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ABSTRACT

Mathematical models were formulated to investigate lipoprotein based MHD fluid flow through a stenosed arterial channel with hematocrit. The governing nonlinear partial differential equations have been transformed into linear partial differential equations, which are solved analytically. The numerical simulations was done using Mathematica 12, results are presented graphically in the form of velocity and concentration profiles. The effects of various parameters such as the Solutal Grashof number, Schmidt number and other parameters such as Hematocrit parameter, lipoprotein external source parameter on the velocity and concen- tration have been examined with the help of the graphs. The present results have an important bearing on the therapeutic procedure of hyper- thermia, particularly in understanding/ regulating blood flow and lipid profile in the blood.

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ABSTRACT

Mathematical models were formulated to investigate lipoprotein based MHD fluid flow through a stenosed arterial channel with hematocrit has been studied. The governing non-linear partial differential equations are transformed into linear partial differential equations, solved analytically. The numerical simulations was done using Mathematica 12, results are presented graphically in the form of velocity and concentration profiles. The effects of various parameters such as the Solutal Grashof number, Schmidt number and other parameter such as Hematocrit parameter, lipoprotein external source parameter on the velocity and concentration have been examined with the help of the graphs. The present results have an important bearing on the therapeutic procedure of hyperthermia, particularly in understanding/regulating blood flow and lipid profile in the blood.

Keywords: Lipoprotein, Artery, Blood, Hematocrit, Mhd, Fluid and Flow.

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INTRODUCTION

The investigation of blood flow through arteries is considerably very important in many cardiovascular diseases. The poor circulation of blood in the body due to occlusion/blockage in arteries is one major health risks. Arteries channels oxygenated blood with nutrients from the heart to the tissues of the body, in the circulatory system of the human body. Blood is a viscous fluid circulating in the artery/vein. It has a strong nourishing effect on the human body and serves as one of the basic substances constituting the human body. Blood is a wonderful fluid which is an important factor of life.

Atherosclerosis is hardening of a blood vessel from a buildup of plaque as a result of excessive cholesterol (lipoprotein) intake. Plaque is made of fatty deposits, cholesterol, and calcium. Plaque buildup causes the artery to narrow and harden which is a serious risk human living.

Plaque buildup can slow and even stop blood flow. This means the tissue supplied by the artery is cut off from its blood supply and as such humans must watch the quantity of cholesterol they consume. It often leads to pain or decreased function. This condition can cause a number of serious health problems. Over the last two decades there has been theoretical and experimental studies of blood flow through the circulatory system of living mammals, has been the subject of scientific research and literatures available such as: characteristics of blood flow through an artery in the presence of multi-stenosis were studied by Chakravarty and Sannigrahi [1] the investigation of basic BFD flow problems attracts interest due to the numerous proposed applications in bioengineering and medical sciences.

Bio-fluids in the presence of a magnetic field with dissipation finds its applications in various upcoming fields like innovative drug targeting, surgical operations, etc. Haik *et al.* [2] reported a 30 percent

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decrease in blood flow rate when subjected to a high magnetic field of 10 T while Yadav *et al*. [3] found a similar reduction in blood flow rate but at a much smaller magnetic field of 0.002 T. Sharma *et al*. [4] formulated a mathematical model for the hydro-magnetic bio-fluid flow in the porous medium with Joule effect. A theoretical analysis of blood flow and heat transfer in a permeable vessel in the presence of an external magnetic field was made by Sinha *et al*. [5]. Shit and Roy [6] investigated the effect of induced magnetic field on blood flow through a constricted channel, and demonstrated that increasing the values of the magnetic field reduces the velocity of the blood flow at the center. Rahbari *et al*. [7] carried out an analytical study on blood flow containing nanoparticles through porous blood vessels in the presence of the magnetic field using the Homotopy Perturbation Method (HPM). Blood flow in a large blood vessel has a profound influence on the efficiency of thermal therapy treatment. Electromagnetic heat, such as short waves and microwaves, sends heat up to 2 inches into the tissue and muscles. It works best for injuries in joints, muscles, and tendons. Moreover, hyperthermia treatment is found to be effective during cancer therapy in recent years. Its objective is to raise the temperature of pathological tissues above cytotoxic temperatures (41–450C) without overexposing healthy tissues [8]. Heat and mass transfer of blood flow considering its pulsatile hydro-magnetic rheological nature under the presence of viscous dissipation, Joule heating and a finite heat source was discussed by Sharma *et al*. [9]. Sinha and Shit [10] investigated the combined effects of thermal radiation and MHD heat transfer blood flow through a capillary. Thermal radiation effect on inclined arterial blood flow through a non-Darcian porous medium with magnetic field was discussed by Sharma *et al*. [11]. Bunonyo et al [12] investigated blood flow a stenosed artery with heat in the presence of magnetic field. In their investigation it is observed that magnetic field increase inhibit blood flow as a result of Lorentz force.

