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A Multiple Contracts Version of the SACRE

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ABSTRACT

Taking into consideration that the SACRE (system of increasing amortization in real terms), as originally instituted by "Caixa Econômica Federal", is not financially consistent, an exact procedure, denoted as SACRE, was proposed in de Faro and Lachtermacher (2022). The present paper submits a multiple contract version of SACRE*. It is shown that, taking into account the financial institution cost of capital, it is always better to implement the multiple contracts approach.*

Keywords: *Amortization Systems, Crescent Amortization System (SACRE)*

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I. INTRODUCTION

In 1996, the "Caixa Econômica Federal" (CEF), which is the main institution for housing financing in Brazil, introduced a debt amortization scheme named "Sistema de Amortizações Reais Crescentes" – SACRE (system of increasing amortizations in real terms).

In its original version, this very peculiar amortization system is not financially consistent. Namely, even if all contractual payments are dutifully made, a residual debt remains, which must be paid in full by the borrower, usually one month after the end of the term of the contract.

Given that de Faro and Lachtermacher (2022) proposed a financially consistent variant of the SACRE, the purpose of this paper is to formulate a multiple contracts version of this system. Similar to cases of the adoption of either the constant payments scheme or the constant amortization scheme of debt financing, which were considered in De-Losso et al (2013) and in de Faro (2022), it will be shown that the financial institution granting the loan, depending on its cost of capital, may derive substantial income tax reductions in terms of present values.

II. THE CASE OF A SINGLE CONTRACT

Denoting by F the loan amount, and by i the periodic rate of compound interest , suppose that, in the case where a single contract is considered, it is stipulated by the financing institution granting the loan that the debt must be repaid in n periodic payments, in accordance with the SACRE scheme.

Since the SACRE scheme is a combination of the constant payments scheme with the constant amortization scheme, the number n of payments is divided into ℓ subperiods, each with m payments. The numbers n, ℓ and m are integer numbers with $n = \ell \times m$, and with m constant payments in each of the first $\ell - 1$ sub-periods.

Specifically, denoting by S_k the outstanding debt immediately after the k-th payment p_k has been made, where $k = 1, 2,..., n$ and $S_0 = F$, it is established that:

$$P_1 = p_1 = A_1 + J_1 = \frac{S_0}{n} + i \times S_0 = S_0 \times \left(\frac{1}{n} + i\right) = \frac{S_0}{n} \times (1 + n \times i) \quad (1)$$

where $P_1 = p_1 = p_k$, $for\ k = 1, 2, ..., m$ with $A_1 = F/n$ denoting the parcel of amortization of the first payment p_1, and $J_1 = i \times S_0$ denoting the corresponding parcel of interest.

For the evolution of the outstanding debt, S_k, it is convenient to recall that, cf. de Faro e Lachtermacher (2012, p. 240):

$$S_k = (1 + i) \times S_{k-1} - p_k, \quad \text{for } k = 1, 2, ..., n \quad (2)$$

Therefore, using the presumed recurrence method to determine the debtor's balance, we have:

$$S_m = S_0 \times (1+i)^m - \left\{P_1 \times \left[(1+i)^m - 1\right]/i\right\} = S_0 \times (1+i)^m - (P_1 \times \alpha) \quad (3)$$

where $\alpha = \left[(1 + i)^m - 1\right]/i$.

This relationship, in view of the value of P_1 presented in Equation 1, can be rewritten as:

$$S_m = \frac{S_0}{n} \times \left\{n - \left[(1+i)^m - 1\right]/i\right\} = \frac{S_0}{n} \times (n - \alpha) \quad (3')$$

b) For the second sub period of constant installments, equal to P_2, that is, for k equal to m+1, m+2,...,2m , we have:

$$A_{m+1} = S_m / (n - m) \quad \text{and} \quad J_{m+1} = i \times S_m \quad (4)$$

with

$$p_{m+1} = A_{m+1} + J_{m+1} = S_m \times \left[1 + i \times (n - m)\right] / (n - m) \quad (5)$$

or

$$P_2 = p_k = S_m \times \left[1 + i \times (n - m)\right] / (n - m), \quad \text{for} \quad k = m+1, m+2, ..., 2m \quad (5')$$

