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Repeatedly, the Brazilian Judicial System has determined that home-financing contracts written in terms of compound interest, both in the case of constant payments and in the case of the system of constant amortization, should be substituted by contracts specifying simple interest. This has resulted, for the case of the system of constant amortization, in the adoption of a variant of a procedure that has been named the "Gauss' Method". It is shown that the implementation of a multiple contracts' version may imply substantial fiscal gains, depending on the financial institution opportunity cost.

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Repeatedly, the Brazilian Judicial System has determined that home-financing contracts written in terms of compound interest, both in the case of constant payments and in the case of the system of constant amortization, should be substituted by contracts specifying simple interest. This has resulted, for the case of the system of constant amortization, in the adoption of a variant of a procedure that has been named the "Gauss' Method". It is shown that the implementation of a multiple contracts' version may imply substantial fiscal gains, depending on the financial institution opportunity cost.

Keywords: multiple contracts scheme; amortization constant system in simple interest.

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I. INTRODUCTION

Similar to the case of constant payments, which in terms of multiple contracts, was recently analyzed in de Faro and Lachtermacher (2023), the Brazilian Judicial System, cf. Jusbrasil (2023), has repeatedly determined that the System of Constant Amortization (SAC, with the acronym as written in Portuguese), which was introduced in the Brazilian System of House Financing (SFH) in 1971, and which is based on compound interest, should also be substituted by a version based on simple interest.

For this purpose, Rovina (2009) proposed a variant of the so called "Gauss' Method," cf. Nogueira (2013), to consider that in the system of constant amortization, the periodic payments follow an arithmetic progression.

Although promulgated in several sentences, the above-mentioned procedure violates a law established in 1964. For this reason, besides considering the afore-mentioned variant of the so called "Gauss' Method," which considers the end of the contract as the focal date, cf. Ayres (1963), we will also consider the case where the focal date, complying with the 1964 law, specifies the beginning of the contract as the focal date.

II. THE ROVINA'S PROPOSAL

Consider a house-financing contract of F units of capital, with terms of n periods, specifying the periodic rate i of compound interest, in accordance with the system of constant amortization.

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Denoting by $\overline{P}_k$ the k-th payment, it can be seen, for instance in de Faro and Lachtermacher (2012, p. 267), that we will have:

$$\overline{P}_k = F \times (i + 1/n) - F \times i \times (k-1)/n, \quad \text{for } k = 1, 2, \dots\ n \tag{1}$$

That is, the periodic payments will follow an arithmetic progression, with initial payment $\overline{P}_1 = F \times (i + 1/n)$ and negative ratio equal to $-\ i \times F/n$.

Consequently, the outstanding debt at the end of the k-th period, just after the k-th payment, denoted $\overline{S}_k$, will linearly decrease. That is:

$$\overline{S}_k = F \times (1 - k/n), \quad \text{for } k = 1, 2 \dots\ , n \tag{2}$$

with $S_0 = F$.

On the other hand, in the case where the interest rate, *i*, is of simple interest, we must specify a particular focal date.

In general, if the end of the contract is specified as the focal date, denoting by P_k the k-th periodic payment, we must have:

$$F \times (1 + i \times n) = \sum_{k=1}^{n} P_k \times \{1 + i \times (n-k)\} \tag{3}$$

According with Rovina (2009), who extended the so called "Gauss' Method," as in Nogueira (2013), to the case where the periodic payments follow an arithmetic progression, the first step is to introduce a weight factor, *I*, defined as:

$$I = 3i \times F / \{n \times (2n \times i - 2i + 3)\} \tag{4}$$

Consequently, the k-th periodic payment will be:

$$P_k = (F/n) + [(n-k+1) \times I], \quad \text{for } k = 1, 2, \dots\ , n \tag{5}$$

That is, the periodic payments follow an arithmetic progression with first payment $P_1 = (F/n) + [(n - k + 1) \times I]$ and with ratio $-\ I$, which can be shown to satisfy equation (2).

Furthermore, as in the SAC with compound interest, we will have constant amortization, with the k-th parcel of interest, J_k , being made equal to:

$$J_k = (n-k+1) \times I, \quad \text{for } k = 1, 2, \dots\ , n \tag{6}$$

As shown in de Faro (2014), we have $\overline{P}_1 > P_1$, if $n \geq 2$. Therefore, the recipient of the loan is always benefited if the contract originally written in terms of compound interest, is substituted by one, at the same rate *i*, written in terms of simple interest.