In spite of all these studies, the investigation of lipoprotein based MHD blood flow through a stenosed artery with haematocrit was given little attention. Hence, the main object of the present investigation is to study the importance of the haematocrit and external lipoprotein through human daily meals which prevents the body from producing cholesterol.

MATHEMATICAL FORMULATION

We consider a blood flow through an artery by assuming the artery to be a channel, and blood as an incompressible Newtonian fluid, viscous and electrically conducting. The viscous nature of blood is assumed to be due to the percentage of red blood and lipid. The lipoprotein is the protein in the blood which causes some impediment of the flow with an increase in concentration. The flow is caused by the pumping action of the heart. In addition, we assume u to be the velocity of the fluid, C_w & C_∞ as the concentration of lipoprotein at the wall and far field, D_0 as the molecular diffusivity, S is the external lipid source and H is the hematocrit. The governing equation for the flow of the fluid through an artery is stated as coupled partial differential equations as stated below.

$$\rho\frac{\partial u^*}{\partial t^*}=-\frac{\partial P^*}{\partial x^*}+\frac{\partial}{\partial y^*}\left(\mu^*(H)\frac{\partial u^*}{\partial y^*}\right)+\rho g\beta_C\left(C^*-C_\infty\right) \tag{1}$$

$$\frac{\partial C^*}{\partial t^*}=\frac{\partial}{\partial y^*}\left(D^*\frac{\partial C^*}{\partial y^*}\right)+S\left(C^*\right) \tag{2}$$

Subject to the corresponding boundary conditions are:

$$u^*=0\ ,\qquad C^*=C_\infty^* \qquad \text{at}\quad y=0 \tag{9}$$

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$u^* = 0$, $C^* = C_w^*$ at $y^* = R_0$ (10)

We assumed that the lipoprotein concentration dependent on the fluid viscosity, mass diffusion and external lipoprotein–C source respectively as:

$$\mu^* = \frac{\mu_0}{(1+2.5H)},\ S = Q\left(b_3\left(C^* - C_\infty\right)\right) \text{ and } D^* = D_0, \quad (5)$$

We consider the following dimensionless parameters:

$$\left.\begin{aligned} y = \frac{y^*}{R_0}, x = \frac{x^*}{R_0}, u = \frac{u^* R_0}{\upsilon}, t = \frac{\upsilon t^*}{R_0^2}, \mu = \frac{\mu^*}{\mu_0}, \phi = \frac{C^* - C_\infty}{C_w - C_\infty} \\ P = \frac{P^* R_0^2}{\rho \upsilon^2}, Gc = \frac{g R_0^3 \beta_C \left(C_w - C_\infty\right)}{\upsilon^2}, Sc = \frac{\upsilon}{D_0}, \upsilon = \frac{\mu_0}{\rho}, \lambda = \frac{Q b_3 R_0^2}{\upsilon} \end{aligned}\right\} \quad (6)$$

Transforming equation (1) and (2) using equation (6), we obtain the following:

$$(1+2.5H)\frac{\partial u}{\partial t} = -(1+2.5H)\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} + Gc(1+2.5H)\phi \quad (7)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} + \lambda\phi \quad (8)$$