Thus, recurrently, taking into account (2), it follows that:

$$S_{2m} = S_m \times (1+i)^m - P_2 \times \left[(1+i)^m - 1\right] / i = S_m \times (1+i)^m - P_2 \times \alpha \quad (6)$$

This relationship, in view of the value of P_2 presented in Equation 5', can be rewritten as:

$$S_{2m} = \frac{S_m}{(n-m)} \times \left\{(n-m) - \left[\frac{(1+i)^m - 1}{i}\right]\right\} = \frac{S_m}{(n-m)} \times (n - m - \alpha) \quad (6')$$

or

$$S_{2m} = \frac{S_0 \times (n-\alpha) \times [(n-m)-\alpha]}{n \times (n-m)} \tag{6''}$$

c) Similarly, making $A_{2m+1} = S_{2m}/(n - 2m)$ and $J_{2m+1} = i \times S_{2m}$, we will have:

$$P_3 = p_k = S_{2m} \times \left[\frac{1+i \times (n-2\times m)}{n-2\times m}\right], \text{ for } k = 2m+1, 2m+2, \dots, 3m \tag{7}$$

Hence, recursively, considering (2), it follows that:

$$S_{3m} = S_{2m} \times (1+i)^m - P_3 \times \left[(1+i)^m - 1\right]/i = S_{2m} \times (1+i)^m - P_3 \times \alpha \tag{8}$$

Where P_3 denotes the constant value of the m payments in the third subperiod.

Therefore:

$$S_{3m} = \frac{S_{2m}}{(n-2\times m)} \times \left\{(n-2\times m) - \left[\frac{(1+i)^m - 1}{i}\right]\right\} = \frac{S_{2m}}{(n-2\times m)} \times (n-2\times m-\alpha) \tag{8'}$$

or

$$S_{3m} = \frac{S_0 \times (n-\alpha) \times (n-m-\alpha) \times (n-2\times m-\alpha)}{n \times (n-m) \times (n-2\times m)} \tag{8''}$$

d) Proceeding in a similar manner, it can be depicted, as in de Faro and Lachtermacher (2022), that after $\ell - 1$ sub periods, the outstanding debt will be:

$$S_{(\ell-1)m} = S_0 \times \left\{\prod_{j=1}^{(\ell-1)\times m} \left[\frac{n-(j-1)\times m-\alpha}{n-(j-1)\times m}\right]\right\} \tag{9}$$

At this point, as suggested in de Faro and Lachtermacher (2022), rather than being constant, the last m payments should decrease linearly in accordance with an arithmetic progression of ratio equal to $i \times S_{(\ell-1)}/m$, with $p_{(\ell-1)m+1} = S_{(\ell-1)m} \times (1 + i \times m)/m$, which is a procedure assured to be financially consistent whenever the interest rate, i, is less than 10% per month, and which is far above the current rates charged in the Brazilian house-financing system. Currently, the monthly rate is reflected at 1.5%.

In summary, the sequence of the first $n - m$ payments will be as follows:

$$p_k = \begin{cases} P_1 = S_0 \times (1+n\times i)/n, \text{ for } k=1,2,\ldots,m \\ P_2 = \dfrac{S_0\times(n-\alpha)\times[1+i\times(n-m)]}{n\times(n-m)}, \text{ for } k=m+1,m+2,\ldots,2m \\ P_3 = \dfrac{S_0\times(n-\alpha)\times(n-m-\alpha)\times[1+i\times(n-2m)]}{n\times(n-m)\times(n-2m)}, \text{ for } k=2m+1,2m+2,\ldots,3m \\ \vdots \\ P_{(\ell-1)m} = \dfrac{S_0\times\{1+i\times[n-(\ell-2)\times m]\}}{n-(\ell-2)\times m}\times\left[\prod_{j=1}^{\ell-2}\dfrac{n-(j-1)\times m-\alpha}{n-(j-1)\times m}\right] \\ \quad \text{for } k=(\ell-1)m+1,\ldots,n-m \end{cases} \tag{10}$$

With the last m payments given as:

$$p_k = S_{(\ell-1)m}\times\{1+i(n-k+1)\}/n, \text{ for } k=n-m+1,n-m+2,\ldots,n \tag{11}$$

With regard to the sequence of the parcels of amortization, it should be noted that, as shown in de Faro and Lachtermacher (2012, p. 243), and similar to the case of the constant payments scheme, the parcels of amortization, in each set of constant payments, follow a geometric sequence of ratio equal to 1+i.