Before proceeding, it should be noted, as pointed out by De-Losso et al. (2020), that the specification of the end term of the contract as the focal date, violates a Brazilian law of 1964, which stipulates that the focal date must be the beginning of the contract. This point will be further addressed in section 6.

Notwithstanding, although this peculiar variant of the "Gauss' Method" is plagued by several financial deficiencies, as discussed in de Faro (2014), it is still being judicially supported.

III. A SIMPLE NUMERICAL EXAMPLE

Fixing at 1% the periodic interest rate, *i,* of simple interest, consider a loan of 10,000 units of capital with a single contract specifying 12 periodic payments in accordance with this variant of the "Gauss' Method." The weight factor, *I*, in this case is equal to 7.76397516.

In Table 1, using formulas (4) and (5), and the fact that the parcel of amortization is constant, we present the evolution of the debt S_k, as given, recursively, as $S_k = S_{k-1} - A_k$ with $S_0 = F$. Consequently, as in the case where *i* is a rate of compound interest, S_k also decreases linearly.

Table 1: Evolution of the Debt According with the Rovina's Variant of the "Gauss' Method" – Focal Date = *n*

k	J_k	A_k	P_k	S_k
0				10,000.00
1	93.17	833.33	926.50	9,166.67
2	85.40	833.33	918.74	8,333.33
3	77.64	833.33	910.97	7,500.00
4	69.88	833.33	903.21	6,666.67
5	62.11	833.33	895.45	5,833.33
6	54.35	833.33	887.68	5,000.00
7	46.58	833.33	879.92	4,166.67
8	38.82	833.33	872.15	3,333.33
9	31.06	833.33	864.39	2,500.00
10	23.29	833.33	856.63	1,666.67
11	15.53	833.33	848.86	833.33
12	7.76	833.33	841.10	0.00
$\sum$	605.59	10,000.00	10,605.59	

Before proceeding, it is imperative to point out that the determination of the outstanding debt at time k, S_k, as given by the recursion mentioned above, does not agree with the results that would be derived by the well-established concepts of either the retrospective method or by the prospective method, which, following Kellison (1991), states that:

a) according to the prospective method, the outstanding loan balance at any point in time is equal to the present value on the date of the remaining payments.

For instance, at time 10, just after the 10th payment, as we are using simple interest, we would have:

$$S_{10} = \frac{848.86}{1+0.01} + \frac{841.10}{1+2\times 0.01} = 1,665.06$$ units of capital;

while, according to Table 1 we would have S_{10}=1,666.67 units of capital.

According to the retrospective method, the outstanding loan balance at any point in time is equal to the original amount of the loan accumulated to that date less the accumulated value on the date of all payments previously made.

Thus, for instance, the outstanding loan balance just after the second payment, would have to be:

$$S_2 = 10,000 \times (1+2\times 0.01) - 926.50 \times (1+1\times 0.01) - 918.74 = 8,345.50$$ units of capital.

On the other hand, the results presented in Table 1 would imply that $S_2 = 8,333.33$ units of capital.

To remedy this incongruence, which may result in judicial arguments, Lachtermacher and de Faro (2023) extended the work of Forger (2009) providing a financially consistent procedure. This point will be further addressed in section 6.

However, given that the parcels of interest are not affected, we will proceed with the analysis accordingly.

IV. IMPLEMENTING MULTIPLE CONTRACTS

Rather than engaging in a single contract, the financial institution has the option of requiring the borrower to adhere to n subcontracts - one for each of the n payments that would be associated with the case of a single contract.

If the rate i were of compound interest, the principal of the k-th subcontract, as proposed in De-Losso et al. (2013), in what can be deemed as an adaptation of the master thesis of Sandrini (2007), the principal of the k-th subcontract would be taken to be equal to the present value of the k-th payment of the original single contract – a procedure that was also used in de Faro (2022) and in de Faro and Lachtermacher (2023 a and b).

However, as we are considering the case where the rate i is of simple interest, and the focal date is taken to be the end of the contract, we must make an adaptation, which was suggested in Lachtermacher and de Faro (2023). In this case, the principal of the k-th subcontract, denoted as F_k, will be:

$$F_k = P_k \times \left[1 + i \times (n-k)\right] / (1 + i \times n), \quad \text{for } k = 1,2,\dots,n \tag{7}$$

In this manner we are assured that the sum of the principal of the n subcontracts will be exactly equal to the principal F of the original single contract.