Subject to the corresponding boundary conditions are:

$u = 0$, $\phi = 0$ at $y = 0$ (9)

$u = 0$, $\phi = 1$ at $y = h$ (10)

METHOD OF SOLUTION

In order solve equation (7) and equation (8) subject the boundary conditions in equation (9) – (10), we have to consider the solution in the following form:

$$\left.\begin{aligned} u = u_0 e^{i\omega t} \\ \phi = \phi_0 e^{i\omega t} \end{aligned}\right\} \quad (11)$$

We substitute equation (11) into equation (7) – (8), we obtain the following:

$$\frac{\partial^2 u_0}{\partial y^2} - (1+2.5H)i\omega u_0 = (1+2.5H)P_0 - Gc(1+2.5H)\phi_0 \quad (12)$$

$$\frac{\partial^2 \phi_0}{\partial y^2} + (\lambda - i\omega)Sc\phi_0 = 0 \quad (13)$$

Subject to the corresponding boundary conditions are:

$u_0 = 0$, $\phi_0 = 0$ at $y = 0$ (14)

$u_0 = 0$, $\phi_0 = e^{-i\omega t}$ at $y = h$ (15)

Let $\chi_1 = (1+2.5H)i\omega$, $\chi_2 = (\lambda - i\omega)Sc$, $P = (1+2.5H)P_0$ and $G_1 = Gc(1+2.5H)$ so that equation (12) to (13) can be transformed to:

$$\frac{\partial^2 u_0}{\partial y^2} - \chi_1 u_0 = P - G_1\phi_0 \quad (16)$$

$$\frac{\partial^2 \phi_0}{\partial y^2} + \chi_2 \phi_0 = 0 \tag{17}$$

Equation (16) and (17) are ordinary differential equations, we have to solve them subject to the boundary conditions in equation (14) – (15).

Solving equation (17) which has the general solutions as:

$$\phi_0(y) = A sin\sqrt{\chi_2}\, y + B cos\sqrt{\chi_2}\, y \tag{18}$$

We determine the coefficients in equation (18) using the boundary conditions in equation (14) and (15) as:

$$A = \frac{e^{-i\omega t}}{sin\sqrt{\chi_2 h}}, B = 0 \tag{19}$$

Substitute the values in equation (19) into equation (18), we obtain the following:

$$\phi_0(y) = \left(\frac{e^{-i\omega t}}{sin\sqrt{\chi_2 h}} \right) sin\sqrt{\chi_2}\, y \tag{20}$$

Now, substitute equation (20) into equation (11), we obtain the following:

$$\phi(y) = \left(\frac{sin\sqrt{\chi_2}\, y}{sin\sqrt{\chi_2 h}} \right) \tag{21}$$

To solve the non-homogenous ordinary differential equation, we substitute equation (21) into equation (16) as follows:

$$\frac{\partial^2 u_0}{\partial y^2} - \chi_1 u_0 = P - G_1 \left(\frac{e^{-i\omega t}}{sin\sqrt{\chi_2 h}} \right) sin\sqrt{\chi_2}\, y \tag{22}$$

Solving for equation (22), we have to first obtain the complementary before the particular solution. So, the homogenous solution takes the general form:

$$u_{0c}(y) = A_1 sinh\sqrt{\chi_1}\, y + B_1 cosh\sqrt{\chi_1}\, y \tag{23}$$

The particular solution takes the form:

$$u_{0p}(y) = A_3 + A_4 sin\sqrt{\chi_2}\, y + B_4 cos\sqrt{\chi_2}\, y \tag{24}$$

Differentiate according to the order of the differential equation (22), we obtain:

$$A_4 = \left(\frac{G_1 e^{-i\omega t}}{(\chi_2 + \chi_1) sin\sqrt{\chi_2 h}} \right), B_4 = 0, A_3 = -\frac{P}{\chi_1} \tag{25}$$

Substituting the values in equation (25) into equation (24), we obtain the following:

$$u_{0p}(y) = -\frac{P}{\chi_1} + \left(\frac{G_1 e^{-i\omega t}}{(\chi_2 + \chi_1) sin\sqrt{\chi_2 h}} \right) sin\sqrt{\chi_2}\, y \tag{26}$$