Accordingly, we have:

$$A_k = \begin{cases} S_0\times(1+i)^{k-1}/n, \text{ for } k=1,2,\ldots,m \\ \dfrac{S_0\times(n-\alpha)\times(1+i)^{k-m-1}}{n\times(n-m)}, \text{ for } k=m+1,m+2,\ldots,2m \\ \dfrac{S_0\times(n-\alpha)\times(n-m-\alpha)\times(1+i)^{k-2m-1}}{n\times(n-m)\times(n-2m)}, \text{ for } k=2m+1,2m+2,\ldots,3m \\ \vdots \\ S_0\times\left\{\prod_{j=1}^{(\ell-2)m}\left[\dfrac{n-(j-1)\times m-\alpha}{n-(j-1)\times m}\right]\times(1+i)^{k-(\ell-1)m-1}\right. \\ \quad \text{for } k=(\ell-2)m+1,(\ell-2)m+2,\ldots,(\ell-1)m \end{cases} \tag{12}$$

with the remaining parcels being constant. That is:

$$A_k = S_{(\ell-1)m}/m, \text{ for } k=(\ell-1)m+1,(\ell-1)m+2,\ldots,n \tag{13}$$

for the sequence of the parcels of interest, it suffices to recall that:

$$J_k = p_k - A_k, \text{ for } k=1,2,\ldots,n \tag{14}$$

III. THE MULTIPLE CONTRACTS ALTERNATIVE

Rather than engaging a single contract, the financial institution has the option of requiring the borrower to adhere to n subcontracts; one for each of the n payments that would be associated with the case of a single contract, with the principal of the k-th subcontract being the present value, at the same interest rate i, of the k-th payment of the single contract.

Namely, the principal of the k-th subcontract, denoted by F_k, is:

$$F_k = p_k \times (1+i)^{-k}, \quad k = 1, 2, \ldots, n \tag{15}$$

In this case, the parcel of amortization associated with the k-th payment, which will be denoted by $\hat{A}_k$, will be:

$$\hat{A}_k = F_k = p_k \times (1+i)^{-k}, \quad k = 1, 2, \ldots, n \tag{16}$$

Ergo, the parcel of amortization associated with the k-th subcontract is exactly equal to the value of the corresponding principal.

Conversely, from an accounting point of view, it follows that the parcel of interest associated with the k-th subcontract, which will be denoted by $\hat{J}_k$, is:

$$\hat{J}_k = p_k \times \left\{1-(1+i)^{-k}\right\} = p_k - F_k = p_k - \hat{A}_k \quad , \quad k = 1, 2, \ldots, n \tag{17}$$

From a strict accounting point of view, not taking into consideration the costs that may be associated with the bookkeeping and registration of the subcontracts, the total interest payments is the same in both cases. However, in terms of present values, and depending on the financial institution opportunity cost, it is possible that the financial institution will be better off if it adopts the multiple contracts option.

A simple numerical example

Before presenting a numerical illustration, it is appropriate to give due credit to the one who has introduced the idea of associating a specific contract with each of the payments of the main contract.

As far as we know, the concept was originally proposed by Sandrini (2007), in his Master's thesis for the Federal University of Paraná. However, an actual contract for each of the payments was not effectively proposed. The goal was to imply, specifically for the case of the constant payments scheme of debt amortization, the occurrence of what is named, in legal terms, anatocism – to wit, the charge of interest upon interest.