As for the component of amortization that is associated with the k-th subcontract, we will have:

$$\bar{A}_k = F_k, \quad \text{for } k = 1,2,\dots,n \tag{8}$$

On the other hand, and this is the key factor which justifies the financing institution's gain for substituting a single contract by n subcontracts, the component of interest, associated with the k-th subcontract, denoted as $\bar{J}_k$, will now be:

$$\bar{J}_k = P_k - \bar{A}_k = P_k \times \left[1 - \frac{1+i\times(n-k)}{1+i\times n}\right], \quad \text{for } k=1,2,\ldots,n \tag{9}$$

Therefore, considering our simple numerical example of section 3, Table 2 presents the sequence of the 12 constant payments, which is the same in both the case of a single contract, as well in the case of the 12 individual subcontracts.

Additionally, Table 2 also presents the sequences J_k and $\bar{J}_k$, as well as the sequence of differences $d_k = J_k - \bar{J}_k$.

As previously pointed out, it should be noted that the original debt of 10,000 units of capital is fully amortized, since:

$$\sum_{k=1}^{n} F_k = \sum_{k=1}^{n} \bar{A}_k = F \tag{10}$$

in this case with $n=12$.

Formally, however, we do not have a system of constant amortization anymore as the parcels of amortization $\bar{A}_k$ are not equal.

Table 2: The Sequences of the Parcels of Interest and its Differences (Focal Date = n)

	Multiple Contracts				
k	F_k	$\bar{A}_k$	$\bar{J}_k$	P_k	d_k
1	918.23	918.23	8.27	926.50	84.90
2	902.33	902.33	16.41	918.74	69.00
3	886.57	886.57	24.40	910.97	53.24
4	870.95	870.95	32.26	903.21	37.62
5	855.47	855.47	39.98	895.45	22.14
6	840.92	840.92	47.55	887.68	6.79
7	824.92	824.92	54.99	879.92	-8.41
8	809.86	809.86	62.30	872.15	-23.48
9	794.93	794.93	69.46	864.39	-38.40
10	780.14	780.14	76.48	856.63	-53.19
11	765.49	765.49	83.37	848.86	-67.84
12	750.98	750.98	90.12	841.10	-82.35
Σ		10,000.00	605.59	10,605.59	0,00

Strictly from an accounting point of view, there is no gain for the financial institution granting the loan if a single contract is substituted by multiple contracts, since the sums of the corresponding parcels of interest is the same. That is:

$$\sum_{k=1}^{12} J_k = \sum_{k=1}^{12} \bar{J}_k = 605.59 \quad \text{units of capital.}$$

Yet, depending on the opportunity cost of the financial institution, which will be denoted as ρ, and is usually of compound interest, and which is supposed to be relative to the same period of the simple interest rate, *i*, that is being charged, the financial institution may derive substantial gains in terms of tax deductions.

In other words, it is possible that:

$$V_1(\rho)=\sum_{k=1}^{n} J_k \times (1+\rho)^{-k} > V_2(\rho)=\sum_{k=1}^{n} \bar{J}_k \times (1+\rho)^{-k} \quad (11)$$

where $V_1(\rho)$ denotes the present value, at the rate ρ of the sequence of the parcels of interest in the case of a single contract, and $V_2(\rho)$ denotes the corresponding present value in the case of the adoption of multiple contracts.

Moreover, at least in the case of our simple numerical example, as the sequence d_k of differences has only one change of sign, thus characterizing what is termed a conventional financing project, cf. de Faro (1974), whose internal rate of return is unique, and in this particular case is null, it follows that $\Delta = V_1(\rho) - V_2(\rho) > 0$ if $\rho > 0$.

Figure 1 outlines the evolution of Δ , for $0 \leq \rho \leq 5\%$ per period, for F = 10.000 units of capital and n = 12. Additionally, we also have the evolution of Δ, where the simple interest rate, *i,* is equal to 0.5%, 1%, 1.5%, 2%, 2.5% and 3% per period.

