrt{N}}\right)^{0.36} \Delta^3 \cot^{1.5}\alpha} \tag{13}$$

where $K' = 2100$ for spilling breakers and $K' = 700$ for plunging breakers.

Table 8. Range of test conditions for data in Figure 13

Data Source	D_{n50} (m)	H_s/h_{toe} (g)	T (s)	Damage (%)	m (-)	$\cot\alpha$ (-)	Breaker type
Gordon (1973)	0.029	–	1.3	1–18	0.033	1.25, 1.5, 2.0	Plunging
van Gent et al. (2003)	0.026	0.42–0.59	1.2–3.0	2–30	0.033	2, 4	Spilling
Nielsen & Gordon (2025)	0.024	0.42	1.2	0.5–10	0.000	2.5	Spilling

3.3 Overtopping Wave Energy Considerations

Equation 7 has been modified for wave energy considerations incorporating wave period and the surrogate wave height for design $H_{\text{design}} = h_{L/2}$ in Equation 14:

$$\frac{q}{\left(g(g h_{L/2})^{0.5} T_p h_{L/2}^2\right)^{0.5}} = \frac{0.0000137}{\sqrt{\tan\alpha_{sf}}} \left(\frac{0.55 T_p}{\sqrt{R_c + h_{\text{toe}}}}\right)^{8.544} \times \exp\left[-\left(1.29 \left(\frac{0.55 T_p}{\sqrt{R_c + h_{\text{toe}}}}\right)^{1.26} \times \frac{R_c}{\gamma_f \gamma^* \left((g h_{L/2})^{0.5} T_p h_{L/2}^2\right)^{0.33}}\right)^{1.3}\right] \tag{14}$$

with $\gamma_f = 0.55$ and $\gamma^* = 0.60$ for the revetment configuration in Figure 10.

The factors a and b for Equation 14 were derived from model data, requiring the scaling of periods as shown (Nielsen 2025). Measured overtopping rates *versus* predictions of the flume test results of Ahrens *et al.* (1986) using the current practice formula (Equation 7) and the wave energy formula incorporating wave period (Equation 14) and the surrogate wave height for design, $H_{\text{design}} = h_{L/2}$, are presented in Figure 14, which shows that normalising the flume data with wave energy factors results in a significant improvement in the coherence of the results.

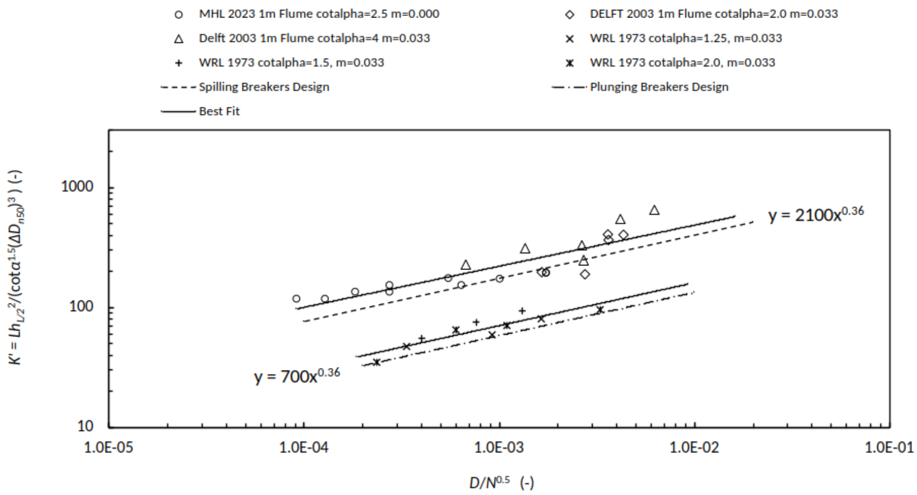

Figure 13. Wave energy damage coefficient versus damage by a number of waves for double layered standard rough angular rock-armoured revetments placed pell-mell over impermeable underlayers for waves breaking in shallow water

