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ABSTRACT

The minimal spanning tree (MST) algorithms by using the edges weights were presented mainly by Prim's and Kruskal's algorithms. In this article we use the weights for the bipolar neutrosophic edges by using the score functions with the new model algorithms namely bipolar neutrosophic Prim's algorithm and bipolar neutrosophic Kruskal's algorithm. Further, we use the score functions to get the more appropriate results based on the algorithms.

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The minimal spanning tree (MST) algorithms by using the edges weights were presented mainly by Prim's and Kruskal's algorithms. In this article we use the weights for the bipolar neutrosophic edges by using the score functions with the new model algorithms namely bipolar neutrosophic Prim's algorithm and bipolar neutrosophic Kruskal's algorithm. Further, we use the score functions to get the more appropriate results based on the algorithms.

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I. INTRODUCTION

The idea of Neutrosophic technique was derived by Smarandache [16]. These can abduct the ordinary circumstance of both deception as well as ambiguity that endure in genuine existence plots.

The concept of the triplet was derived by using a fuzzy set and intuitionistic fuzzy set. The triplet contains truth, indeterminacy and false numbers lies between 0 and 1. By using Fuzzy of type 1, intuitionistic fuzzy and conventional sets are NS. In NS there are three components called truth assign number indicate T and indeterminate assign number I, false assign number indicate F, separately.

Furthermore, in real life problems we can apply BNSs for the reference see [15]. Next they got an idea of BNS. By using this BNSs properties described by Wang et al.[19]. In order to deal with uncertainty, fuzzy concept proposed by Zadeh [22]

A rank method on IVIFSs with rank method studied by Nayagam et al. [9] and Mitchell [7]. The generalizations of score function on IVFSs introduced by Garg [3]. The weighted accuracy function and score function in IVIFSs studied by Xu [20] and Liu and Xie [6] the IVIFSs.

There different types of algorithms for finding the minimal sapping tree see the reference [12], in that mainly Prim's algorithm and Kruskal's algorithm.

The number of types of spanning tree problems contained to residential through a lot of researchers who assign the weights to edges are not accurate in that an uncertainty is there for that see the references [2, 4,5, 8 10, 11, 21,23].

In this manuscript the main aim is to find a minimal spanning tree from BNSCWG by using algorithms like Prim's and Kruskal's with different score functions.

II. PRELIMINARIES

The authors are researching some results based on the bipolar neutrosophic theory [1, 14, 13, 14, 16, 17, 18].

Explanation 2.1

Neutrosophic set contains the triplet numbers which are lies between 0 and 1. The summation of these three values present between the 0 and 3.

Example: $\langle 0.2, 0.5, 0.6 \rangle$ *is the neutrosophic set and the sum also shows* $0 \le 1.3 \le 3$.

Explanation 2.2

A Bipolar Neutrosophic set contains the six numbers which lie between the 0 and 1,-1 and 0. The summation of these three values present between the 0 and 3,-3 and 0 respectively.

Example: $\langle 0.1, 0.2, 0.3; -0.3, -0.1, -0.5 \rangle$ *is the bipolar neutrosophic set and the sum also shows* $0 \le 0.6 \le 3$ *and* $-3 \le -0.9 \le 0$

Explanation 2.3

The bipolar neutrosophic sets is denoted as following $A = < T^+, I^+, F^+; T^-, I^-, F^- >$ *and defined as the functions* $0 \le T^+ \le 1$ *is the truth number* $0 \le I^+ \le 1$ *is the indeterminacy number and* $0 \le F^+ \le 1$ *falsity number. The* T^+ , I^+ *and* F^+ *satisfies the condition* $0 \le T^+ + I^+ + F^+ \le 3$ *.Similarly the functions* $-1 \le T^- \le 0$ *is the truth number* $-1 \le I^- \le 0$ *is the indeterminacy number and* $-1 \le F^- \le 0$ *falsity number. The* T^- , I^- *and* F^- *satisfies the condition* $-3 \le T^- + I^- + F^- \le 0$