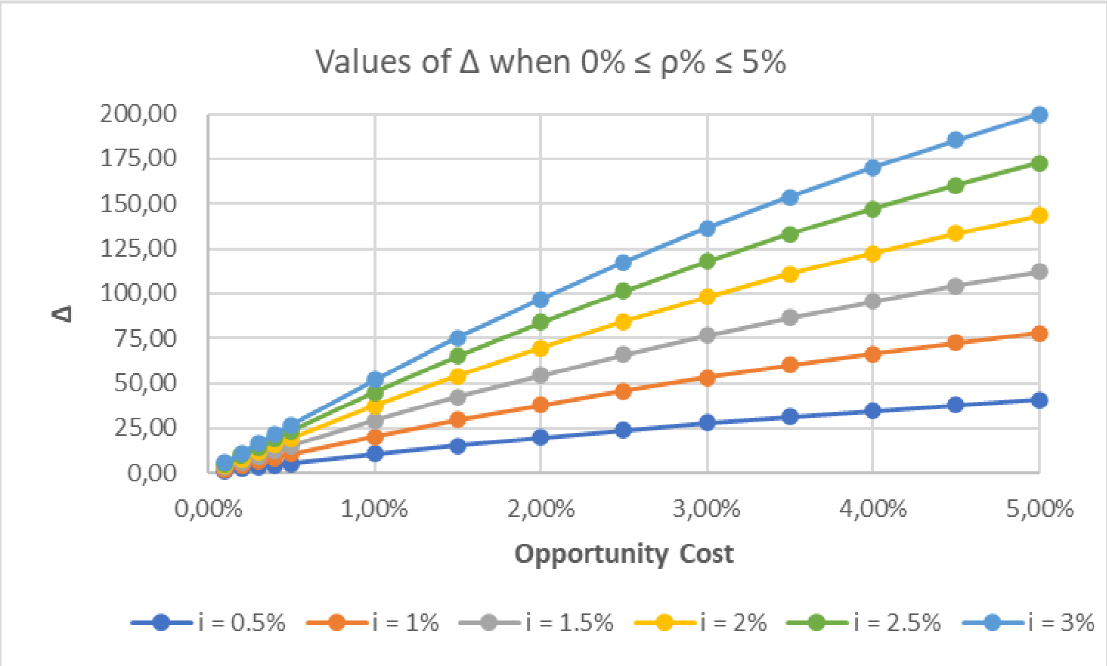

Figure 1: Values of Δ , F =10,000 units of capital and n =12

V. GENERAL ANALYSIS - DATA FOCAL AT TIME *N*

In the previous section, focusing attention on our simple example, with only 12 periods, it was verified that the sequence of differences of the interest payments present just one change of sign, thereby assuring us of the uniqueness of the corresponding internal rate of return, which is known to be null.

Furthermore, this inference appears to always be true, as supported by the evidence provided in Figure 2, which presents the evolution of the sequence d_k for the case where $F = 1,200,000$ units of capital of a single contract with 180 periods and with the simple interest rate, *i,* being as high as 3% per period.

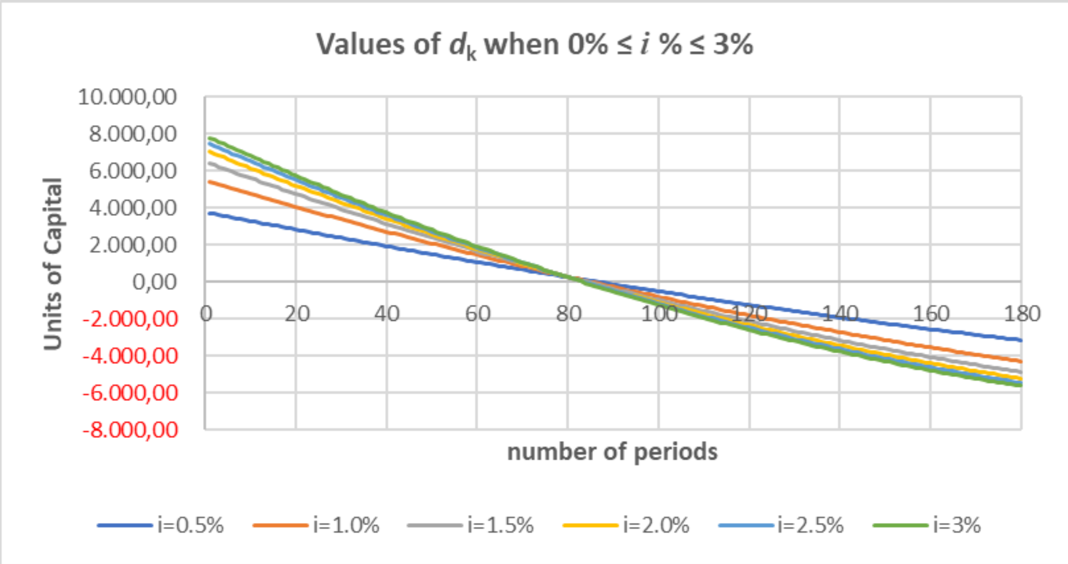

Figure 2: Values of d_k , F =1,200,000 units of capital and n =180.