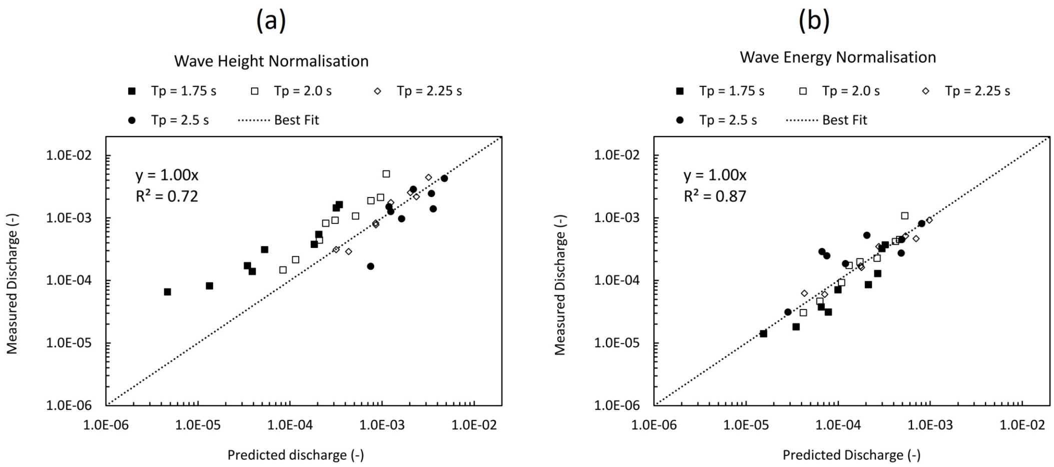

Figure 14. Measured versus predictions of flume test results of Ahrens et al. (1986) using (a) the current practice formula (Equation 7) and (b) the wave energy formula (Equation 14) incorporating wave period and $h_{L/2}$ for design wave height

4 Discussion

4.1 Scale Modelling

Results from physical scale modelling may not represent prototype behavior correctly because of experimental model effects and/or scale effects (Nielsen 2025). There is little research published on rock armour stability at shallow foreshores (van der Meer 2024), and what data is available (van Gent et al., 2003) was used herein. Flume data for stability testing under random waves plunging onto rock armoured revetments were not found. Testing under regular waves, as presented herein, does not schematise infragravity waves that are generated in the surf zone under random wave conditions. However, the objective of the testing was to develop formulae for stability based on depth, it being up to the designer to determine a depth for design, considering non-linear surf zone processes such as infragravity waves and wave setup. The narrow distribution of random wave heights in the (very) shallow waters of a surf zone fronting a shoreline revetment subjected to depth-limited wave breaking is well-represented by regular wave testing, with plunging waves being rare and a conservative condition for concept design.

Depending on the scale applied for concept design, prototype conditions could lay beyond some of the limits of the modelling parameters used for concept design formulae. Extrapolating model results should consider experimental constraints including:

- The fronting seabed slope affects wave shoaling and breaking. Seabed slopes of the scour tests varied widely from $m = 0.013 - 0.05$, allowing the development of formulae that included m. However, for armour mass only very flat beds and one bed slope of $m = 0.033$ were modelled, as was the case for overtopping with $m = 0.01$.

- Revetment slopes varied little for scour with $\cot\alpha = 1.8 - 2.0$, for armour mass $\cot\alpha = 2, 3, 4$ and 6 but for overtopping considered herein only one slope of $\cot\alpha = 3.0$ was modelled. However, the overtopping modelling included a large range of equivalent revetment slopes from 1:7 to 1:20.

- Model wave periods varied from 1 s to 3 s. This would limit a 9:1 Froude scale model to schematise prototype wave periods from 3 s to 9 s. For a smaller scale model of 50:1, say, the prototype periods schematised by the modelling would range from 7 s to 21 s. Scaling up to prototype dimensions using equations developed from scale modelling may include test results that have schematised unrealistic prototype wave periods. Modelling benefits from testing with the broadest range of wave period, optimising extrapolation and interpolation for prototype conditions.

- Much of the data used to develop concept design formulae are based on around 1,000 zero-crossing waves. Revetments in depth-limited situations are likely to be subjected to a much larger number of waves, in which case average overtopping discharges and requisite armor mass may be underestimated.

- Results of two-dimensional flume studies do not replicate any three-dimensional processes that occur in the nearshore surf zone fronting revetments.

Apart from experimental constraints, Reynolds number scale effects are possible. The drag coefficient for spheres increases for Re $< 2 \times 10^3$ but decreases for Re $> 2 \times 10^5$ (Yager 2017). Should that apply to rock armour, if the model data are applied at normal scales for prototype concept design, for the likely prototype ranges of Reynolds number the drag force coefficient of the flow on rock armor would be relatively lower in the prototype than that in the model, leading to conservative estimates for requisite armour mass (Thompson & Shuttler 1975 p.6) but non-conservative estimates for runup and overtopping (Nielsen 2025).

Site-specific scale modelling is recommended for the detailed design of coastal revetments.

4.2 Toe Scour

The goodness of fit of predictions made using wave energy factors and with using factors incorporating bed slope are depicted in Figure 15, which shows an improvement specifically for plunging waves with Equation 5.