The general solution to the momentum equation (22), we have the following:

$$u_0(y) = A_1 sinh\sqrt{\chi_1}\, y + B_1 cosh\sqrt{\chi_1}\, y - \frac{P}{\chi_1} + \left(\frac{G_1 e^{-i\omega t}}{(\chi_2 + \chi_1) sin\sqrt{\chi_2 h}} \right) sin\sqrt{\chi_2}\, y \tag{27}$$

We can solve for the constant coefficients in (27) using the boundary condition in equations (14) and (15), we obtain the following:

$$A_1 = \left[\frac{P}{\chi_1} - \left(\frac{P}{\chi_1}\right)\frac{\cosh\sqrt{\chi_1}h}{\sinh\sqrt{\chi_1}h} - \left(\frac{G_1 e^{-i\omega t}}{(\chi_2+\chi_1)\sin\sqrt{\chi_2}h}\right)\frac{\sin\sqrt{\chi_2}h}{\sinh\sqrt{\chi_1}h}\right], B_1 = \frac{P}{\chi_1} \quad (28)$$

Then the general solution for the velocity profile is:

$$u(y) = \left[\begin{array}{l}\left[\frac{P}{\chi_1}\left(1 - \frac{\cosh\sqrt{\chi_1}h}{\sinh\sqrt{\chi_1}h}\right) - \left(\frac{G_1 e^{-i\omega t}}{(\chi_2+\chi_1)\sinh\sqrt{\chi_1}h}\right)\right]\sinh\sqrt{\chi_1}y \\ + \left(\frac{P}{\chi_1}\right)\left(\cosh\sqrt{\chi_1}y - 1\right) + \left(\frac{G_1 e^{-i\omega t}}{(\chi_2+\chi_1)\sin\sqrt{\chi_2}h}\right)\sin\sqrt{\chi_2}y\end{array}\right] e^{i\omega t} \quad (29)$$

RESULTS PRESENTATION

In this section, numerical simulation is carried out for equation (21) and (29) using the following parameter values to investigate the effect of haematocrit, oscillatory parameter, mass lipoprotein source parameter and Schmidt number on the velocity and concentration functions respectively. We considered variable parameters such as: $H = 3, Gc = 10, Sc = 0.22, \omega = 2$ and $t = 1$. The graphical results presented as follows:

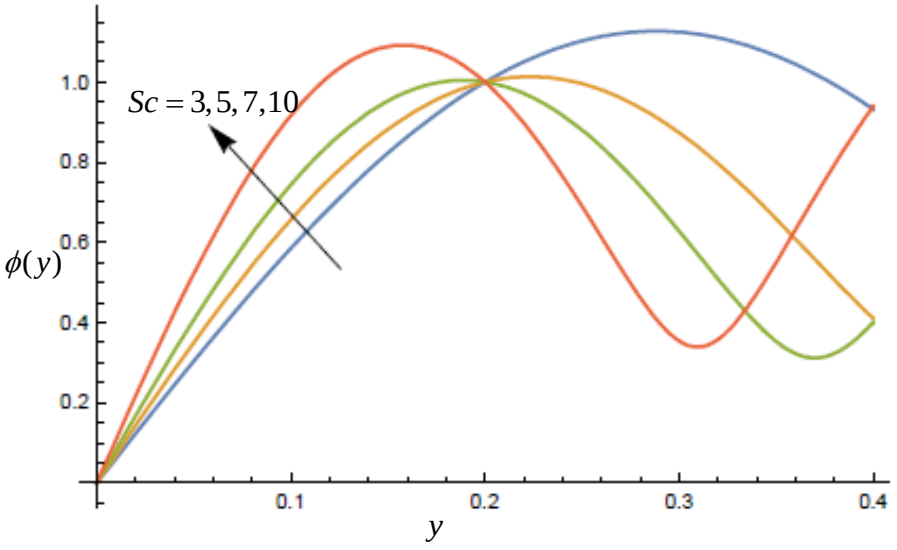

Fig 1 influence of Sc on concentration profile

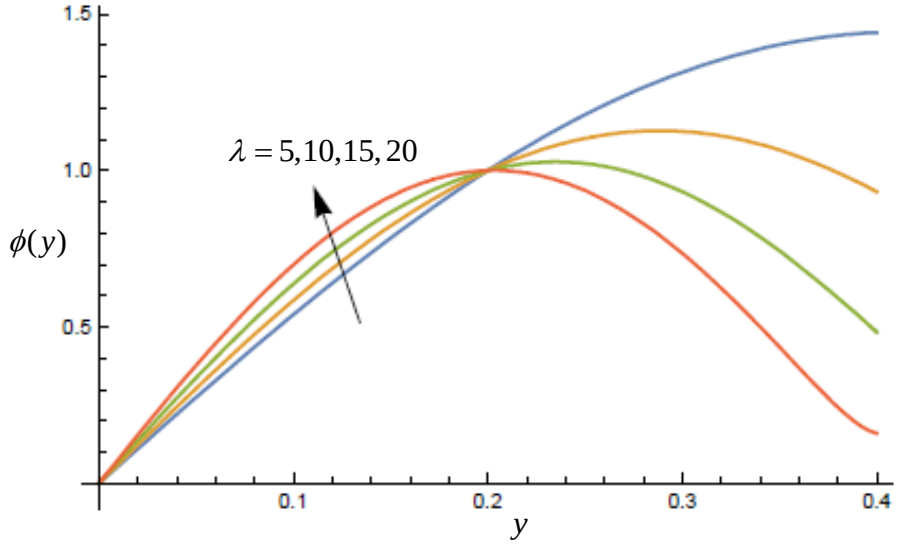

Fig 2 influence of λ on concentration profile

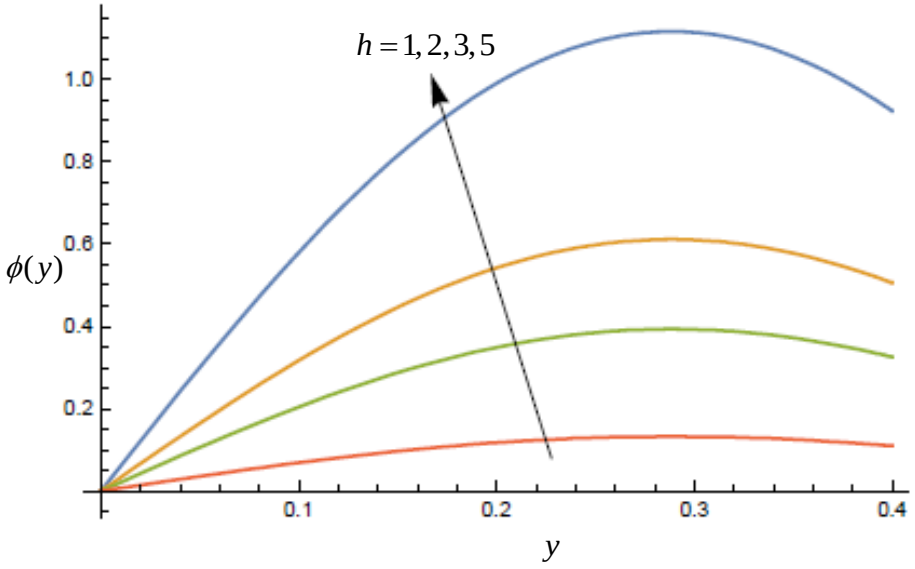

Fig 3 influence of h on concentration profile

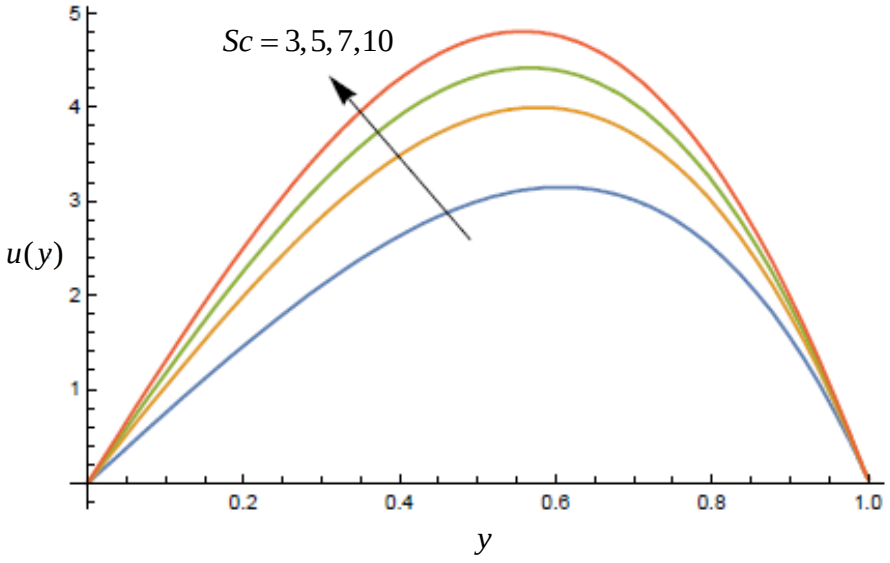

Fig 4 influence of Sc on concentration profile

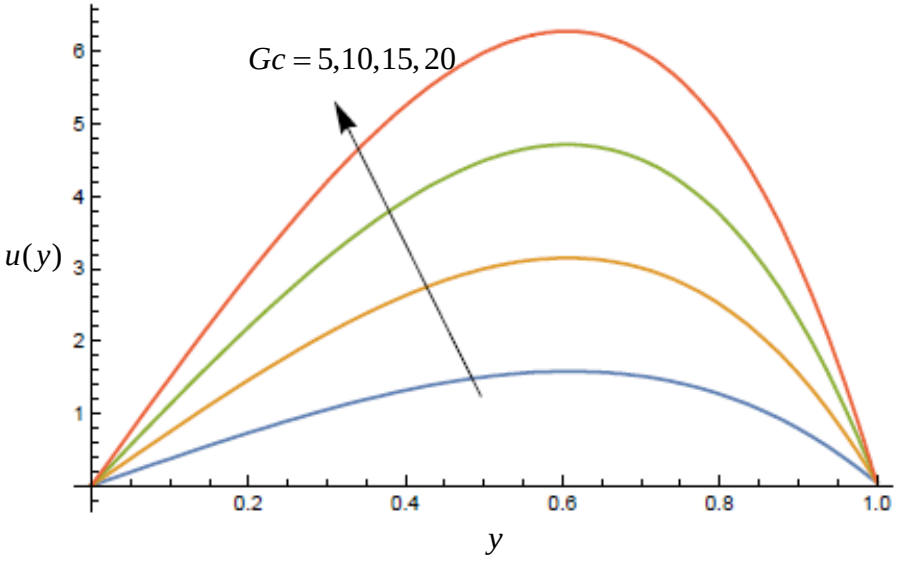

Fig 5 influence of Gc on velocity profile

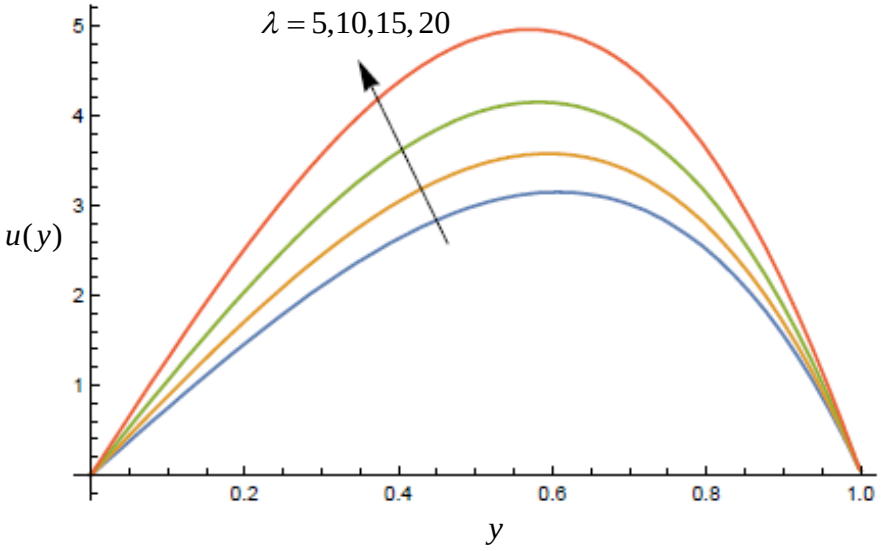

Fig 6 influence of λ on velocity profile

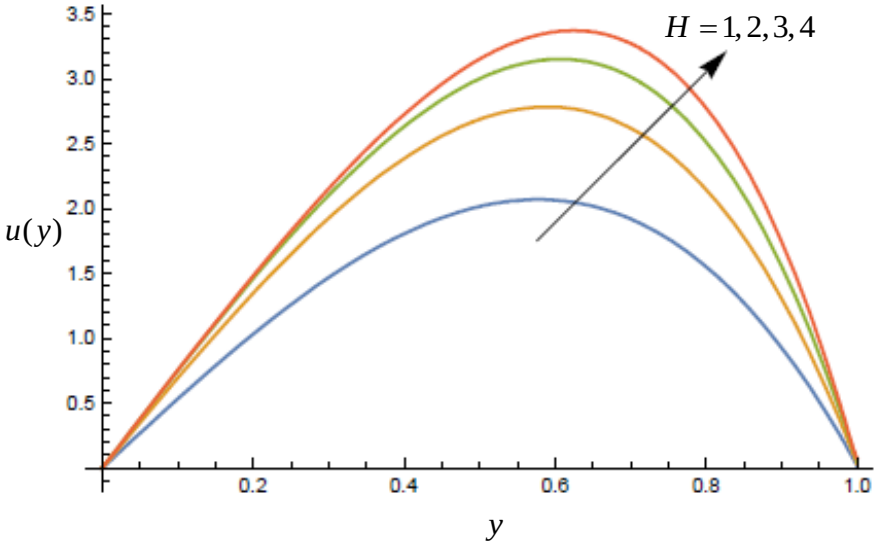

Fig 7 influence of H on velocity profile

DISCUSSION

The Schmidt number clearly influenced the concentration profile as can seen in **Fig 1**. This however converged at a point for different values of Schmidt number increase before the concentration profile diverged.