Explanation 2.4

A bipolar neutrosophic graph (BN-graph) with N_V *is explained by to be a two of a kind* (P, Q) *everywhere [0, 1] indicates the interval B.*

The functions $T_P : N_V \to B, I_P : N_V \to B$ *and* $F_P : N_V \to B$ *and* $0 \le T_P + I_P + F_P \le 3$ *for all vertices in* N_V *. Further,* *The functions* $T_Q : N_V \times N_V \to B, I_Q : N_V \times N_V \to B$ *and* $F_Q : N_V \times N_V \to B$ *are explained by* $T_Q(a_i,a_j) \le \min[T_Q(a_i), T_Q(a_j)], I_Q(a_i,a_j) \ge \max[I_Q(a_i), I_Q(a_j)]$

and $F_Q(a_i,a_j) \ge \max[F_Q(a_i), F_Q(a_j)]$ *with the condition* $0 \le T_B(a_i,a_j) + I_B(a_i,a_j) + F_B(a_i,a_j) \le 3$ *for all* $(a_i,a_j) \in E.$

Explanation 2.3

Let $A = (T^+, I^+, F^+; T^-, I^-, F^-)$ *be a BNs. Then a score function S is explained by* $S(A) = \frac{1}{6}\left[T^+ + 1 - I^+ + 1 - F^+ + 1 + T^- - I^- - F^-\right]$, *where* T^+, I^+ *and* F^+; T^-, I^- *and* F^- *corresponds to the truth number, indeterminacy number and falsity number lies between 0 and1, -1 and 0 respectively.*

III. MINIMAL SPANNING TREE ALGORITHM OF BIPOLAR NEUTROSOPHIC WEIGHTED GRAPH

This segment, an innovative description of minimum spanning tree algorithms, comes within reach of a new technique presented by using bipolar neutrosophic graph theory with edge weight.

In the subsequent, we put forward MST algorithm, with step by steps process given below:

We discuss all bipolar neutrosophic graphs that are connected with n vertices.

3.1 Minimal Spanning tree by using Prim's algorithm

Input: Bipolar Neutrosophic weighted graph.
Output: Minimal Spanning tree from the given bipolar neutrosophic weighted graph.

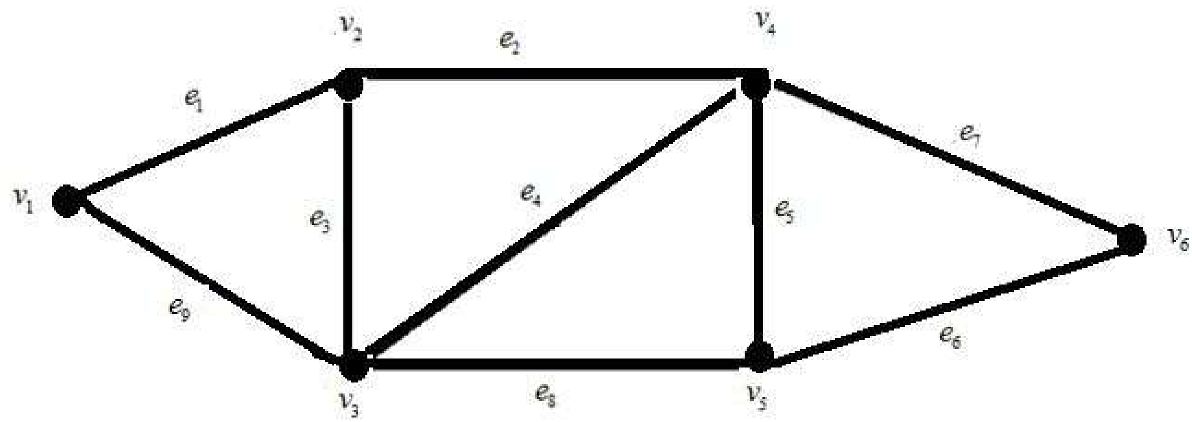
3.2 Bipolar Neutrosophic prim's algorithms

Step: 1 Compute the Adjacency matrix of the given bipolar neutrosophic graph.
Step:2 With help of the score function change the adjacency matrix to assign the scores of each edge.