Later, De-Losso et al. (2013) presented a formalization of the concept of multiple contracts. Focusing on the case of the constant payments scheme. Later de Faro (2022) extended the analysis to consider the Constant Amortization System.

Now, as a numerical illustration, consider a loan of 12,000 units of capital, for the case of $n = 12$ periodic payments, with $m = 3$, and $\ell = 4$, with the periodic rate of interest, i, being equal to 1% per period.

Table 1 presents the sequence of the 12 payments, which is the same both in the case of a single contract, as well as in the 12 individual contracts.

Also, in Table 1, we have the sequences of values of J_k and of $\hat{J}_k$ in addition to the sequence of differences $d_k = J_k - \hat{J}_k$

Table 1: The Sequences of the Parcels of Interest and their Differences

k	p_k	J_k	$\hat{J}_k$	d_k
1	1,120.00	120.00	11.09	108.91
2	1,120.00	110.00	22.07	87.93
3	1,120.00	99.90	32.94	66.96
4	1,086.35	89.70	42.39	47.31
5	1,086.35	79.73	52.73	27.01
6	1,086.35	69.67	62.96	6.71
7	1,051.16	59.50	70.72	-11.22
8	1,051.16	49.58	80.43	-30.85
9	1,051.16	39.57	90.04	-50.48
10	1,011.16	29.45	95.77	-66.32
11	1,001.34	19.63	103.82	-84.18
12	991.52	9.82	111.60	-101.78
$\sum$	12,776.55	776.55	776.55	0.00

Strictly from an accounting point of view, there is no gain if a single contract is substituted by multiple contracts since the sums of the corresponding parcels of interest are the same. Hence,

$$\sum_{k=1}^{n} J_k = \sum_{k=1}^{n} \hat{J}_k = 776.55$$

Yet, depending on the opportunity cost of the financial institution, which will be denoted as ρ, the financial institution may derive substantial financial gains in terms of income tax deductions.

In other words, it is possible that:

$$V_1(\rho) = \sum_{k=1}^{n} J_k \times (1+\rho)^{-k} > V_2(\rho) = \sum_{k=1}^{n} \hat{J}_k \times (1+\rho)^{-k} \tag{18}$$

where the interest rate ρ is supposed to be relative to the same period of the interest rate *i*.

Moreover, as the sequence of differences d_k has only one change of sign, thus characterizing what is termed a conventional financing project, cf. de Faro (1974), whose internal rate of return is unique, and in this particular case null, it follows that $\Delta = V_1(\rho) - V_2(\rho) > 0 \; for \; \rho > 0$.

Figure 1 outlines the evolution of $\Delta, \; for \; 0 \leq \rho \leq 5\%$. Additionally, we also have the evolution of , when the interest rate, i , is equal to 0.5%, 1%, 1.5%, 2% and 3%.