As we can see in **Fig 2**, the increasing level of lipoprotein from a source can actually increase the level of the concentration profile on the downstream as indicated in the figure. However, we observed some level of divergence in concentration profile as the boundary layer is increased.

Fig 3 illustrates the influence of the height of stenosis on concentration profile through an artery while other valuable parameters are kept as the same. We noticed that as h increased from 1 to 4 units, there is a corresponding increase in flow concentration profile. It clearly seen that $R > R_0$ which is a reduction in stenosis.

In **Fig 4** we noticed an increase in velocity profile as the value of the Schmidt number increases from 0.22 to 0.9. It clear shows that the porosity remained constant while the dynamic viscosity of the fluid is improved. However, the flow attained different peaks for different values of the Schmidt number before decelerating to zero, which is an indication that for us to maintain some level of flow is have to consider the dynamic viscosity of the fluid as it relates to the molecular diffusivity.

Fig 5 depicts a clear case of an improved flow as the concentration Grashof number is increased from 5 to 20. This researched result is of the opinion that the concentration difference between the wall and that of the far field is greater that dynamic viscosity of the fluid can actually improve the flow field due

Often times high level of cholesterol actually increase the viscosity of the fluid because it adds to the protein level in the blood plasma fluid thereby causing some sought of slow movement of the fluid towards the downstream and causing the heart rate to increase as seen in **Fig 6.** But as indicated in the figure, we can say that flow gets a peak for different values of λ before decelerating to zero in an unimaginable fashion.