Step:3 Starting from any arbitrary vertex v_i, $(i \ne j = 1, 2, \cdots n)$ *and connect it to its nearest neighbor vertex (that is to be vertex which has the smallest weight) say,* v_j $(i \ne j = 1, 2, \cdots n)$. *Now consider the edge* $\{v_i, v_j\}$ *and connect it to its closest neighbor (that is to a vertex other than* v_i *and* v_j, *that has the smallest weight among all entities) without forming a loop.*
Let this vertex be v_k *say.*
Step: 4 start from the vertex v_k *and repeat the process of step:3. Terminate the process after all n vertices connected with n-1 edges. These n-1 edges form a minimal bipolar neutrosophic spanning tree.*

3.3 Numerical example:

The following graph is the given bipolar neutrosophic connected weighted graph G



Weights of the given BNCWG are given in the table form,

Edges	*End vertices*	*weight of the edges*
e_1	v_1v_2	$(0.6,0.5,0.4;-0.1,-0.2,-0.3)$
e_2	v_2v_4	$(0.2,0.3,0.4;-0.2,-0.4,-0.3)$
e_3	v_2v_3	$(0.4,0.2,0.3;-0.3,-0.5,-0.7)$
e_4	v_3v_4	$(0.1,0.3,0.5;-0.4,-0.3,-0.5)$
e_5	v_4v_5	$(0.3,0.2,0.5;-0.1,-0.3,-0.5)$
e_6	v_5v_6	$(0.3,0.2,0.4;-0.3,-0.6,-0.7)$
e_7	v_4v_6	$(0.2,0.3,0.1;-0.4,-0.2,-0.6)$
e_8	v_3v_5	$(0.5,0.6,0.3;-0.4,-0.5,-0.7)$
e_9	v_1v_3	$(0.3,0.2,0.1;-0.3,-0.4,-0.6)$

Now according to the step: 1 process we construct the adjacency matrix of the given BNSCWG.
The adjacency matrix of the given neutrosophic connected weighted graph is

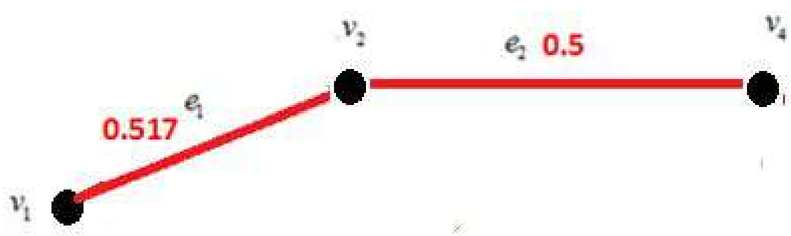
$$\begin{bmatrix} 0 & (0.6,0.5,0.4;-0.1,-0.2,-0.3) & (0.3,0.2,0.1;-0.3,-0.4,-0.6) & 0 & 0 & 0 \\ (0.6,0.5,0.4;-0.1,-0.2,-0.3) & 0 & (0.4,0.2,0.3;-0.3,-0.5,-0.7) & (0.2,0.3,0.4;-0.2,-0.4,-0.3) & 0 & 0 \\ (0.3,0.2,0.1;-0.3,-0.4,-0.6) & (0.4,0.2,0.3;-0.3,-0.5,-0.7) & 0 & (0.1,0.3,0.5;-0.4,-0.3,-0.5) & (0.5,0.6,0.3;-0.4,-0.5,-0.7) & 0 \\ 0 & (0.2,0.3,0.4;-0.2,-0.4,-0.3) & (0.1,0.3,0.5;-0.4,-0.3,-0.5) & 0 & (0.3,0.2,0.5;-0.1,-0.3,-0.5) & (0.2,0.3,0.1;-0.4,-0.2,-0.6) \\ 0 & 0 & (0.5,0.6,0.3;-0.4,-0.5,-0.7) & (0.3,0.2,0.5;-0.1,-0.3,-0.5) & 0 & (0.3,0.2,0.4;-0.3,-0.6,-0.7) \\ 0 & 0 & 0 & (0.2,0.3,0.1;-0.4,-0.2,-0.6) & (0.3,0.2,0.4;-0.3,-0.6,-0.7) & 0 \end{bmatrix}$$