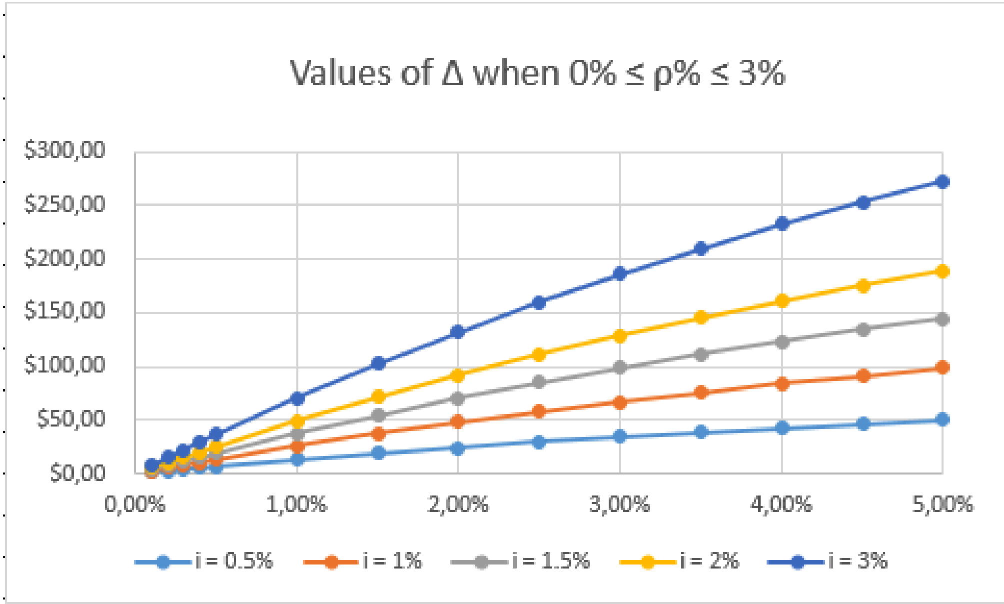

Figure 1

For instance, if $i = 1\%$ per period, and if $\rho = 2\%$ per period, we will have $\Delta = V_1(2\%) - V_2(2\%) = 709.38 - 661.56 = 47.82$ units of capital. Namely, the financing institution will have a non-trivial income tax gain, in terms of present values, if a single contract is substituted by 12 individual contracts, one for each of the 12 payments.

The difference Δ is substantially greater if $i = 3\%$ per period and $\rho = 5\%$ per period. That is, $\Delta = V_1(5\%) - V_2(5\%) = 1,860.77 - 1,587.79 = 272.98$ units of capital.

IV. GENERAL ANALYSIS

In the previous section, focusing attention on the case of a contract with only 12 payments, it was verified that the sequence, d_k, of differences of the interest payments yielded just one change of sign, thereby assuring us of the uniqueness of the corresponding internal rate of return, which was known to be zero.

However, when the number of payments is increased, it is possible to have instances wherein more than one change of sign can occur.

This possibility is illustrated in Figure 2, which refers to the case where a loan of 1,200,000 units of capital has a term of 15 years (180 months), with $\ell = 15$, monthly payments, and with the monthly interest rate, i, going from 0.5% up to 3%.

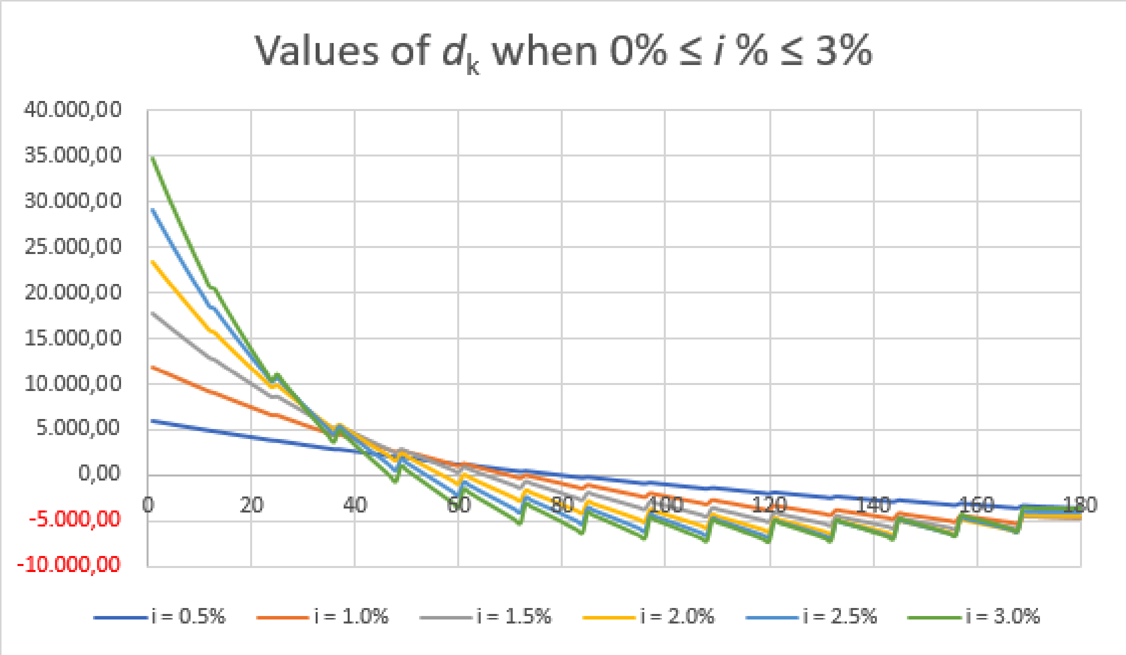

Figure 2

Wherefore, for the cases where the monthly interest rate i assumes the values of 1%, 2% and 3%, we have three changes of sign in the sequences of differences d_k with only one change of sign in the other three cases.