Consequently, it can be inferred that the financing institution is always better off if a single contract is substituted by multiple contracts - one for each one of the n payments of the original contract.

Taking into account that in Brazil the monthly interest rates charged do not exceed 2% per month, in real terms, we are going to analyze the behavior of the percentage increase of the fiscal gain $\delta = \left[V_1(\rho)/V_2(\rho) - 1\right] \times 100$ for some values of the corresponding annual opportunity cost ρ_a , with each contract with a term of n_a years.

Table 3: Fiscal gain δ ,the end term of the contract as the focal date , i = 0.5%p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	7.9719	16.1314	24.4507	32.9031	41.4632	50.1072
10	16.2389	33.9052	52.8273	72.8069	93.6337	115.0988
15	24.8052	53.2265	84.7246	118.6077	154.1421	190.6437
20	33.7337	74.1080	119.7820	169.0355	220.2051	271.9532
25	43.0539	96.4714	157.4421	222.6120	289.1879	355.3285
30	52.7773	120.1807	197.0807	278.0298	359.2563	438.7897

Table 4: Fiscal gain δ , the end term of the contract as the focal date , i = 1.0%p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	7.7552	15.6718	23.7228	31.8826	40.1269	48.4330
10	15.6890	32.6551	50.7238	69.7007	89.3852	109.5827
15	23.9269	51.1075	80.9942	112.9219	146.2102	180.2417
20	32.5457	71.0942	114.3113	160.5803	208.3928	256.5663
25	41.5726	92.5563	150.2096	211.4310	273.7177	335.4564
30	51.0157	115.3712	188.1381	264.3247	340.5672	415.1392

Table 5: Fiscal gain δ , the end term of the contract as the focal date , i = 1.5%p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	7.6074	15.3594	23.2297	31.1934	39.2270	47.3088
10	15.3771	31.9511	49.5475	67.9753	87.0402	106.5558
15	23.4807	50.0420	79.1365	110.1156	142.3264	175.1843
20	31.9849	69.6895	111.7900	156.7216	203.0461	249.6486
25	40.9094	90.8278	147.0548	206.6009	267.0851	326.9859
30	50.2576	113.3324	184.3928	258.6363	332.8607	405.4327

Table 6: Fiscal gain δ , the end term of the contract as the focal date , i = 2.0%p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	7.5001	15.1333	22.8736	30.6966	38.5797	46.5019
10	15.1762	31.4996	48.7962	66.8775	85.5535	104.6434
15	23.2107	49.4007	78.0243	108.4433	140.0218	172.1943
20	31.6584	68.8766	110.3394	154.5123	199.9972	245.7168
25	40.5332	89.8540	145.2875	203.9075	263.3997	322.2919
30	49.8357	112.2056	182.3341	255.5224	328.6544	400.1461

As shown in Tables 3 to 6, the fiscal gain is substantial, although it decreases when the interest rate, i , increases.

VI. AN ALTERNATIVE

As pointed out in section 2, the Rovina's Proposal, even though supported by several judicial sentences, violates a still prevailing Brazilian law promulgated in 1964.

Therefore, it seems more adequate to also consider the case where the focal date coincides with the date of the contract, which is the date stipulated in the 1964 law.

In this case, the value of the k-th payment, now denoted as $\widehat{P}_k$, must satisfy the following equation:

$$F = \sum_{k=1}^{n} \frac{\hat{P}_k}{1 + i \times k} \qquad (12)$$

Before proceeding, it should be pointed out, as shown in de Faro (2014), that we will have $\hat{P}_1 > P_1$, if $n \geq 2$.

For the special case under scrutiny, where we are assuming constant amortization, it is appropriate to make use of the work of Forger (2009), which introduced the concepts of capitalized and non-capitalized components.

Following the above-mentioned reference, the parcel of interest at time k, now denoted as $\hat{J}_k$, is:

$$\hat{J}_k = F \times f \times i \times (n - k + 1) / n, \text{ for } k = 1, 2, \ldots, n \qquad (13)$$

where f is a weigh factor which decomposes de principal F in a capitalized component, F^C, and in a non-capitalized component, F^N; $0 \leq f \leq 1$.

For our case where the focal date is time zero, there is, in general, no analytical solution to equation (12). It is necessary to make use of an algorithm as the one suggested in de Faro (2014), or the one proposed in Lachtermacher and de Faro (2022), which also provides the value of f.

