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Extract

Dependence of Fuel Economy
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Analysis of FM/FG/1 Retrial
Queue

Single Server Fuzzy Queue
with Reneging



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IMAGE: OBSERVATORY WITH STAR
TRAILS ON MOUNTAINS FOR
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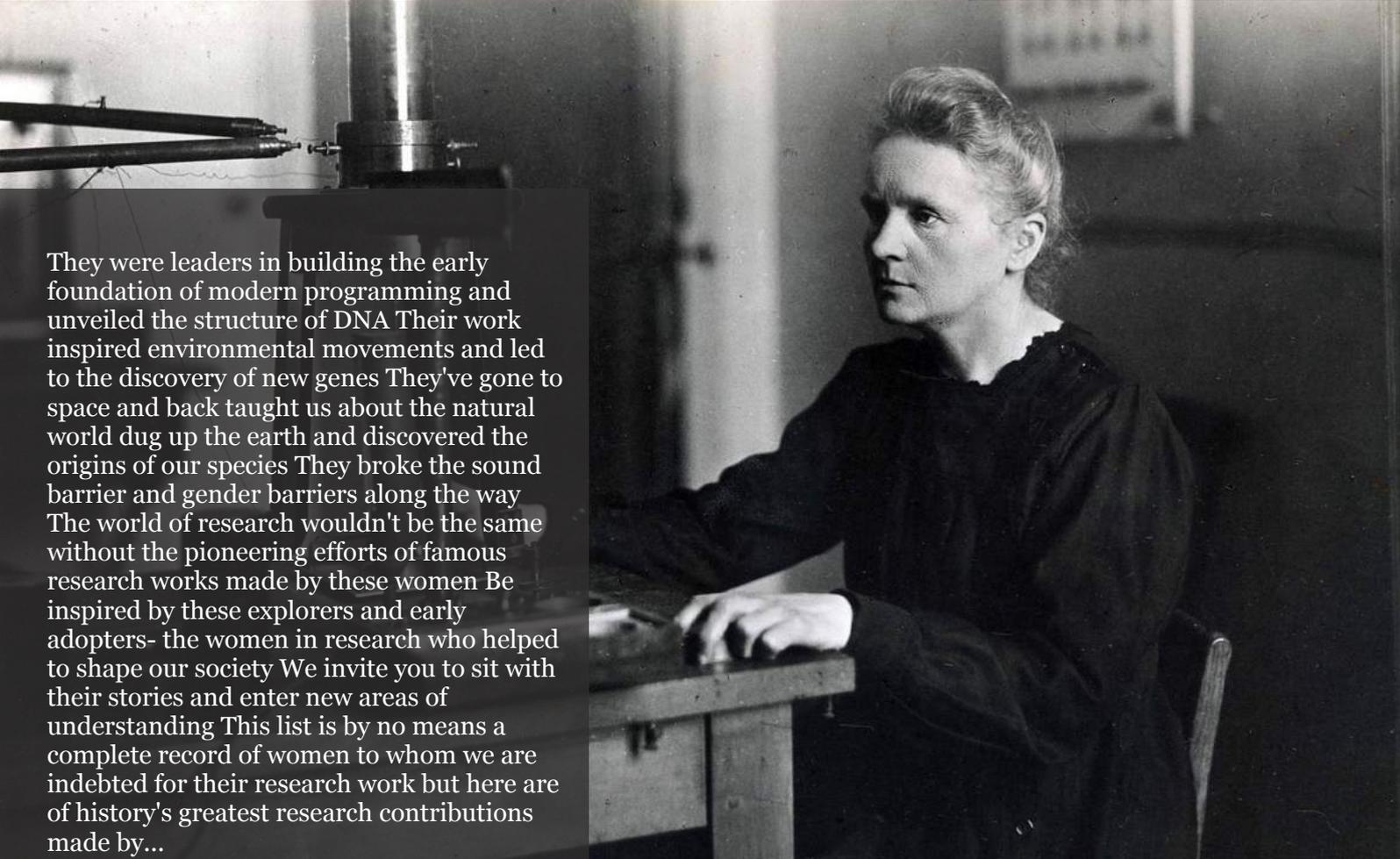
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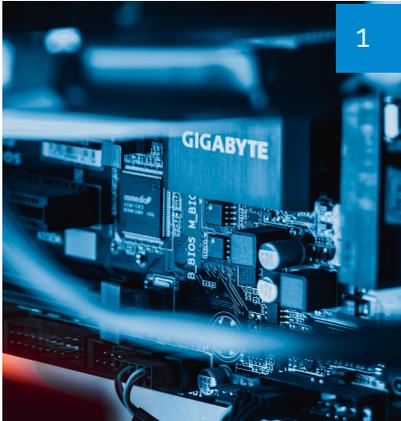
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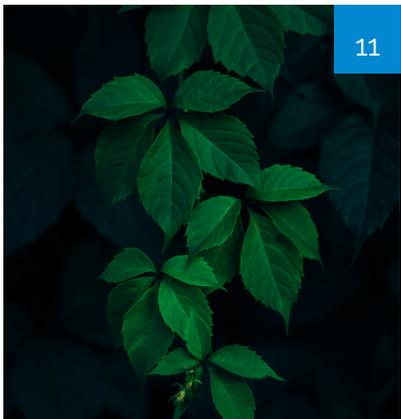


