Physical Properties of Linear Ranging Solution of Path Difference Localization Equation with two Baseline

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Author: China Academy of Management Science, Beijing, China.

I. INTRODUCTION

For the planar multi-station positioning problem, the previous solution method was to construct a group of nonlinear hyperbolic equations [1-3] about the location of the radiation source by using path difference measurement. However, recent studies have shown that linear equations can be obtained by further constructing auxiliary equations using plane geometric relations on the basis of path difference measurement [4].

If only from the perspective of mathematical calculation, with the help of linear solution, it is easier to analyze the positioning accuracy with the classical error theory, and further more helpful to further study and optimize the layout configuration of plane multi-station positioning system. In fact, if there is no explicit solution, the research can only arrange mathematical equations from the perspective of solving unknown quantities. Based on numerical operation, it is difficult to deeply study the internal correlation between various parameters, and even more difficult to discuss the basic physical characteristics of solutions.

On the basis of retelling the linear solution of the plane double-base path difference localization equation, the general physical characteristics of the ranging solution are given, which are directly proportional to the square of the transverse projection component and inversely proportional to the longitudinal first-order difference component.

1.1 Linear solution of one-dimensional double base linear array

For the one-dimensional double-base array shown in figure 1, the path difference between the two adjacent baselines is:

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$$\Delta r_i = r_i - r_{i+1} \qquad (i=1,2) \tag{1}$$

In where: r is radial distance; Δr_i is path difference.

If the midpoint of the entire array is taken as the coordinate origin, the following two geometric auxiliary equations can be listed by the cosine theorem:

$$\begin{aligned} r_i^2 &= r_{i+1}^2 + d_i^2 - 2d_i r_{i+1}\cos(90+\theta_{i+1}) \\ &= r_{i+1}^2 + d_i^2 + 2d_i r_{i+1}\sin\theta_{i+1} \qquad (i=1,2) \end{aligned} \tag{2}$$

In where: d_i is baseline; θ_{i+1} is target azimuth.

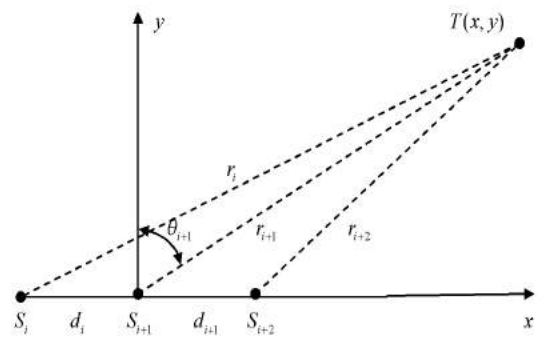

Fig.1: One dimensional double baseline array

Because of: $x = r_{i+1}\sin\theta_{i+1}$, the geometric auxiliary equation can be rewritten as:

$$r_i^2 = r_{i+1}^2 + d_i^2 + 2d_i x \tag{3}$$

$$r_{i+2}^2 = r_{i+1}^2 + d_{i+1}^2 - 2d_{i+1} x \tag{4}$$

In where: x is the abscissa of the rectangular coordinate system.

At this point, if the path differences (1) of the two adjacent baselines are substituted into geometric auxiliary equations (3) and (4), the following binary linear equations can be obtained after the transfer arrangement:

$$2d_i x - 2\Delta r_i r_{i+1} = -d_i^2 + \Delta r_i^2 \tag{5}$$

$$2d_{i+1} x - 2\Delta r_{i+1} r_{i+1} = d_{i+1}^2 - \Delta r_{i+1}^2 \tag{6}$$

From this, the radial distance of the target can be directly solved:

$$r_{i+1} = \frac{\left(d_i^2 - \Delta r_i^2\right)d_{i+1} + \left(d_{i+1}^2 - \Delta r_{i+1}^2\right)d_i}{2\left(\Delta r_i d_{i+1} - \Delta r_{i+1} d_i\right)} \tag{7}$$

If two adjacent baselines are equal, that is $d = d_i = d_{i+1}$, then:

$$r_{i+1} = \frac{\left(2d^2 - \Delta r_i^2 - \Delta r_{i+1}^2\right)}{2\left(\Delta r_i - \Delta r_{i+1}\right)} \tag{8}$$

1.2 Approximate simplification of the ranging solution

For one-dimensional double-base ranging formula:

$$r_2 = \frac{2d^2 - \Delta r_1^2 - \Delta r_2^2}{2(\Delta r_1 - \Delta r_2)} \tag{9}$$