According to the Step:2 the score matrix as follows:
By using the score function, the adjacency matrix is converting to the following matrix

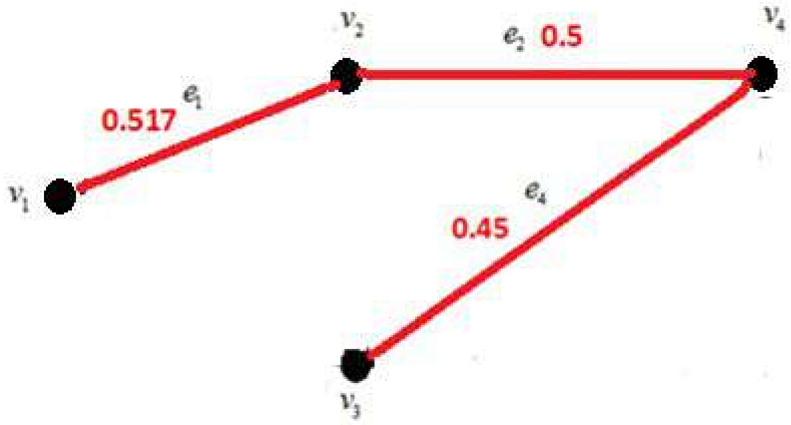
$$\begin{bmatrix} 0 & 0.517 & 0.617 & 0 & 0 & 0 \\ 0.517 & 0 & 0.633 & 0.5 & 0 & 0 \\ 0.617 & 0.633 & 0 & 0.45 & 0.567 & 0 \\ 0 & 0.5 & 0.45 & 0 & 0.55 & 0.533 \\ 0 & 0 & 0.567 & 0.55 & 0 & 0.317 \\ 0 & 0 & 0 & 0.533 & 0.317 & 0 \end{bmatrix}$$

Now, consider step: 3 the arbitrary vertex is v_1 *and connect it to its nearest neighbor vertex which has smallest weight is* v_2 *with the weight 0.517.*

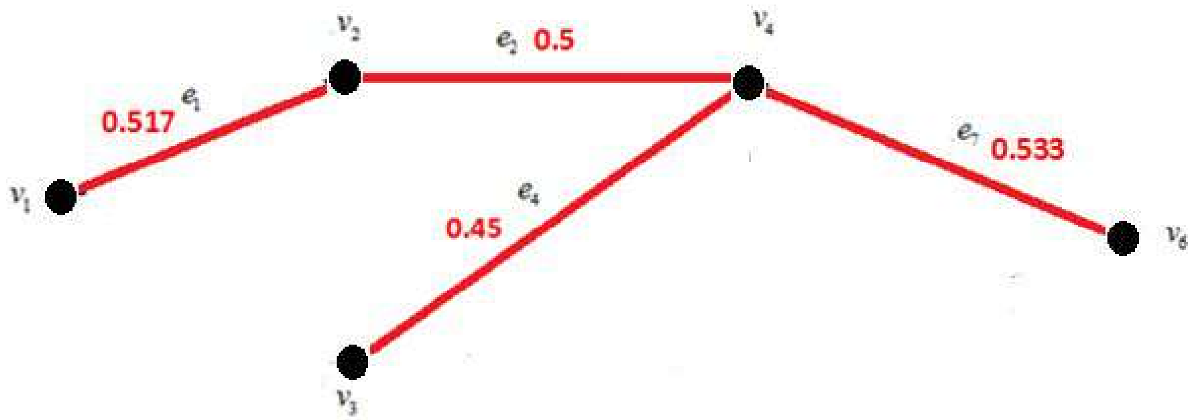
Next, consider the two end vertices on the edge $\{v_1, v_2\}$ *and the nearest neighbor vertex is* v_4 *add to the edge* $\{v_2, v_4\}$ *at* v_2 .