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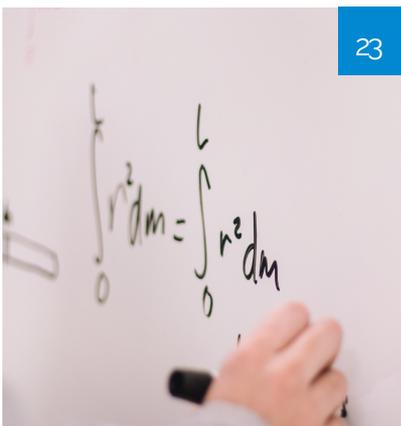
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A Single Server Fuzzy Queue with Reneging and Retention of Reneged Customers

G. Kannadasan & D. Devi

Annamalai University

ABSTRACT

This paper deals with an FM/FM/1 queueing system with reneging and retention of reneged customers and fuzzy parameters. We discuss fuzzy queueing model and performance measures. We are using α -cuts method in the pentagonal fuzzy numbers. Finally numerical results are presented to show the effects of system parameter.

Keywords: FM/FM/1 queue, Multiple working vacations, Reneging, Retention Membership values pentagonal fuzzy numbers.

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Author a o: Department of Mathematics, Annamalai University, Annamalainagar-608002, India. Kamban college of arts and science for women, Department of Mathematics, Tiruvannamalai - 606 603.

I. INTRODUCTION

Queueing system with server vacations and impatience of customers have been broadly studied because of their broad applications in day-to-day life congestion problems such as telecommunication systems, service systems and manufacturing systems. Zhang and shi analyzed M/M/1 queue with Bernoulli schedule vacation and vacation interruptions. Selvaraju and Goswami presented impatience customers in single server Markovian queue with single an multiple working vacations. Abou-EI-Ata and Hariri considered multiple servers queueing system M/M/c/N with balking reneging. Kumar and Sharma presented a single server queueing system with retention of reneged customers. Zadeh analyzed membership function which is having more relevance when there is an uncertain situation. Aburba panda and Mahumangalpal presented the pentagonal fuzzy number. Kaufmann introduced an Introduction to the Theory of fuzzy Subsets and Zimmermann explained the Fuzzy Set Theory and Its Applications

In this paper we describe the FM/FM/1 queue with reneging and retention of reneged customers. In crisp model, given in section 2. In fuzzy environment and performance of measure, we discuss the fuzzy model with the mean number of customers in the system during working vacation period and normal busy period are studied in fuzzy environment respectively. In section 5 includes numerical study about the performance measures.