However, considering a classical result first stated by Norstrom (1972), which is based on the sequence of the accumulated values of the sequence d_k, we can still guarantee the uniqueness of the corresponding internal rate of return, and which we already know is null. Moreover, we are also assured that the difference of present values Δ is positive whenever the opportunity cost ρ is greater than zero.

Taking into consideration that in Brazil the monthly interest rates charged in house-financing contracts do not exceed 2% per month, in real terms, Tables 2-5 present the percentage increase of the fiscal gain $\delta = \left[V_1(\rho_a)/V_2(\rho_a) - 1\right] \times 100$, for some values of the corresponding annual opportunity cost ρ_a, with each contract with a term of n_a years, subdivided in $\ell = n_a$ subperiods, and with each one at $m = 12$ monthly payments.

Table 2: Percentual Values of δ for $i = 0.5\%\ p.m.$

i=0.5%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	7.6032	15.3468	23.2045	31.1515	39.1646	47.2225
10	14.7195	30.4499	47.0152	64.2312	81.9173	99.9056
15	21.1799	44.5397	69.5395	95.6177	122.2627	149.0522
20	27.0049	57.3260	89.7844	123.2682	156.9112	190.1311
25	32.2006	68.6281	107.2472	146.4066	185.0633	222.6793
30	36.8074	78.4424	121.9342	165.2770	207.4811	248.1846

Table 3: Percentual Values of δ for $i = 1.0\% \, p.m.$

i=1%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.9818	14.0369	21.1427	28.2780	35.4240	42.5637
10	12.6403	25.8461	39.4650	53.3526	67.3789	81.4325
15	17.1122	35.2698	54.0650	73.1370	92.1981	111.0376
20	20.6713	42.6924	65.3157	87.9659	110.2641	131.9929
25	23.5050	48.4791	73.8287	98.8507	123.1814	146.6734
30	25.7789	52.9968	80.2719	106.8814	132.5477	157.2110

Table 4: Percentual Values of δ for $i = 1.5\% \, p.m.$

i=1.5%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	6.4400	12.9049	19.3754	25.8347	32.2678	38.6619
10	11.0264	22.3577	33.8719	45.4608	57.0333	68.5156
15	14.2645	29.0323	44.0129	58.9738	73.7455	88.2143
20	16.6191	33.8043	51.0807	68.1246	84.7455	100.8487
25	18.3616	37.2551	56.0467	74.3817	92.1025	109.1625
30	19.6818	39.8015	59.6129	78.7830	97.2111	114.8975

Table 5: Percentual Values of δ for $i = 2.0\% \, p.m.$

i=2%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	5.9649	11.9192	17.8476	23.7367	29.5750	35.3530
10	9.7461	19.6393	29.5842	39.5008	49.3250	59.0074
15	12.1807	24.5843	37.0028	49.2816	61.3156	73.0401
20	13.8378	27.8861	41.8294	55.4706	68.7046	81.4871
25	15.0065	30.1631	45.0706	59.5276	73.4576	86.8487
30	15.8621	31.7896	47.3310	62.3081	76.6821	90.4695

The results presented in Tables 2 to 5 are self-evident. They illustrate a compelling support for the substitution of a single contract by multiple contracts.

For instance, if the interest rate i charged by the financial institution granting the loan is 0.5% per month, the percentual value of δcan be as high as 47% when its opportunity cost is 30% annually, the contract has a 5-year term, and with a percentage fiscal gain over 248%, if the contract is of 30 years, and ρ_a=30%per year.