If the higher-order term of path difference is approximated: $\Delta r_1^2 \approx \Delta r_2^2$, then:

$$r_2 \approx \frac{d^2 - \Delta r_2^2}{\Delta r_1 - \Delta r_2} \tag{10}$$

As shown in figure 2, further using Pythagorean theorem, based on the relationship between hypotenuse and right angle side, approximately:

$$d^2 - \Delta r_2^2 \approx d^2\cos^2\theta_2 \tag{11}$$

Based on the relationship between the two right sides, it is approximately:

$$d^2 - \Delta r_2^2 \approx \Delta r_2^2 ctg^2\theta_2 \tag{12}$$

So are:

$$r_2 \approx \frac{\Delta r_2^2\cos^2\theta_2}{\Delta r_1 - \Delta r_2} \approx \frac{\Delta r_2^2 ctg^2\theta_2}{\Delta r_1 - \Delta r_2} \tag{13}$$

The above equation indicates that the square of the item on the molecule can be expressed as the projection of the baseline or the projection of the higher-order path difference.

Further, from the point of view of differentiation, for the differential of two path difference in the denominator, it should have the property of rate of change.

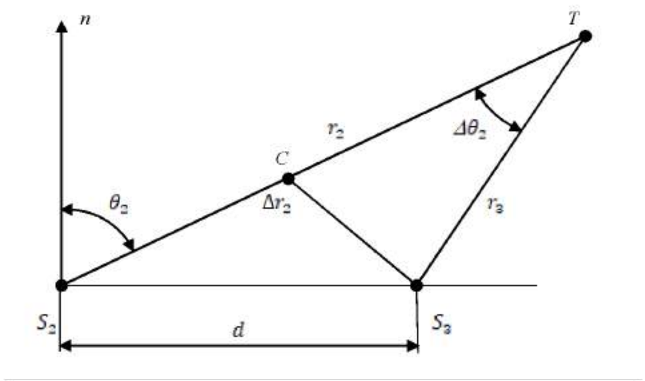

Figure 2: Projection of the baseline

II. BASIC FEATURES

2.1 Physical properties based solely on length measurements

In order to express concisely, the radial direction is demarcated as the longitudinal direction, and the direction perpendicular to the longitudinal direction is called the transverse direction.

According to the first equation of equation (13), property I can be directly obtained:

$$r \cdot (\Delta r_1 - \Delta r_2) = (d \cdot \cos\theta)^2 \qquad (\text{I})$$

Characteristic I indicates that the longitudinal distance is determined by the difference of the path difference after the baseline length and target orientation are determined. In other words: after the baseline length is determined, the longitudinal distance is determined by the path difference and the target orientation.

According to the second equation of (13), property II is directly derived:

$$r \cdot (\Delta r_1 - \Delta r_2) = (\Delta r \cdot ctg\theta)^2 \qquad (\text{II})$$

Property II shows that the longitudinal distance is the ratio of the square of the projection of the transverse high-order path difference to the differential of the first-order path difference in two approximately orthogonal directions, and the magnitude value in both approximately orthogonal directions may change dynamically.

2.3 Physical properties associated with motion parameters

First of all, define:

Baseline average moving speed:

$$v_d = \frac{d}{\Delta t}$$

Average moving speed of path difference:

$$v_r = \frac{\Delta r}{\Delta t}$$

Average acceleration of path difference:

$$a = \frac{\Delta v}{\Delta t}$$

Difference of average acceleration:

$$a_1 - a_2 = \frac{\Delta r_1 - \Delta r_2}{\Delta t^2} = \frac{v_1 - v_2}{\Delta t}$$

Divide both sides of physical property (I) by time interval to get property III:

$$r \cdot (a_1 - a_2) = (v_d \cos\theta)^2 \qquad (\text{III})$$

Property III shows that the longitudinal distance is proportional to the square of the horizontal projection of the average velocity of the baseline, and inversely proportional to the difference of the average acceleration of the path difference.
If we further divide both sides of property II by time intervals, then we have property IV:

$$r \cdot (a_1 - a_2) = (v_r ctg\theta)^2 \qquad (\text{IV})$$

The fourth property shows that the longitudinal distance is proportional to the square of the lateral projection of the average moving speed of the path difference, and inversely proportional to the difference of the average acceleration of the path difference.

III. CONCLUSION

Without a simplified analysis, the description of the property would be more verbose, and the right-hand side of all the equations describing the physical property would be described as the sum of the two projected components.

Although the present analysis in this paper may be flawed and imperfect, it would be more

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meaningful to observe the physical properties of solutions.

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