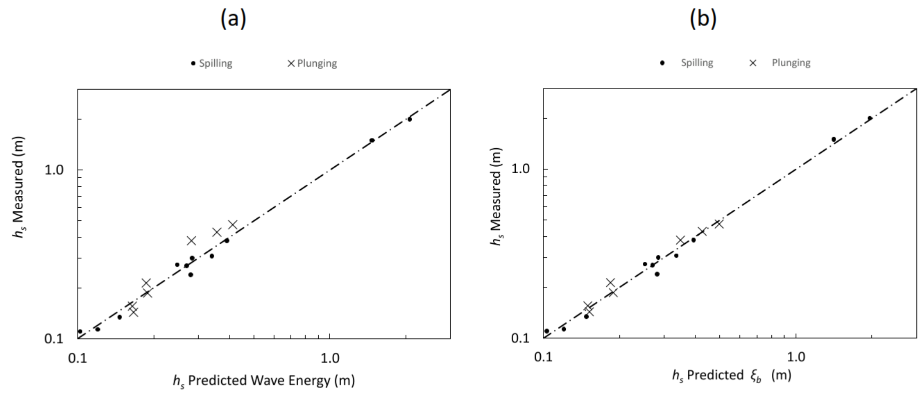

Figure 15. Goodness of fit of scour depth predictions using (a) wave energy factors and (b) factors developed with the Iribarren number, which includes seabed slope

4.3 Armour Mass

The formulae of Hudson (1958), Foster and Gordon (1973) and van Gent et al. (2003) have no parameter for wave period. For this reason, the shallow water formulae of van der Meer, as modified by van Gent (CIRIA 2007), are used almost universally, although not condoned by van der Meer *et al.* (2024), although the original Hudson (1958) formula is still current.

Further significant advantages of the van der Meer formulae are the inclusion of the parameter for structure porosity, which distinguishes revetments (impermeable core $P = 0.1$) from breakwaters (permeable core $P = 0.5$), and the surf similarity parameter or Iribarren Number (Battjes 1974; see Figure 3), which distinguishes the type of breaking wave (Figure 16).

The van der Meer (1988) testing was undertaken in water depths for which waves were either not breaking but surging onto the armour, or plunging onto steep armour slopes. Van der Meer (2021) presents the shallow water equations incorporating wave breaking using data from van Gent *et al.* (2003). However, these formulae are not condoned by van der Meer due to non-linearities that develop in the surf zone under wave breaking (van der Meer *et al.* 2024). Testing by van der Meer (1988) was undertaken in relatively deep water (Figure 20a) with non-breaking waves either surging on or plunging onto the revetment, while that of Gordon (1973), van Gent *et al.* (2003) (Figure 20b) and Nielsen & Gordon 2025 (Figure 20c) was undertaken in very shallow waters with breakers either spilling or plunging onto the revetments. Normalising the shallow water test data of Gordon (1973), van Gent *et al.* (2003) and Nielsen and Gordon (2025), the stability coefficients were more coherent using a wave energy factor, with a coefficient of Determination $R^2 = 0.68$, than with using a wave height factor, which had a Coefficient of Determination $R^2 = 0.31$ (Figure 17).

4.4 Overtopping

Significant improvements to the coherence of the test results of Ahrens *et al.* (1986) was achieved by normalising discharge and freeboard with wave energy factors in lieu of wave height factors as demonstrated in Figure 18 (Nielsen 2025). The average Coefficient of Determination (R^2) for the data normalised with wave energy factors was 0.75 compared with 0.38 for the data normalised with wave height factors.

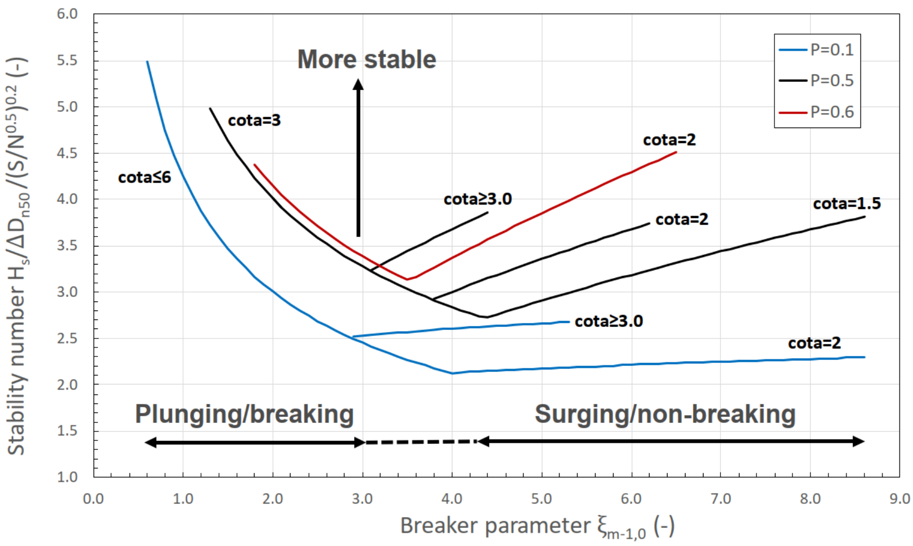

Figure 16. Scope of application for the van der Meer (1988) rock armour stability formulae (van der Meer et al. 2024)