Furthermore, even though the fiscal gain decreases when the interest rate, i , being charged is increased, the percentage gain is no less than 35% in every case.

Accordingly, one can conclude that the financial institution is well advised whenever it substitutes a single contract by multiple contracts, one for each of the payments of the single contract, whenever using our version of SACRE scheme.

V. A COMPARISON WITH TWO ALTERNATIVE SYSTEMS OF AMORTIZATION

Given that the financial institution granting the loan may have the option of choosing an alternative system of amortization, this section addresses two such possibilities, since both alternatives have also been considered in the Brazilian House-Financial program.

The first one is the system of constant payments. In this case, as shown in De-Losso et al. (2013) and also in de Faro (2022), the present value of the sequence of interest payments, if multiple contracts are adopted, is equal to:

$$V_3(\rho) = p \times \left\{ \frac{1-(1+\rho)^{-n}}{\rho} - \frac{1-(1+\hat{\rho})^{-n}}{\hat{\rho}} \right\} \tag{19}$$

where $p = i \times F/\left[1 - (1 + i)^{-n}\right]$ and $\hat{\rho} = \rho + i + (\rho \times i)$.

Tables 6 to 9 illustrate the percentage increase of the fiscal gain $\delta' = \left[V_1(\rho_a)/V_3(\rho_a) - 1\right] \times 100$, wherein the financial institution adopts the multiple contracts version of the SACRE instead of the constant payments scheme.

Table 6: Percentual Values of δ' for $i = 0.5\%\ p.m.$

i=0.5%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	5.7794	5.2285	4,6985	4.1891	3.7003	3.2316
10	9.4904	7.6829	5.9746	4.3761	2.8930	1.5266
15	12.4358	8.8406	5.5845	2.7002	0.1883	-1.9736
20	14.6772	9.0254	4.2075	0.2399	-2.9578	-5.5094
25	16.3350	8.5602	2.4188	-2.2365	-5.7050	-8.2918
30	17.4969	7.7185	0.6241	-4.3241	-7.7642	-10.2021

Table 7: Percentual Values of δ' for $i = 1.0\%\ p.m.$

i=1.0%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	11.7027	10.6266	9.6013	8.6255	7.6976	6.8157
10	19.1158	15.7858	12.7269	9.9396	7.4162	5.1429
15	24.8388	18.6177	13.2481	8.6888	4.8592	1.6614
20	28.9490	19.7683	12.4276	6.6867	2.2392	-1.2076
25	31.7211	19.8263	11.1033	4.8366	0.3318	-2.9539
30	33.3774	19.2161	9.7229	3.4250	-0.8325	-3.8088

Table 8: Percentual Values of δ' for $i = 1.5\%\ p.m.$

i=1.5%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	17.7320	16.1494	14.6545	13.2434	11.9119	10.6558
10	28.5432	23.8819	19.6904	15.9451	12.6137	9.6597
15	36.2426	27.9434	21.0064	15.2744	10.5670	6.7070

20	41.0125	29.3097	20.3074	13.4737	8.2943	4.3416
25	43.5284	28.9674	18.7241	11.5730	6.5250	2.8817
30	44.3848	27.6427	16.8807	9.9207	5.2787	2.0524

Table 9: Percentual Values of δ' for $i = 2.0\%\, p.m.$

i=2.0%p.m.	$\rho_a(\%)$					
n(years)	5%	10%	15%	20%	25%	30%
5	23.8328	21.7572	19.8114	17.9876	16.2783	14.6761
10	37.5385	31.6843	26.5071	21.9496	17.9496	14.4441
15	46.2311	36.2146	28.0327	21.4001	16.0368	11.6928
20	50,6002	36,9708	26.7575	19.1564	13.4773	9.1860
25	52.0719	35.6097	24.3397	16.6165	11.2283	7.3658
30	51.7253	33.2421	21.6769	14.3198	9.4569	6.0912

As indicated in the overwhelming majority of the cases, the financial institution should not choose the multiple contracts version of the SACRE. That is, if possible, the best option is to adopt the multiple contracts version of the constant payments scheme.