Considering the simple numerical example of section 3, and taking into account that the value of the weigh factor can be determined to be $f = 0.966126423$, Table 5 presents the sequence of payments that represents the solution of equation (12), as well as the sequence of the parcels of interest, as given by expression (13).

It should be noted that, as the parcel of amortization $\hat{A}_k$ is equal to the difference $\hat{P}_k - \hat{J}_k$, for all k, we are satisfying the requisite of constant amortization as shown in Table 5.

Additionally, Table 7 also presents the evolution of the outstanding debt $\hat{S}_k$ which decreases linearly.

Table 7: Evolution of the Payments and of the Parcels of Interest (Focal Date = 0)

k	$\hat{J}_k$	$\hat{A}_k$	$\hat{P}_k$	$\hat{S}_k$
0				10,000.00
1	96.61	833.33	929.95	9,166.67
2	88.56	833.33	921.89	8,333.33
3	80.51	833.33	913.84	7,500.00
4	72.46	833.33	905.79	6,666.67
5	64.41	833.33	897.74	5,833.33
6	56.36	833.33	889.69	5,000.00
7	48.31	833.33	881.64	4,166.67
8	40.26	833.33	873.59	3,333.33
9	32.20	833.33	865.54	2,500.00
10	24.15	833.33	857.49	1,666.67
11	16.10	833.33	849.44	833.33

12	8.05	833.33	841.38	0.00
$\sum$	627.98	10,000.00	10,627.98	

Before proceeding, it should be noted that, as shown in Lachtermacher and de Faro (2022), the Forger (2009) procedure also satisfies the concept of financial consistency as proposed in de Faro (2014). That is, the determination of the outstanding debt at any point in time can be achieved by any of the classical methods.

VII. MULTIPLE CONTRACTS IN THE CASE OF FOCAL DATE AT TIME ZERO

In this case, analogously to the case where the interest rate *i* is of compound interest, the principal of the *k*-th subcontract, now denoted as $\widehat{F'}_k$, is taken to be equal to the present value, now at the rate *i* of simple interest of the *k*-th payment of the single contract.

That is:

$$\hat{F}'_k = \frac{\hat{P}_k}{1+i\times k} \quad , \qquad \text{for } k=1,2\ldots\ ,n \tag{14}$$

with the corresponding parcel of amortization, now denoted as $\widehat{A'}_k$, being exactly equal to the principal of the subcontract. Namely:

$$\hat{A}'_k = \hat{F}'_k \quad , \quad \text{for } k=1,2,\ldots\ ,n \tag{15}$$

as for the corresponding parcel of interest, denoted as $\widehat{J'}_k$, since $J'_k = \widehat{P}_k - \widehat{A'}_k$, we will have:

$$\hat{J}'_k = \frac{\hat{P}_k \times i \times k}{1+i\times k} \quad , \quad \text{for } k=1,2\ldots\ ,n \tag{16}$$

In Table 8, still considering our simple numerical example, we show the sequences of values of the constant payment $\widehat{P}_k$, the sequence of the interest payments $\widehat{J'}_k$, as well as the sequence $\widehat{J}_k$, and the sequence of the values of the differences $d'_k = \widehat{J}_k - \widehat{J'}_k$.

Table 8: The Sequences of the Parcels of Interest and its Differences (Focal Date = 0)

	Multiple Contracts				
k	$F'_k = \hat{A}'_k$	$\hat{P}_k$	$\hat{J}'_k$	$\hat{J}_k$	d'_k
1	920.74	929.95	9.21	96.61	87.41
2	903.82	921.89	18.08	88.56	70.49
3	887.23	913.84	26.62	80.51	53.89
4	870.95	905.79	34.84	72.46	37.62
5	854.99	897.74	42.75	64.41	21.66
6	839.33	889.69	50.36	56.36	6.00
7	823.96	881.64	57.68	48.31	-9.37
8	808.88	873.59	64.71	40.26	-24.45

9	794.07	865.54	71.47	32.20	-39.26
10	779.53	857.49	77.95	24.15	-53.80
11	765.26	849.44	84.18	16.10	-68.08
12	751.24	841.38	90.15	8.05	-82.10
$\sum$	10,000.00	10,627.98	627.98	627.98	0,00

Comparatively to the case where the focal date is the end period of the contract, we have the same amount for the total of the interest payments.

However, the financing institution may likewise derive substantial gains in terms of tax deductions.