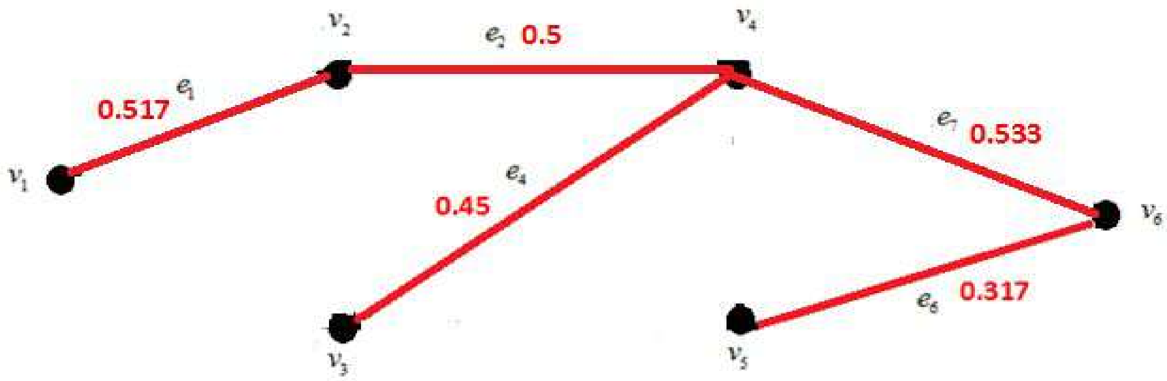
Now, consider the vertices $\{v_1, v_2, v_4\}$ *, the nearest vertex with the small weight is* v_3 *add the vertex to the vertex sets and edge* $\{v_4, v_3\}$ *the new vertex set contains 4 vertices, i.e.,* $\{v_1, v_2, v_3, v_4\}$.



We have 4 vertices now we consider the vertex set $\{v_1, v_2, v_3, v_4\}$ *, repeat the same process we get nearest vertex having smallest weight is not visited already is* v_6 *, the new vertex set is* $\{v_1, v_2, v_3, v_4, v_6\}$.



We have 5 vertices now we consider the vertex set $\{v_1, v_2, v_3, v_4, v_6\}$ *, repeat the same process we get nearest vertex having smallest weight is not visited already is* v_5 *, the new vertex set is* $\{v_1, v_2, v_3, v_4, v_6, v_5\}$ *.Then stop the process because all the vertices are appeared.*



Now, the required sapping tree is $v_1 - v_2 - v_4 - v_3 - v_5 - v_6$.

The minimal weight is 0.517+0.5+0.45+0.533+0.317= 2.317

This minimal spanning tree weight by Prim's algorithm is compared with Kruskal's algorithm.

Input: Bipolar Neutrosophic weighted graph.

Output: Minimal Spanning tree of the given bipolar neutrosophic weighted graph with n-1 edges.

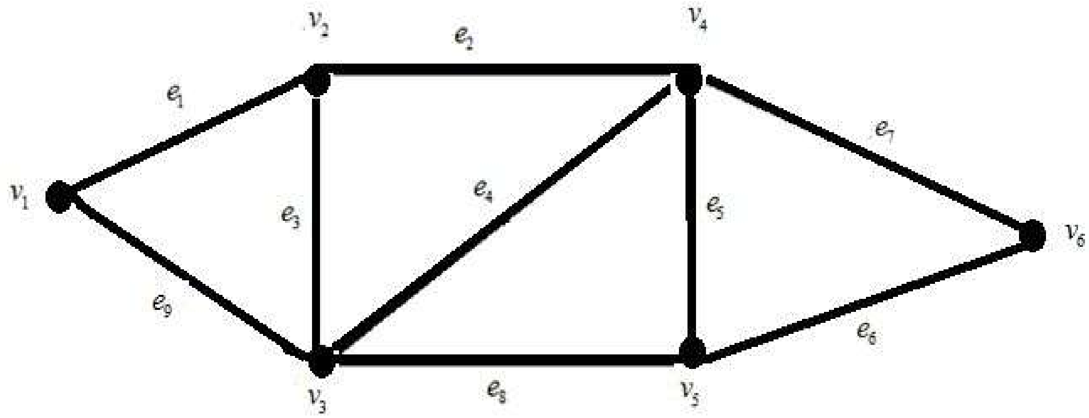
3.3 Bipolar Neutrosophic Kruskal's algorithm