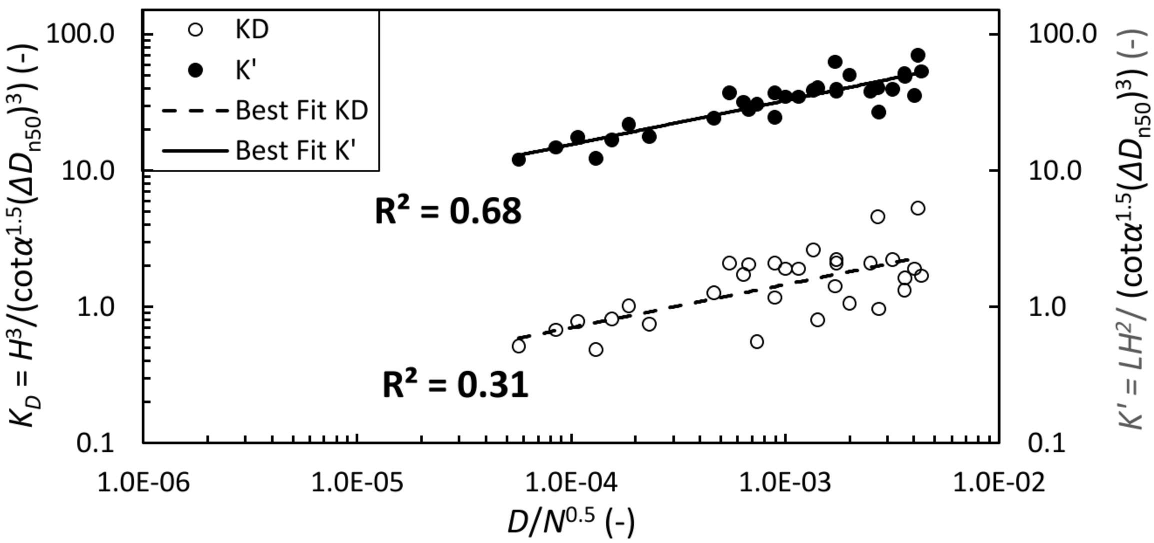

Figure 17. Coefficients of Determination (R^2) for stability formulae damage coefficients based on normalisation with a wave energy factor ($K' = f(H^2L)$) compared with the formula based on normalising with a wave height factor ($K_D = f(H^3)$), indicating greater coherence of the former with the flume data (Nielsen & Gordon 2025)

Predictions of the flume test results for Configuration 10 of Ahrens *et al.* (1986) using the current practice formula (Equation 7) and the wave energy formula incorporating wave period (Equation 14) are presented in Figure 19, which shows that incorporating wave period into the equation improves the accuracy of predictions markedly, although generous factors of safety would be required still for design. However, the formula presented herein was developed for only one foreshore slope ($m = 0.01$) and it applies specifically only to Configuration 10 of Ahrens et al. (1986), although a large but mild range of equivalent revetment slopes was modelled. The Overtopping Manual (EurOtop 2018 pp.28-31) gives guidance for overtopping with shallow water wave conditions on a range of foreshore slopes.

For these tests, the nearshore depth $< 1.5H_{m0}$, resulting in waves spilling across a surf zone before incidence onto the revetment. This precluded distinguishing the results with plunging wave data.