This is because, denoting by $V'_1(\rho)$ the present value, at the interest rate ρ in the case of a single contract, and by $V'_2(\rho)$ the corresponding present value in the case of multiple contracts, it is possible to have:

$$V_1'(\rho)=\sum_{k=1}^{n} J_k' \times (1+\rho)^{-k} > V_2'(\rho)=\sum_{k=1}^{n} \hat{J}_k' \times (1+\rho)^{-k} \tag{17}$$

Moreover, in this case of our simple numerical example, and in several others cases, with different values of i, n and F tested, the sequence d'_k of differences also characterizes a conventional project, whose internal rate of return is unique, and which in this particular case is null, it follows that $\Delta = V'_1(\rho) - V'_2(\rho) > 0$ if $\rho > 0$.

Figure 3 outlines the evolution of Δ', not only when $i = 1\%$ per period, but also when i assumes the values of 0.5%, 1.5%, 2% and 3%, and when $0 \le \rho \le 5\%$per period.

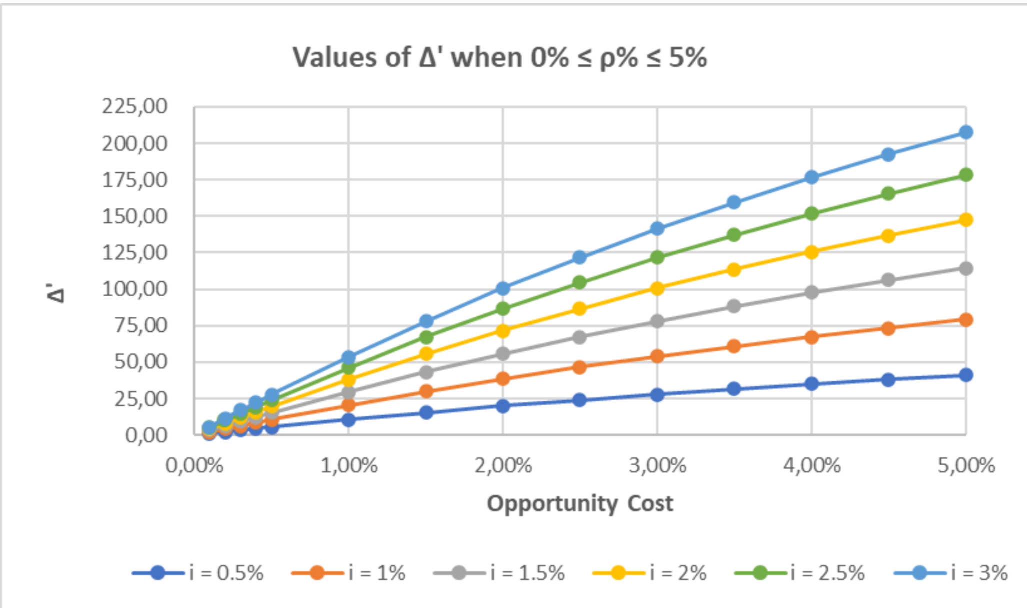

Figure 3: Values of Δ' , F =10,000 units of capital and n =12

Hence, as it was seen in the case where the focal date is time n, the value of Δ' increases both regarding i and ρ.

VIII. GENERAL ANALYSIS - DATA FOCAL AT TIME ZERO

As illustrated in Figure 4, which concerns the case where $n = 180$ periods, we also have just one change of sign in the sequence d'_k.