Step:1 Compute the Adjacency matrix of the given bipolar neutrosophic weighted graph.
Step:2 With help of the score function of BNCWG, change the adjacency matrix to scores matrix by replacing edge weight with its score value.
Step: 3 List the edges of BNCWG in the order of non decreeing weights
Step: 4 Start with a small weighted edge, proceed sequentially by selecting one edge at a time such that no cyclic is formed.
Step: 5 Stop the process of step:4 when n-1 edges are selected.

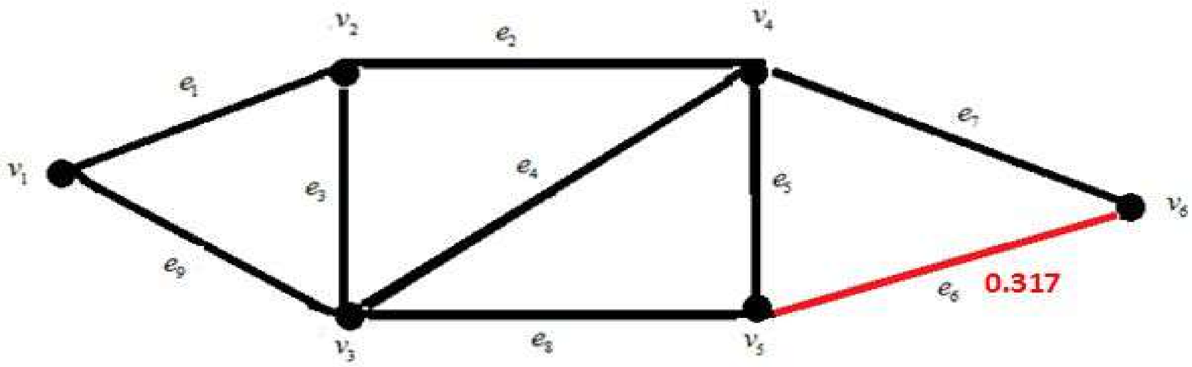
These n-1 edges make up a minimal bipolar neutrosophic spanning tree of bipolar neutrosophic weighted graph G.

Remark: The process of step: 4 is called greedy process.

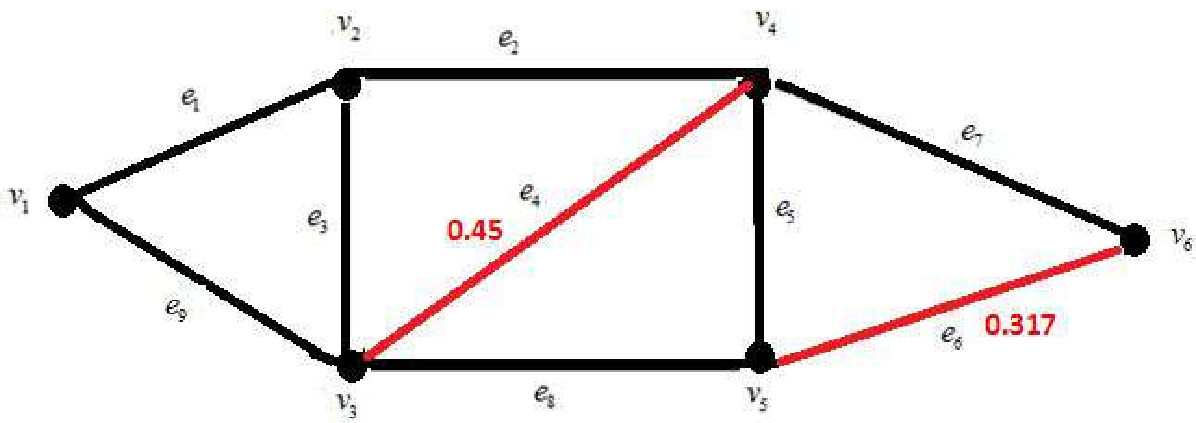
By using the same numerical example, we consider the adjacency matrix and also score function both are same. (ref. above example)