Haematocrit as earlier stated is the percentage of red blood cells in the blood volume, which means it has quite a substantial amount of hemoglobin in the bloodstream. However, we noticed in **Fig 7** that as the haematocrit level is increased from 1 to 4 through a supplement there is increase in the general blood flow but different level of H results to decrease in due to an increased viscosity.

CONCLUSION

In this paper, investigation of lipoprotein based MHD fluid flow through an arterial channel with haematocrit is analyzed. The conclusions of the present analysis are as follows:

1. The velocity profile was raised for the increasing values of Solutal Grashof number, Schmidt number.
2. An increase in Schmidt number increases the concentration profile
3. With increasing values of the haematocrit initially increased the velocity to a peak before it decelerated to zero.

4. Lipoprotein source parameter also influences the flow profile to increase to a peak before decelerating to zero as clearly seen.

REFERENCES

1. Chakravarty S. and Sannigrahi A. (1999): *A nonlinear mathematical model of blood flow in a constricted artery experiencing body acceleration*. – Math. Comput. Model, vol.29, pp.9-25.
2. Haik Y., Pai V. and Chen C.-J. (2001): *Apparent viscosity of human blood in a high static magnetic field*. –J. Magn. Magn. Mater., vol.225, No.1–2, pp.180-186.
3. Yadav R.P., Harminder S. and Bhoopal S. (2008): *Experimental studies on blood flow in stenosis arteries in presence of magnetic field*. – Ultra Sci., vol.20, No.3, pp.499-504.
4. Sharma B.K., Mishra A. and Gupta S. (2013): *Heat and mass transfer in magneto-biofluid flow through a non- Darcian porous medium with Joule effect*. – J. Eng. Phys. and Thermo Phy., vol.86, No.4, pp.716-725.
5. Sinha A., Mishra J.C. and Shit G.C. (2016): *Effect of heat transfer on unsteady MHD flow of blood in a permeable vessel in the presence of non-uniform heat source*. – Alexandria Engineering Journal, vol.55, No.3, pp.2023-2033.
6. Shit G.C. and Roy M. (2016): *Effect of induced magnetic field on blood flow through a constricted channel. An analytical approach*. – Journal of Mechanics in Medicine and Biology, vol.16, No.03, pp.1650030.
7. Rahbari A., Fakour M., Hamzeh Nejad A., Vakilabadi M.A. and Ganji D.D. (2017): Heat transfer and fluid flow of blood with nanoparticles through porous vessels in a magnetic field: A quasi-one dimensional analytical approach. – Mathematical Biosciences, vol.283, pp.38-47.
8. Levin W., Sherar M.D., Cooper B., Hill R.P., Hunt J.W. and Liu F.-F. (1994): Effect of vascular occlusion on tumour temperatures during superficial hyperthermia. – International Journal of Hyperthermia, vol.10, No.4, pp.495-505.
9. Sharma B.K., Sharma M., Gaur R.K. and Mishra A. (2015): Mathematical modeling of magneto pulsatile blood flow through a porous medium with a heat source. – International Journal of Applied Mechanics and Engineering, vol.20, No.2, pp.385-396.
10. Sinha A. and Shit G.C. (2015): Electromagnetohydrodynamic flow of blood and heat transfer in a capillary with thermal radiation. –Journal of Magnetism and Magnetic Materials, vol.378, pp.143-151.
11. Sharma B.K., Sharma M. and Gaur R.K. (2015): Thermal radiation effect on inclined arterial blood flow through a non-Darcian porous medium with magnetic field. – Proceeding: First Thermal and Fluids Engineering Summer Conference, ASTFE Digital Library, vol.17, pp.2159-2168, DOI: 10.1615/TFESC1.bio.013147.
12. Bunonyo, K. W., Israel-Cookey, C., & Amos, E. (2018). Modeling of Blood Flow through Stenosed Artery with Heat in the Presence of Magnetic Field. *Asian Research Journal of Mathematics*, 1-14.