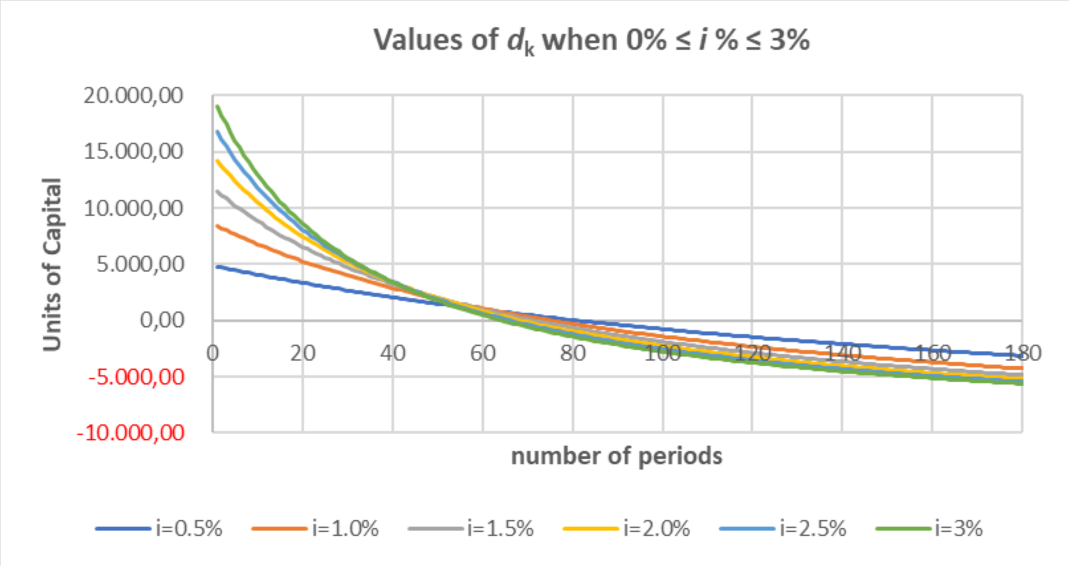

Figure 4: Values of d'_k F =1,200,000 units of capital and n =180.

Consequently, we have a clear indication that we always have $\Delta' > 0$. This means that, also in this case, the financing institution should prefer to use multiple contracts instead of a single contract.

To give a numerical evidence of the values of the fiscal gain, Tables 9, 10, 11, and 12 provide the percentage values of the increase of the fiscal gain $\delta' = \left[V'_1(\rho)/V'_2(\rho) - 1\right] \times 100$, where ρ_a expresses the annual value of the opportunity cost, and where n_a expresses the length of the contract in years.

Table 9: Fiscal gain δ', beginning of the contract as the focal date , i = 0.5% p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	7.5385	15.2110	22.9912	30.8542	38.7769	46.7380
10	14.4790	29.9006	46.0843	62.8441	80.0004	97.3886
15	20.8174	43.6344	67.8898	93.0180	118.5191	143.9946
20	26.6334	56.2731	87.6870	119.7630	151.6785	182.9162
25	31.9818	67.7404	105.1330	142.5263	178.9776	214.0710
30	36.9046	78.0270	120.2101	161.5131	201.1316	238.8904

Table 10: Fiscal gain δ', beginning of the contract as the focal date , i = 1.0% p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	6.9406	13.9511	21.0079	28.0897	35.1771	42.2530
10	12.6124	25.7559	39.2682	52.9979	66.8105	80.5926

15	17.3770	35.7323	54.6099	73.6175	92.4539	110.9110
20	21.4769	44.1958	67.2778	90.0806	112.2184	133.5060
25	25.0559	51.3787	77.6256	103.0167	127.2335	150.2221
30	28.2097	57.4769	86.0521	113.1885	138.7521	162.8518

Table 11: Fiscal gain δ' , beginning of the contract as the focal date , i = 1.5% p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	6.4621	12.9496	19.4423	25.9218	32.3724	38.7801
10	11.2941	22.8849	34.6338	46.4190	58.1392	69.7143
15	15.1473	30.7727	46.5152	62.0946	77.3177	92.0672
20	18.3449	37.1686	55.8321	73.9320	91.2690	107.7805
25	21.0602	42.4137	63.1610	82.8826	101.4772	118.9960
30	23.4010	46.7501	68.9679	89.7395	109.1230	127.2862

Table 12: Fiscal gain δ' , beginning of the contract as the focal date , i = 2 % p.m.

	$\rho_a(\%)$					
n_a	5%	10%	15%	20%	25%	30%
5	6.0665	12.1266	18.1627	24.1594	30.1038	35.9848
10	10.2954	20.7404	31.2173	41.6279	51.8947	61.9600
15	13.5477	27.2897	40.9374	54.2848	67.2032	79.6259
20	16.1830	32.4468	48.3122	63.5134	77.9472	91.6081
25	18.3820	36.5870	53.9823	70.3341	85.6390	99.9876
30	20.2520	39.9559	58.4040	75.4830	91.3246	106.1087

Similar to the case of the focal date at the end of the financing period, the fiscal gains are also substantial, although they also decrease when the interest rate, i,is increased.

IX. CONCLUSIONS

Given that, repeatedly, the Brazilian judicial system has determined that home-financing contracts written in terms of compound interest should be substituted by contracts making use of simple interest, we have analyzed the possibility that the financing institution granting the loan decides to substitute a single contract by n subcontracts - one for each one of the payments of the single contract.

Focusing attention on the case of constant amortization, which is one of the most employed, and which in Brazil is known as "SAC," we have concluded that the financing institution granting the loan should always prefer the multiple contracts option since this can result in significant tax gains.

It was shown that the tax gains are higher if the focal date is the end period of the contract, which is the usual case, even though it violates a Brazilian law of 1964.

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