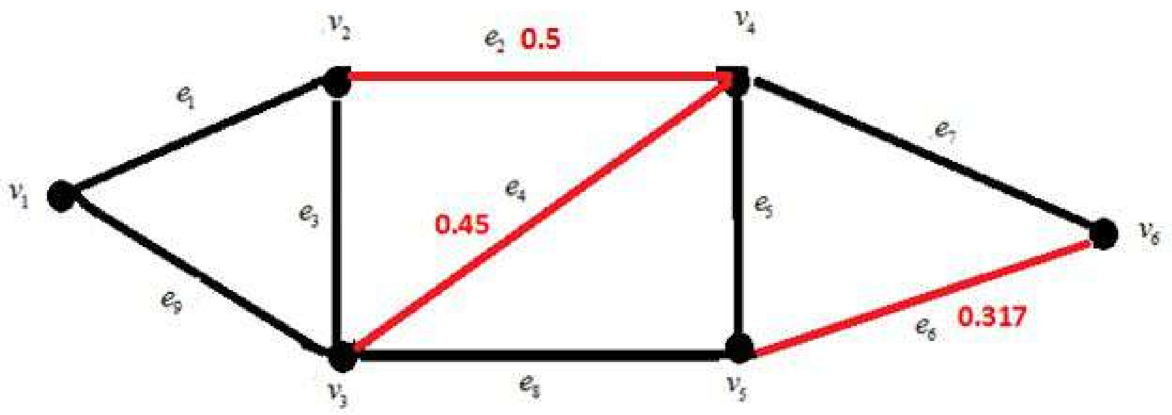
Now, according to the Kruskal's algorithm, we chose minimum weight edge first, the edge is $\{v_5, v_6\}$ *with minimum weight is 0.317.*



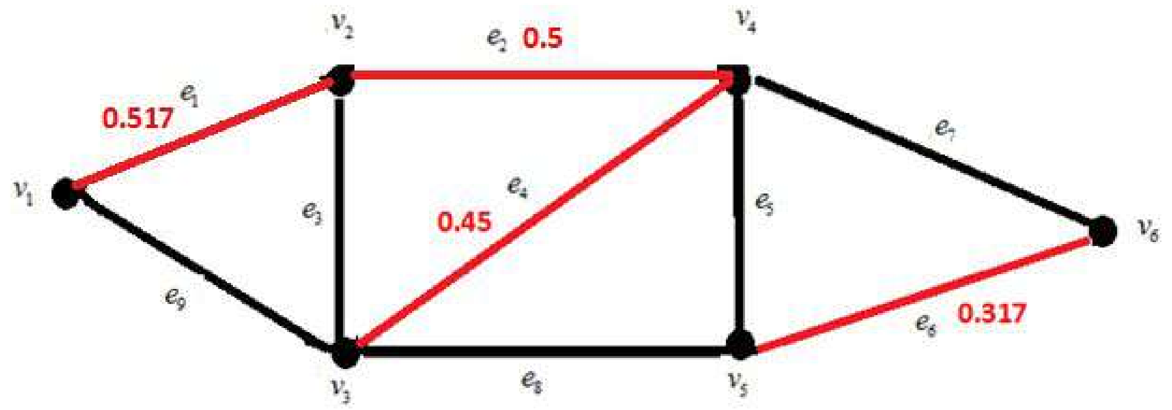
After that, we consider the next smallest weight edge is $\{v_3, v_4\}$ *with weight 0.45.*