II. THE CRISP MODEL

We consider an FM/FM/1 queueing model with multiple working vacations, vacations interruptions, reneging, retention of reneged customers an Bernoulli feedback. Customers arrivals to a poisson distribution with parameter λ . The service times during a normal service period η , the service time during a working vacation period η_1 and vacation time σ are according to exponential distribution. During the multiple working vacations the customers are assumed to be impatient, the customer stimulate an impatient timer t_1 , which is exponentially distributed with parameter ν . The reneged customer can be retained in the system with probability δ_1 or the customer abandon the queue with complementary probability $\delta = (1 - \delta_1)$. The server continue a regular busy period with probability β or remains in the vaction with probability $\beta_1 = (1 - \beta)$. After conclusion of each service, the customer can either join the end of the queue with probability ζ or the customer can leave the system with probability γ where $\zeta + \gamma = 1$.

let $M(t)$ denote the number of customer in the system at time t, and let $N(t)$ denote the state of the server at time t with

$$N(t) = \begin{cases} 1, & \text{if the server is in normal busy period} \\ 0, & \text{if the server is in working vacation period} \end{cases}$$

Then $\{(M(t), N(t)); t \geq 0\}$ is a continuous time Markov process with state space $\Omega = \{(0, 0) \cup (i, j), i = 1, 2, 3, \dots, j = (0, 1)\}$. and steady state condition also satisfied

III. THE MODEL IN FUZZY ENVIRONMENT

In this section, the arrival rate, service for normal busy period, service for working vacation period, working vacation time, impatient of customers, abandon the queue, leave the system after completion of each service and server resumes a regular busy period are assume to be fuzzy numbers $\bar{\lambda}, \bar{\eta}, \bar{\eta}_1, \bar{\sigma}, \bar{\nu}, \bar{\delta}, \bar{\beta}, \bar{\gamma}$ respectively. Now

$$\begin{aligned} \bar{\lambda} &= \{(x, \mu_{\bar{\lambda}}(x)); x \in S(\bar{\lambda})\}, \\ \bar{\eta} &= \{(y, \mu_{\bar{\eta}}(y)); y \in S(\bar{\eta})\}, \\ \bar{\eta}_1 &= \{(y_1, \mu_{\bar{\eta}_1}(y_1)); y_1 \in S(\bar{\eta}_1)\}, \\ \bar{\sigma} &= \{(z, \mu_{\bar{\sigma}}(z)); z \in S(\bar{\sigma})\}, \\ \bar{\nu} &= \{(u, \mu_{\bar{\nu}}(u)); u \in S(\bar{\nu})\}, \\ \bar{\delta} &= \{(v, \mu_{\bar{\delta}}(v)); v \in S(\bar{\delta})\}, \\ \bar{\beta} &= \{(r, \mu_{\bar{\beta}}(r)); r \in S(\bar{\beta})\}, \\ \bar{\gamma} &= \{(s, \mu_{\bar{\gamma}}(s)); s \in S(\bar{\gamma})\}, \end{aligned}$$

where $S(\bar{\lambda}), S(\bar{\eta}), S(\bar{\eta}_1), S(\bar{\sigma}), S(\bar{\nu}), S(\bar{\delta}), S(\bar{\beta})$ and $S(\bar{\gamma})$ are the universal set's of the arrival rate, service for normal busy period, service for working vacation period, working vacation time, impatient of customers, abandon the queue, leave the system after completion of each service and server resumes a regular busy period respectively. Define $f(x, y, y_1, z, u, v, r, s)$ as the system performance measure related to the above defined fuzzy queuing model, which depends on the fuzzy membership function $S(\bar{\lambda}), S(\bar{\eta}), S(\bar{\eta}_1), S(\bar{\sigma}), S(\bar{\nu}), S(\bar{\delta}), S(\bar{\beta})$ and $S(\bar{\gamma})$. Applying Zadeh's extension principle (1978) the membership function of the performance measure $S(\bar{\lambda}), S(\bar{\eta}), S(\bar{\eta}_1), S(\bar{\sigma}), S(\bar{\nu}), S(\bar{\delta}), S(\bar{\beta})$ and $S(\bar{\gamma})$ can be defined as

