The Underlying Vectors and Surfaces with the Same Timescales as Causes of Order Emergence in the Turbulent Flows

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The paper shows that vectors of the integrated mass and energy flow set the surfaces, on which the oscillatory processes have the same timescales or frequencies. Consequently, there is introduced the concept of synchronous surfaces. The correlation vector of density and velocity lies on the same surface, which indicates that chaotic motions are conceived and relatively steadily exist on such surfaces. The partially ordered (coherent) structures may also be conceived on the similar surfaces.

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The paper shows that vectors of the integrated mass and energy flow set the surfaces, on which the oscillatory processes have the same timescales or frequencies. Consequently, there is introduced the concept of synchronous surfaces. The correlation vector of density and velocity lies on the same surface, which indicates that chaotic motions are conceived and relatively steadily exist on such surfaces. The partially ordered (coherent) structures may also be conceived on the similar surfaces.

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I. OBTAINING THE DIFFERENTIAL EQUATIONS OF CONSERVATION BY INTRODUCING THE INTEGRATED FLOWS.

With the known values of the flow thermodynamic parameters (pressure, temperature, density) and rate (W), at an arbitrary point in space, the instantaneous values of the mass and energy flow vectors shall be set:

$$g = \rho W \ , \tag{1.1}$$

$$e_{\Sigma} = \rho W[c_V T + P/\rho] + \rho W W^2/2 - W\sigma(W) - \lambda grad T = e_i + e_K + e_\mu + q \tag{1.2}$$

where

e_i - the instantaneous enthalpy flow;

e_K - the instantaneous kinetic energy flow;

e_μ - the instantaneous flow of energy of viscose surface force;

q - the instantaneous heat flow.

$\sigma(W)$ - viscous stress tensor.

By integrating (1.1-1.2) during a time interval τ_0, it is possible to determine the values of the integrated flows or the averaged vectors of mass and energy:

$$\overline{g_\tau} = \frac{1}{\tau_0}\int_0^{\tau_0} \rho W dt \ , \tag{1.3}$$

$$\overline{e_{\Sigma\tau}} = \frac{1}{\tau_0}\int_0^{\tau_0} e_\Sigma dt \ . \tag{1.4}$$

If we have stationary turbulence, then, after the completion of one cycle, all flow parameters should take their initial value and, accordingly, the condition of conservation of the integrated (or averaged) fluxes must be met, which has the following form:

$$div\overline{g_\tau} = 0 \ , \tag{1.5}$$

$$div\overline{e_{\Sigma\tau}} = 0 \ . \tag{1.6}$$

Thus, on the basis of the condition of conservation of the integrated flows, we obtain the averaged differential conservation equations in the form of 1.5–1.6. At the same time, the averaged conservation equations are also obtained by direct integration of differential equations of a continuous medium.

II. OBTAINING THE AVERAGED DIFFERENTIAL EQUATIONS OF CONSERVATION BY INTEGRATING THE DIFFERENTIAL EQUATIONS OF MASS AND ENERGY CONSERVATION.

Consider the differential equations of mass and energy conservation:

$$\partial\rho/\partial t + div\rho W = 0 , \qquad (2.1)$$

$$\partial[\rho c_V T + \rho W^2/2]/\partial t + div[e_\Sigma] = 0 . \qquad (2.2)$$

If we integrate these equations within a cyclic process, under conditions of variables over the space of timescales

$(A = -grad \ln\tau_0 = grad \ln f \neq 0)$, we obtain the different equations:

$$div\overline{g}_\tau = \overline{g_\tau A} , \qquad (2.3)$$

$$div\overline{e_{\Sigma\tau}} = \overline{e_{\Sigma\tau} A} .$$

A comparison of these equations with equations 1.5-1.6. leads to the conclusion:

$$\overline{g}_\tau A = 0 , \qquad (2.5)$$

$$\overline{e}_{\Sigma\tau} A = 0 \qquad (2.6)$$

As will be seen later, the equations obtained can play an important role in the search for a closed equation system for solving the problem of turbulence.