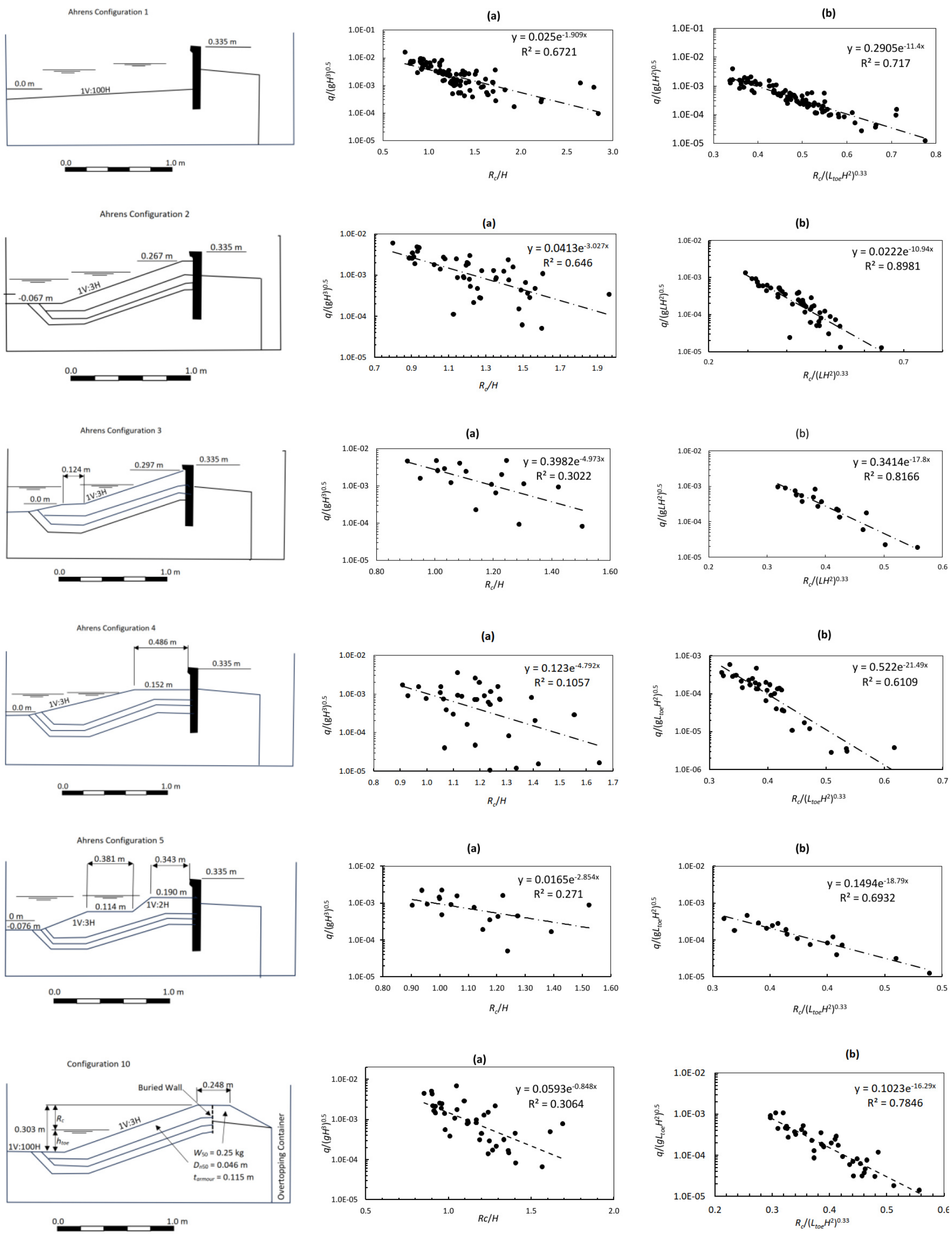

Figure 18. Discharge versus freeboard normalised with (a) wave height factors and (b) wave energy factors for the revetment/seawall model schema of Ahrens et al. (1986). Data are in model units. (Nielsen 2025)

