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M. A. Gopalan, S. Vidhyalakshmi & J. Shanthi

Shrimati Indira Gandhi College

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On the Diophantine Equation $x^2+axy+by^2=z^2$

M. A. Gopalan[α], S. Vidhyalakshmi[σ] & J. Shanthi[ρ]

I. ABSTRACT

A new and different set of solutions is obtained for the ternary quadratic diophantine equation $x^2+axy+by^2=z^2$ *through representing it as a system of double equations.*

Keywords: ternary quadratic, system of double equations, integer solutions.

Author α σ: Professor, Department of Mathematics, Shrimati Indira Gandhi College, Trichy-620 002, Tamil Nadu, India.
ρ: Associate Professor, Department of Mathematics, Shrimati Indira Gandhi College, Trichy-620 002, Tamil Nadu, India.

II. INTRODUCTION

The diophantine equation of the form $cx^2+axy+by^2=dz^2$ where a, b, c, d are non-zero integers has been discussed by several authors [1-3]. In [4-14], integer solutions to the above equation are presented when a, b, c, d take particular numerical values. In this communication, different sets of integer solutions to the above equation are obtained when $c=d=1$ by representing it as a system of double equations involving trigonometric functions. It seems that they have not been presented earlier.

III. METHOD OF ANALYSIS

The diophantine equation under consideration is

$$x^2+axy+by^2=z^2 \tag{1}$$

The above equation is represented as the system of double equations as below:

System	1	2	3
$z+x$	$(ax+by)\sec\theta$	$y\cot\theta$	$(ax+by)\cot\theta$
$z-x$	$y\cos\theta$	$(ax+by)\tan\theta$	$y\tan\theta$

Consider system: 1

Elimination of z leads to

$$x=t\left(b-\cos^2\theta\right),\quad y=t\left(2\cos\theta-a\right) \tag{2}$$

Case: 1

Assume

$$\cos\theta=\frac{2pq}{\sqrt{t}},\quad t=\left(p^2+q^2\right)^2 \tag{3}$$

Substituting (3) in (2), we have

$$x=b\left(p^4+q^4\right)+2p^2q^2(b-2)$$

$$y=\left(p^2+q^2\right)\left(4pq-a\left(p^2+q^2\right)\right)$$

and from the given system

$$z=b\left(p^4+q^4\right)+2p^2q^2(b+2)-2apq\left(p^2+q^2\right)$$

Case: 2

Assume

$$\sin\theta = \frac{2pq}{\sqrt{t}}\ ,\ t = (p^2+q^2)^2 \qquad (4)$$

Substituting (4) in (2), we have

$$x = (b-1)(p^4+q^4) + 2p^2q^2(b+1)$$

$$y = (2-a)p^4 - (2+a)q^4 - 2ap^2q^2$$

and from the given system

$$z = (b-a+1)p^4 + (b+a+1)q^4 + 2p^2q^2(b-1)$$

Consider system: 2

Elimination of z leads to

$$x = t(\cos^2\theta - b\sin^2\theta)$$

$$y = t(2\sin\theta\cos\theta + a\sin^2\theta)$$

Case: 3

Assuming (3), the corresponding values of x, y, z are given by

$$x = 2p^2q^2(b+2) - b(p^4+q^4)$$

$$y = (p^2-q^2)\left(a(p^2-q^2)+4pq\right)$$

$$z = b(p^4+q^4) + 2p^2q^2(2-b) + 2apq(p^2-q^2)$$

Case: 4

The assumption (4) leads to

$$x = p^4+q^4-2p^2q^2(1+2b)$$

$$y = 4pq(p^2-q^2+apq)$$

$$z = p^4+q^4+2p^2q^2(2b-1) + 2apq(p^2-q^2)$$

Consider system: 3

Elimination of z leads to

$$\left.\begin{aligned} x &= t(b\cos^2\theta - \sin^2\theta) \\ y &= t(2\sin\theta\cos\theta - a\cos^2\theta) \end{aligned}\right\} \qquad (5)$$

Case: 5

Substituting (3) in (5), we have

$$x = 2p^2q^2(2b+1) - (p^4+q^4)$$

$$y = 4pq(p^2-q^2-apq)$$

and from the given system

$$z = p^4+q^4+2p^2q^2(2b-1) - 2apq(p^2-q^2)$$

Case: 6

Assuming (4), the corresponding values of x, y, z are found to be

$$x = b(p^4+q^4) - 2(b+2)p^2q^2$$

$$y = (p^2-q^2)\left(4pq - a(p^2-q^2)\right)$$

$$z = b(p^4+q^4) + 2p^2q^2(2-b) - 2apq(p^2-q^2)$$

It is worth to mention that the above solutions are different from the solutions presented in [15].

IV. GENERATION OF SOLUTION

If (x_0, y_0, z_0) is a given solution of (1), then a second solution is found by setting

$$x_1 = x_0 + h\ ,\ y_1 = y_0 + h\ ,\ z_1 = z_0 + h$$

Then h is determined rationally and the second solution is represented by

$$x_1 = (b-2)x_0 - (a+2b)y_0 + 2z_0$$

$$y_1 = -(a+2)x_0 - b\,y_0 + 2z_0$$

$$z_1 = -(a+2)x_0 - (a+2b)y_0 + (a+b+2)z_0$$

The repetition of the above process leads to the generation of sequence of solutions to (1).

In conclusion, one may obtain different choices of solutions of (1) by choosing alternative forms of the system of equations or by any other method.

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