If we determine the instantaneous values of density and velocity as the sum of the average value and pulsation ($\rho = \overline{\rho} + \rho', W = V + w$), then the averaged value of the mass flux vector will be as follows:

$$\overline{g}_\tau = \overline{\rho}V + \frac{1}{\tau_0}\int_0^{\tau_0} \rho' w dt , \qquad (2.7)$$

Or:

$$\overline{g}_\tau = \overline{\rho}V + \overline{\rho}A_\rho , \qquad (2.8)$$

where A_ρ expresses some vectoral value, which reflects mass propagation due to the velocity pulsation - or the velocity of the turbulent mass propagation.

$$A_\rho = \frac{1}{\overline{\rho}\tau_0}\int_0^{\tau_0} \rho' w = \frac{\overline{\rho' w}}{\overline{\rho}} , \qquad (2.9)$$

As we can see, if there is a correlation between the velocity and density pulsations, then the mass flux depends not only on the average velocity vector - V, but also on the velocity vector of the turbulent mass diffusion - A_ρ. In the incompressible fluid flows, this term is understandably eliminated, but, as will be seen later, in the turbulent compressible fluid flows, this quantity has a clearly defined value.

The orthogonality of the velocity vector and timescale gradient **(** $AV = 0$ **)**

Consider a differential equation of continuity in another form:

$$\rho divW + Wgrad\rho = -\partial\rho/\partial t , \qquad (3.1)$$

Imagine the velocity and density in the form of the sum of the average value and pulsation and implement the time integration within a cycle:

$$\overline{\rho div(V+w)} + \overline{\rho' divw} + \overline{\rho' divV} + V\overline{grad\rho} + \overline{wgrad\rho} = 0 , \qquad (3.2)$$

We shall take into account that, according to the rules of integration, at $A \neq 0$, the following conditions are correct:

$$\overline{div(V+w)} = divV - AV , \qquad (3.3)$$

$$\overline{\rho' divV} = \overline{\rho'} divV = 0 , \qquad (3.4)$$

$$\overline{wgrad\rho} = \overline{wgrad(\overline{\rho}+\rho')} = \overline{wgrad\rho'} . \qquad (3.5)$$

Respectively, (3.2) will be as follows:

$$\overline{\rho} divV - \overline{\rho} AV + V\overline{grad\rho} + \overline{\rho' divw} + \overline{wgrad\rho'} = 0 \qquad (3.6)$$

or:

$$\overline{\rho} divV + V\overline{grad\rho} - \overline{\rho}AV + \overline{div\rho'w} = 0 \, . \quad (3.7)$$

We shall take into account that:

$$V\overline{grad\rho} = Vgrad\,\overline{\rho} - \overline{\rho}AV \, , \quad (3.8)$$

$$\overline{div\rho'w} = div\,\overline{\rho'w} - \overline{\rho'w}A \, . \quad (3.9)$$

Respectively, (3.7) yields:

$$div\,\overline{\rho}V + div\,\overline{\rho'w} - \overline{\rho'w}A - 2\overline{\rho}AV = 0 \, , \quad (3.10)$$

Applying the definition of the averaged mass flow vector 2.8, the latter expression yields:

$$div\,\overline{g}_\tau - \overline{g}_\tau A - \overline{\rho}AV = 0 \, , \quad (3.11)$$

And with provision for 2.3 - 2.5, we obtain:

$$AV = 0 \, , \quad (3.12)$$

or

$$Vgrad\tau_0 = \frac{d\tau_0}{dt} = 0 \, , \quad (3.13)$$

Thus, on the basis of a fundamentally different approach, a more general result was obtained, than it was shown when analyzing one-dimensional pulsating flows [2], which indicates that the timescale (or main frequency) along the jet remains unchanged.