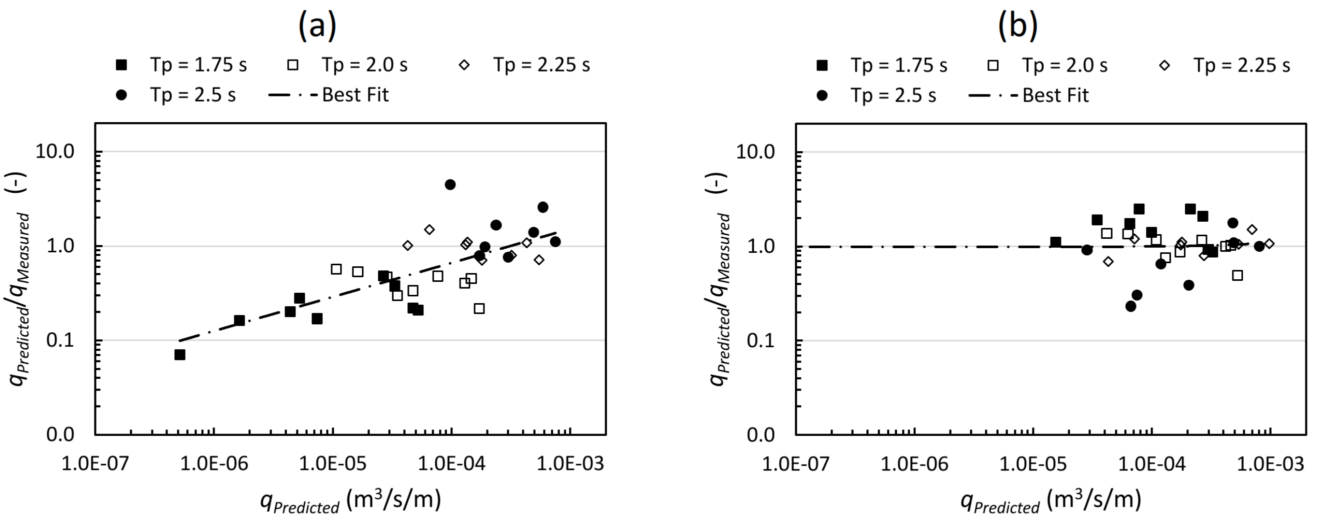

Figure 19. The accuracy of predictions of the flume test results of Ahrens et al. (1986) by using (a) the current practice formula Equation 7 and (b) the wave energy formula Equation 14 incorporating wave period and surrogate design wave height $H = h_{L/2}$

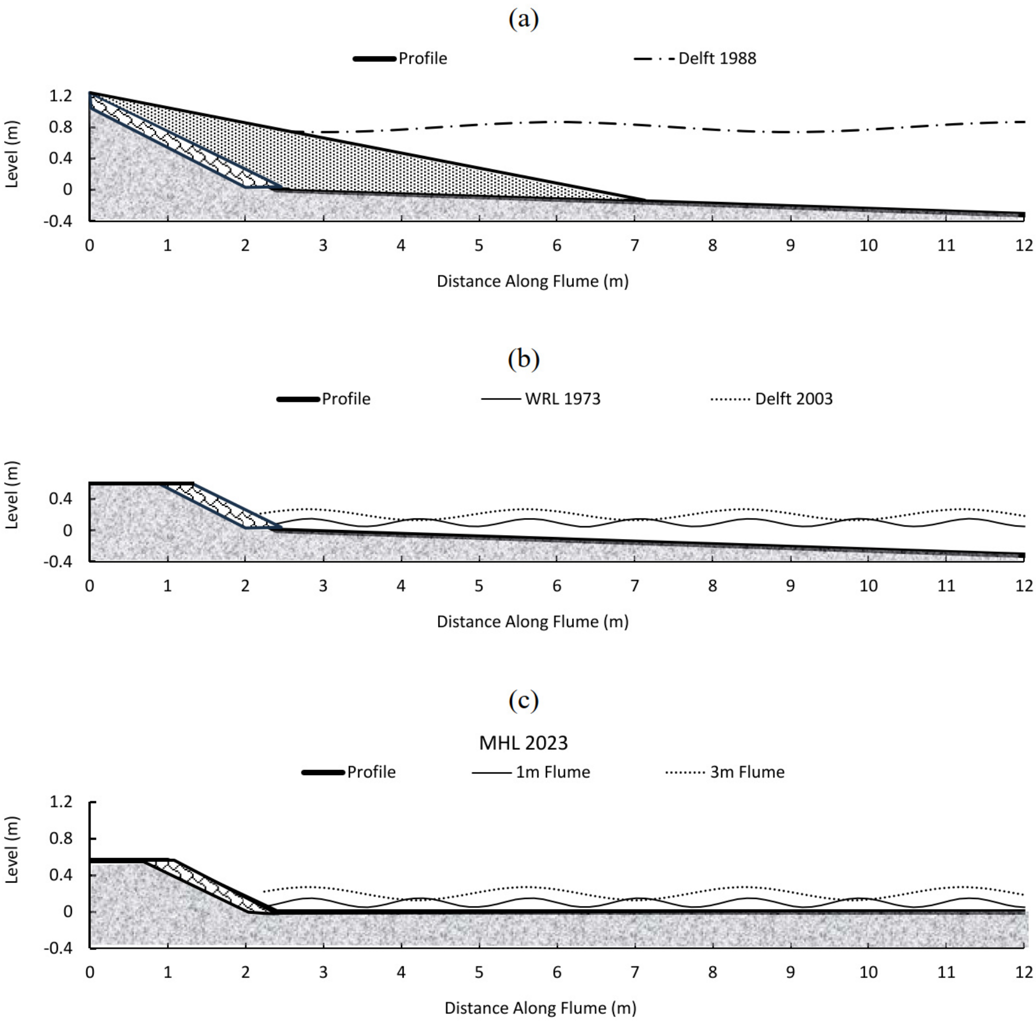

Figure 20. Schematic sections presenting flume setups of revetment armour testing for (a) van der Meer (1988), (b) Foster & Gordon (1973) and van Gent et al. (2003), and (c) Nielsen & Gordon (2025)