On the other hand, in the case of the system of constant amortization, the present value of the sequence of interest payments, where multiple contracts are adopted as shown in de Faro (2022), is equal to:

$$V_4(\rho)=\frac{F}{n}\times\left\{\frac{(i-\rho)\times\left[(1+\rho)^{-n}-1\right]+n\times i\times\rho}{\rho^2}-\frac{(i-\hat{\rho})\times\left[(i+\hat{\rho})^{-n}-1\right]+n\times i\times\hat{\rho}}{\hat{\rho}^2}\right\} \quad (20)$$

Tables 10 to 13 portray the percentage increase of the fiscal gain $\hat{\delta} = \left[V_1(\rho_a)/V_4(\rho_a) - 1\right]\times 100$, when the financial institution adopts the multiple contracts version of the SACRE, instead of the constant amortization version.

Table 10: Percentual Values of $\hat{\delta}$ for $i = 0.5\%\, p.m.$

i=0.5%p.m.	$\rho_a(\%)$					
n(years)	5%	10%	15%	20%	25%	30%
5	1.2310	1.0836	0.9439	0.8114	0.6861	0.5675
10	1.1235	0,8599	0.6193	0.4016	0.2061	0.0315
15	1.0147	0.6658	0.3650	0.1106	-0.1015	-0.2766
20	0.8942	0.5125	0.1968	-0.0552	-0.2521	-0.4043
25	0.7927	0.3987	0.0889	-0.1445	-0.3168	-0.4438
30	0.7084	0.3151	0.0216	-0.1884	-0.3371	-0.4436

Table 11: Percentual Values of $\hat{\delta}$ for $i = 1.0\%\ p.m.$

i=1.0%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	2.5282	2.2448	1.9781	1.7273	1.4915	1.2700
10	2.3441	1.8686	1.4429	1.0645	0.7298	0.4351
15	2.1678	1.5681	1.0653	0.6496	0.3087	0.0302
20	1.9540	1.3198	0.8130	0.4170	0.1106	-0.1265
25	1.7694	1.1305	0.6452	0.2849	0.0184	-0.1807
30	1.6121	0.9843	0.5301	0.2070	-0.0246	-0.1942

Table 12: Percentual Values of $\hat{\delta}$ for $i = 1.5\%\ p.m.$

i=1.5%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	3.8908	3.4788	3.0936	2.7336	2.3972	2.0829
10	3.6425	2.9811	2.3967	1.8829	1.4330	1.0398
15	3,4074	2.5917	1.9182	1.3674	0.9186	0.5529
20	3.0991	2.2443	1.5721	1.0511	0.6482	0.3350
25	2.8242	1.9654	1.3226	0.8473	0.4941	0.2278
30	2.5846	1.7397	1.1366	0.7078	0.3981	0.1689

Table 13: Percentual Values of $\hat{\delta}$ for $i = 2.0\%\ p.m.$

i=2.0%p.m.	ρ_a(%)					
n(years)	5%	10%	15%	20%	25%	30%
5	5.3187	4.7826	4.2842	3.8207	3.3898	2.9890
10	5.0078	4.1727	3.4415	2.8038	2.2487	1.7659
15	4.7111	3.6897	2.8547	2.1760	1.6247	1.1756
20	4.3022	3.2309	2.3969	1.7531	1.2553	0.8670
25	3.9287	2.8489	2.0482	1.4577	1.0179	0.6847
30	3.5992	2.5325	1.7778	1.2421	0.8542	0.5657

Similarly, it is clear that in the overwhelming majority of cases, the financial institution should opt for the multiple contracts version of the constant amortization scheme.

VI. CONCLUSION

In similarity to the cases where either the constant payments system or the constant amortization system is adopted, a financial institution which implements our version of the SACRE, will be well advised if a multiple contract scheme, rather than a single contract, is implemented.

However, if the financial institution has the option of rather than adopting the SACRE, choosing either the constant payment system or the constant amortization one, in the vast majority of cases, SACRE is not the best option.

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