Taking into account the condition (2.3), we obtain that the correlation vector A_ρ is also oriented perpendicular to the frequency gradient:

$$\overline{\rho'w}A = 0 \, , \quad (3.14)$$

III. THE SURFACES OF THE SAME TIMESCALES OR THE SYNCHRONOUS SURFACES, SET BY THE DIRECTIONS OF PROPAGATING THE PRINCIPLE SUBSTANCES - PRIORITY AREAS OF TURBULENT MOTIONS

Thus, we have obtained the equations indicating that the timescale gradients are orthogonal to three vectors simultaneously.

$$VA = 0 \, , \quad (4.1)$$

$$A_\rho A = 0 \, , \quad (4.2)$$

$$\overline{e}_{\Sigma\tau}A = 0 \, , \quad (4.3)$$

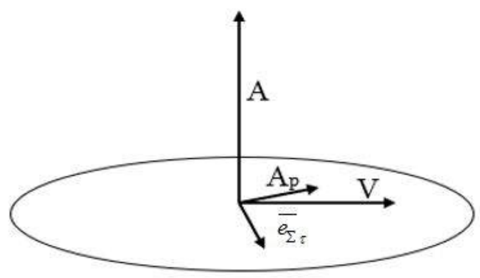

Fig. 1: *Average velocity V, vector of the cumulative flow of energy $\overline{e}_{\Sigma\tau}$ and correlation A_ρ on a plane orthogonal to the timescale gradients A.*

The conditions (4.1-4.3) can be met in the following different cases:

A. $A = 0$ - no timescale gradients. The presence of this condition in a compressible medium can be considered to be the unlikely special case.

B. The vectors V, A_ρ, $\overline{e_{\Sigma\tau}}$ are arranged on the same plane (Fig. 1). Naturally, the vector can be perpendicular to three vectors, simultaneously, only if these vectors are arranged on the same plane. This situation can be considered to be the most common characteristic of the compressible media.

As we can see, vectors of the mass and energy flows set such surfaces, on which the timescales (or the frequencies) are the same. Therefore, they can be called the synchronous surfaces. Obviously, on the surfaces of synchronous oscillations, we should expect synchronization of processes. Consequently, more ordered motions or the turbulent structures with the relatively large scales (and, accordingly, the large timescales) may be conceived on such surfaces. In the theory of turbulence, such structures are called coherent, and the turbulence itself is considered to be a mixture of chaos and order, in which, against the background of regular disordered motions, periodically, spontaneously or regularly, there are appeared the ordered structures, the development of which, to some extent, has an autonomous character.

Consequently, vectors of the mass and energy flows set the special surfaces, which determine the priority directions of motion with identical frequencies, which, in turn, contributes to the appearance of order in the depth of chaotic motion.

The fact that the correlation vector A_ρ lies also on this plane, once again indicates that turbulent oscillations occur mostly in this plane. Moreover, this feature allows for determining the dependence of A_ρ on V and $\overline{e_{\Sigma\tau}}$, what will be shown later.

Analysis of stress tensors, which depend on 6 scalar values, shows that for their mathematical description it is enough to introduce two base vectors (or major axes), which, in the stationary turbulent flows, also set some stable reference surfaces. As is known, in the system of such axes, some components of tensor take a zero value, and this fact also indicates that the structure of turbulence is always associated with anisotropy due to the presence of the priority directions and elements of order in a chaotic process.

Based on the above, it can be said that not only the ordered coherent structures, but also the smallest turbulent vortices, are conceived on the certain surfaces in a partially ordered form, and the deviation from such a pattern can be explained only by the fact that the shape and orientation of the synchronous surfaces can vary not only in space, but also in time, since the stationary turbulent processes, do not exist in their pure form, and, due to variation of V and $\overline{e_{\Sigma\tau}}$ not only in space, but also in time (in the strict sense), a stable process of conceiving of the vortex structures, with the same spatial orientation, at a fixed point of space is obviously difficult to be observed.

The issues of representation of the turbulent stress tensor based on the underlying vectors will be considered in a separate section.

IV. CONCLUSION

The vectors of the integrated mass and energy flow set the surfaces directed orthogonally to the timescale gradients, on which the oscillatory processes have the same timescales or frequencies - the synchronous surfaces. The correlation vector of density and velocity lies on the same surface, which indicates that chaotic motions are conceived and relatively steadily exist on such surfaces. The partially ordered (coherent) structures may also be conceived on the similar surfaces. Similarly to one-dimensional pulsating streams, in more complex flows timescale (or main frequency) along the jet remains unchanged.

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