5 Conclusions

Concept design formulae for revetment toe scour, rock armour mass stability and wave overtopping discharge for rock-armoured coastal revetments have been developed utilising shallow water breaking wave conditions derived from published scale model studies. The laboratory data were normalised using factors based on wave energy, incorporating wave period, resulting in significant improvements to the coherence of the calibrating laboratory data. The depth located at half the nearshore wavelength in front of the structures was adopted as a surrogate for the design wave height. The Iribarren number distinguished formulae for toe scour and armour mass stability between spilling and plunging breaking waves. Predictions from the modelling results using the new formulae were more coherent and accurate than those from formulae in current use. The formulae demonstrated that for shallow water wave breaking conditions, the pertinent design parameters were depth, wave period and seabed slope. Using the formulae should consider their limitations as discussed and this research would benefit from testing under a greater range of seabed slopes and wave periods. Final designs for revetment armour and wave overtopping should be based on site-specific scale modelling.

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Mr Angus Gordon OAM suggested energy gradients as being the determining factor in coastal processes. Dr Benjamin Williams suggested examining the Iribarren number. Professor Jentsje van der Meer suggested depth being a determining factor with very shallow water wave breaking processes. The laboratory work for the Mardie Salt and Potash project was undertaken at NSW Government Manly Hydraulics Laboratory, supervised by Indra Jayewardene.

REFERENCES

[1] Ahrens, J.P., Heimbaugh, M.S., Davidson, D.D. (1986). Irregular Wave Overtopping of Seawall/Revetment Configurations Roughans Point Massachusetts, Coastal Engineering Research Center, Department of the Army, Waterways Experiment Station, Corps of Engineers Report CERC-86-7, September, 64pp.

[2] Altomare, C., Suzuki, T., Chen, X., Verwaest, T., Kortenhaus, A. (2016). Wave overtopping of sea dikes with very shallow foreshores. *Coastal Engineering* 116, pp236–257.

[3] Battjes, J. (1974). Surf similarity. *Coastal Engineering Proc.*, Vol. 1, No.14, 466-480. https://doi.org/10.9753/icce.v14.26

[4] Boers, M., van Geer, P.F.C., van Gent, M.R.A. (2011). Dike and dune revetment impact on dune erosion, Coastal Sediments 2011, Eds: Wang, Rosati & Roberts, *World Scientific*, 810-823.

[5] CIRIA (2007). The Rock Manual. The use of rock in hydraulic engineering (2nd edition). C683, CIRIA, CUR, CETMEF. London, 1389pp.

[6] Coastal Wiki. (2025). Available online: https://www.coastalwiki.org/wiki/Shallow-water_wave_theory Figure 27 (accessed on 7-10-2025).

[7] Dean, R.G. (1986). Coastal Armouring: Effects, Principles and Mitigation. Proc. 20th Int. Conf. on Coastal Engineering, ASCE, Taiwan, 1843–1857.

[8] EurOtop (2018). Manual on wave overtopping of sea defences and related structures. An overtopping manual largely based on European research, but for worldwide application. Van der Meer, J.W., Allsop, N.W.H., Bruce, T., De Rouck, J., Kortenhaus, A., Pullen, T., Schüttrumpf, H., Troch, P., Zanuttigh, B. 2018, http://www.overtopping-manual.com.

[9] Foster, D.N., Gordon, A.D. (1973). Stability of armour units against breaking waves. Proc. 1st Aust. Conference on Coastal Engineering, Sydney, Australia, 98-107.

[10] Goda, Y. (2000). *Random Seas and Design of Maritime Structures*, World Scientific, 443pp.

[11] Gordon, A.D. (1973). Stability of breakwaters under the action of breaking waves, MEngSc thesis, UNSW, 60pp.

[12] Hudson, R.Y. (1958). Design of Quarry-Stone/Cover Layers for Rubble-Mound Breakwaters - Hydraulic Laboratory Investigation, U.S. Army Engineer Waterways Experiment Station, Corps of Engineers, Vicksburg, Mississippi, Research Report No. 2-2 July 1958, 68pp.

[13] Herbich, J.B. (Ed.). (1990). *Handbook of coastal and ocean engineering* (Vols. 1–3). Gulf Publishing Company.