$$\mu_{\bar{f}(\bar{\lambda}, \bar{\eta}, \bar{\eta}_1, \bar{\sigma}, \bar{\nu}, \bar{\delta}, \bar{\beta}, \bar{\gamma})}(H) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y \in S(\bar{\eta}) \\ y_1 \in S(\bar{\eta}_1) \\ z \in S(\bar{\sigma}) \\ u \in S(\bar{\nu}) \\ v \in S(\bar{\delta}) \\ r \in S(\bar{\beta}) \\ s \in S(\bar{\gamma})}} \{ \min \{ \mu_{\bar{\lambda}}(x), \mu_{\bar{\eta}}(y), \mu_{\bar{\eta}_1}(y_1), \mu_{\bar{\sigma}}(z), \mu_{\bar{\nu}}(u), \mu_{\bar{\delta}}(v), \mu_{\bar{\beta}}(r), \mu_{\bar{\gamma}}(s) \} / H \}$$

Where $H = f(x, y, y_1, z, u, v, r, s)$ (1)

If the α - cuts of $f(\lambda, \eta, \eta_1, \sigma, \nu, \delta, \beta, \gamma)$ degenerate to some fixed value, then the system performance is a crisp number, otherwise it is a fuzzy number.

The mean number of customers in the system during working vacation period

$$E[L_1] = \left(\frac{\gamma\eta - \lambda}{\sigma + \beta\gamma\eta_1} \right) - \left[\frac{(\beta\gamma\eta_1)(\sigma + \beta\gamma\eta_1) + (\beta\gamma\eta_1 - (\gamma\eta - \lambda))\eta_1(1 - \beta)\gamma}{(\eta_1(1 - \beta)\gamma)(\sigma + \beta\gamma\eta_1)} \right] A$$

where

$$A = \frac{(1 - \beta)(\delta\nu + \sigma + \beta\gamma\eta_1)(\gamma\eta - \lambda)}{[(1 - \beta)(\delta\nu(\gamma\eta - \lambda) + \gamma(\eta - \eta_1)(\sigma + \gamma\beta\eta_1)) + (\nu\delta\beta + \sigma + \beta\gamma\eta_1)(\sigma + \gamma\beta\eta_1)]}$$

The mean number of customers in the system during normal busy period

$$E[L_2] = \left(\frac{\sigma + \gamma\beta\eta_1}{\gamma\eta - \lambda} \right) \left[\frac{A\lambda - (\delta\nu + \gamma\eta_1 + \sigma - \lambda)B}{\sigma + \gamma\beta\eta_1 + 2\delta\nu} \right] + \left[\frac{(1 - \beta)(\gamma\sigma\eta + \lambda\gamma\beta\eta_1) + [\sigma\beta(\eta_1(1 - \beta)\gamma - (\sigma + \gamma\beta\eta_1)) - (\gamma\sigma\eta + \lambda\gamma\beta\eta_1)]A}{(1 - \beta)(\sigma + \gamma\beta\eta_1)(\gamma\eta - \lambda)} \right]$$

$$\text{where } B = \left(\frac{\gamma\eta - \lambda}{\sigma + \beta\gamma\eta_1} \right) - \left[\frac{(\beta\gamma\eta_1)(\sigma + \beta\gamma\eta_1) + (\beta\gamma\eta_1 - (\gamma\eta - \lambda))\eta_1(1 - \beta)\gamma}{(\eta_1(1 - \beta)\gamma)(\sigma + \beta\gamma\eta_1)} \right] A$$