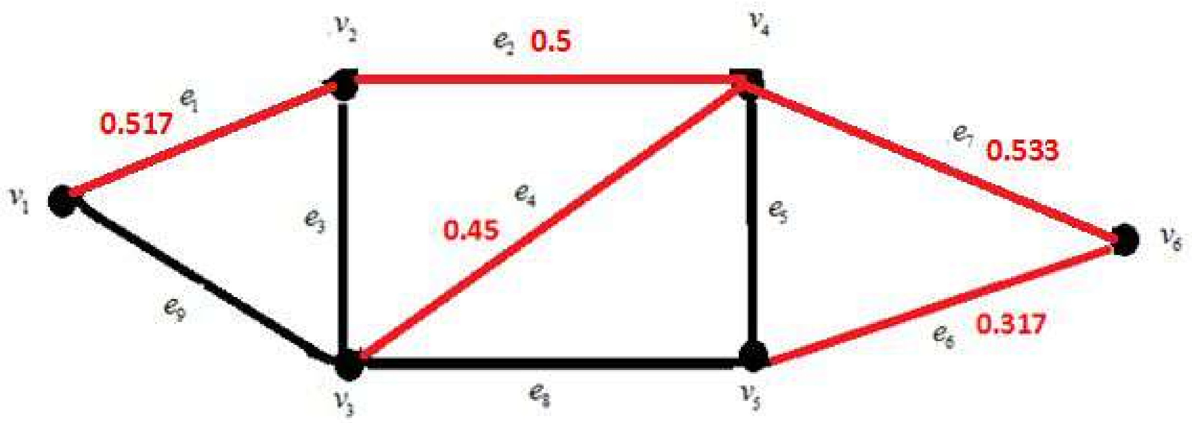
Next, we repeat this processor we get the smallest weight is 0.5 add the edge $\{v_2, v_4\}$.



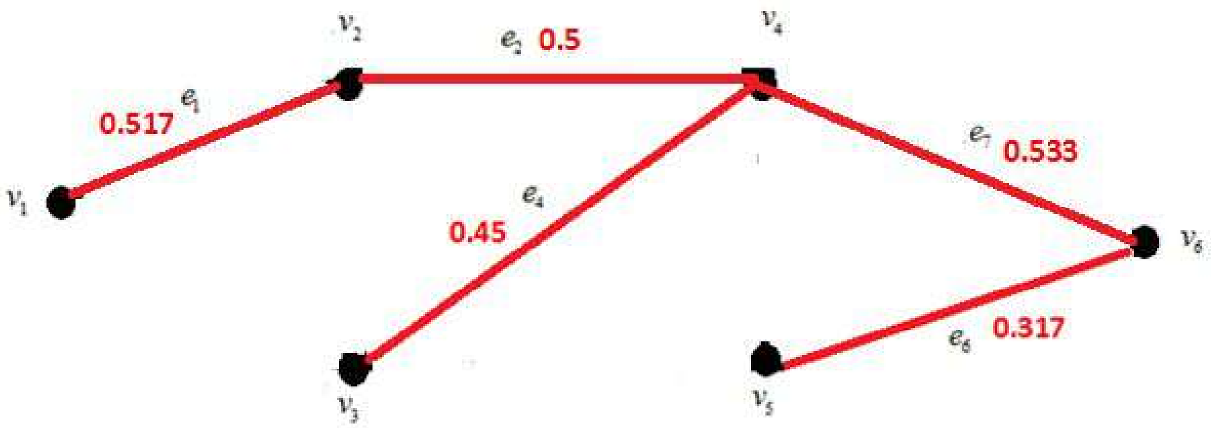
After that, we consider the smallest weight edge is 0.517 $\{v_1, v_2\}$.



Further, we go to the edge $\{v_4, v_6\}$ *with minimum weight is 0.533. Therefore the required sapping tree is*



Now the minimal Spanning tree is



The minimal weight is 0.317+0.45+0.5+0.517+0.533= 2.317

IV. CONCLUSION

This study indicates the extension of the bipolar neutrosophic theory. Here we proposed the MST algorithms and extended to bipolar neutrosophic graphs. We conclude that score functions to derive minimal weights are also equal in bipolar neutrosophic theory by using Prim's & Kruskal's algorithms.

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