[14] Nielsen, A.F. (2023). Design scour levels for dune revetments and seawalls. ASCE J. Waterway, Port, Coastal, Ocean Eng., 149(3): 04023005, 8pp, DOI: 10.1061/JWPED5.WWENG-1963.

[15] Nielsen, A.F. (2025). Wave energy considerations for predicting average overtopping discharges of depth limited waves breaking onto rock-armoured coastal revetments. Taylor and Francis, Australasian Journal of Water Resources, DOI:10.1080/13241583.2025.2599522.

[16] Nielsen A.F., Gordon A.D. (2025). Revetment rock armour stability under depth-limited breaking waves. Proc. 27th Aust. Coasts and Ports Conference, IEAust., Adelaide, 18-21 August 2025, 8pp.

[17] Salauddin, M., Pearson J.M. (2019). Experimental study on toe scouring at sloping walls with gravel foreshores, J. Mar. Sci. Eng., 7,198; doi:103390/jmse7070198, 12pp.

[18] Steetzel, H.J. (1985). Systematic Research into the Functioning of Dune Toe Structures - Model Study into Scour near the Dune Revetment Toe (in Dutch). Delft Hydraulics Technical Report, 135pp.

[19] Steetzel, H.J. (1988). Scour holes – analysis of the factors that determine the shape of a scour hole based on available literature of physical model tests (in Dutch). Delft Hydraulics Technical Report.

[20] Steetzel, H.J. (1993). Cross-shore Transport during Storm Surges. Thesis Tech. Univ. Delft, Published also as Delft Hydraulics Communication No. 476, September, 294pp.

[21] Sutherland, J., Obhrai, C., Whitehouse, R.J.S., Pearce, A.M.C. (2006). Laboratory tests of scour at a seawall, Proc. 3rd Int. Conf. on Scour and Erosion. Amsterdam, CUR Bouwn & Infra, 19pp.

[22] Thompson, D.M., Shuttler, R.M. (1975). Riprap design for wind-wave attack – A laboratory study in random waves. Hydraulics Research Station Wallingford Report EX 707, September 1975, 129pp.

[23] Tsai, C.P., Chen, H.B., You, S.S. (2009). Toe Scour of Seawall on a Steep Seabed by Breaking Waves, *J. Waterway, Port, Coastal, Ocean Eng*., 135(2): 61-68

[24] USACE (1984). *Shore Protection Manual*, Coastal Engineering Research Centre, Waterways Experiment Station, US Army Corps of Engineers, 2 volumes.

[25] USACE (2006). *Coastal Engineering Manual*, Coastal Engineering Research Centre, Waterways Experiment Station, US Army Corps of Engineers, EM 1110-2-1100. 6 volumes.

[26] Van der Meer, J.W. (1988). Rock Slopes and Gravel Beaches Under Wave Attack. Ph.D. Thesis, Delft University of Technology, Delft, The Netherlands, 1988. Delft Hydraulics Communication No. 396, 214pp.

[27] Van der Meer, J.W. (2021). Rock Armour Slope Stability under Wave Attack; the Van der Meer Formula revisited, Journal of Coastal and Hydraulic Structures, Vol. 1, 2021, 8.

[28] Van der Meer, J.W., Lykke Andersen, T., Eldrup, M.R. (2024). Rock armour slope stability under wave attack in shallow water. JCHS, Vol. 4, paper 35. DOI: https://doi.org/10.59490/jchs.2024.0035.

[29] Van Gent, M.R.A. (2004). On the stability of rock slopes, Keynote, NATO-workshop, May 2004, Bulgaria, 20pp.

[30] Van Gent, M.R.A., Pozueta, B. (2004). Rear-side stability of rubble mound structures. In J. M. Smith, and J. M. Smith (Eds.), Proc. 29th Int. Conf. on Coastal Engg., Vol. 2005 January, ASCE, pp. 3481-3493. https://doi.org/10.1142/9789812701916_0281.

[31] Van Gent, M.R.A., Smale, A.J., Kuiper, C. (2003). Stability of Rock Slopes with Shallow Foreshores, ASCE, Proc. Coastal Structures, Portland, January 2003, DOI:10.1061/40733(147)9

[32] Van Rijn, L.C. (2018). Local Scour Near Structures, http://www.leovanrijn-sediment.com, 48pp.

[33] Weggel, J.R. (1972). Maximum Breaker Height. Journal of the Waterways, Harbours and Coastal Engineering Division 98: pp529-548.

[34] Yager, R.J. (2014). Calculating drag coefficients for spheres and other shapes using C++. Report ARL-TN-612, US Army Research Laboratory, Aberdeen Proving Ground, MD 21005-5066, June 2014.