we acquire the membership function some performance measures, namely the mean number of customers in the system during working vacation period $E[L_1]$ and the mean number of customers in the system during normal busy period $E[L_2]$. For the system in terms of this membership function are, as follows

$$\mu_{\overline{[L_1]}}(P) = \sup_{\substack{x \in S(\lambda) \\ y \in S(\eta) \\ y_1 \in S(\eta_1) \\ z \in S(\sigma) \\ u \in S(\bar{\nu}) \\ v \in S(\bar{\delta}) \\ r \in S(\bar{\beta}) \\ s \in S(\bar{\gamma})}} \{ \min \{ \mu_{\bar{\lambda}}(x), \mu_{\bar{\eta}}(y), \mu_{\bar{\eta}_1}(y_1), \mu_{\bar{\sigma}}(z), \mu_{\bar{\nu}}(u), \mu_{\bar{\delta}}(v), \mu_{\bar{\beta}}(r), \mu_{\bar{\gamma}}(s) \} / P \} \quad (2)$$

$$\text{Where } P = f(x, y, y_1, z, u, v, r, s)$$

Where

$$P = \left(\frac{sy - x}{z + rsy_1} \right) - \left[\frac{(rsy_1)(z + rsy_1) + (rsy_1 - (sy - x))y_1(1 - r)s}{(y_1(1 - r)s)(z + rsy_1)} \right] A$$

where

$$A = \frac{(1 - r)(vu + z + rsy_1)(sy - x)}{[(1 - r)(vu(sy - x) + s(y - y_1)(z + rsy_1)) + (uvr + z + rsy_1)(z + rsy_1)]}$$

$$\mu_{\overline{[L_2]}}(Q) = \sup_{\substack{x \in S(\lambda) \\ y \in S(\eta) \\ y_1 \in S(\eta_1) \\ z \in S(\sigma) \\ u \in S(\bar{\nu}) \\ v \in S(\bar{\delta}) \\ r \in S(\bar{\beta}) \\ s \in S(\bar{\gamma})}} \{ \min \{ \mu_{\bar{\lambda}}(x), \mu_{\bar{\eta}}(y), \mu_{\bar{\eta}_1}(y_1), \mu_{\bar{\sigma}}(z), \mu_{\bar{\nu}}(u), \mu_{\bar{\delta}}(v), \mu_{\bar{\beta}}(r), \mu_{\bar{\gamma}}(s) \} / Q \} \quad (3)$$

$$\text{Where } Q = f(x, y, y_1, z, u, v, r, s)$$

$$Q = \left(\frac{z + rsy_1}{sy - x} \right) \left[\frac{Ax - (vu + sy_1 + z - x)B}{z + rsy_1 + 2vu} \right]$$

$$+ \left[\frac{(1 - r)(szy + xsry_1) + [zr(y_1(1 - r)s - (z + rsy_1)) - (szy + xsry_1)]A}{(1 - r)(z + rsy_1)(sy - x)} \right]$$

$$\text{where } B = \left(\frac{sy - x}{z + rsy_1} \right) - \left[\frac{(rsy_1)(z + rsy_1) + (rsy_1 - (sy - x))y_1(1 - r)s}{(y_1(1 - r)s)(z + rsy_1)} \right] A$$

Using the fuzzy analysis technique describe, we can find the membership of $\mu_{\overline{E[L_1]}}$, $\mu_{\overline{E[L_2]}}$ as a function of the parameter α . Thus the α -cut approach can be used to develop the membership function of $\mu_{\overline{E[L_1]}}$, $\mu_{\overline{E[L_2]}}$.

IV. PERFORMANCE OF MEASURE

The following performance measure are studied for this model in fuzzy environment.

The mean number of customers in the system during working vacation period

Based on Zadeh's extension principle $\mu_{\overline{E[L_1]}}(P)$ is the superimum of minimum over $\mu_{\bar{\lambda}}(x), \mu_{\bar{\eta}}(y), \mu_{\bar{\eta}_1}(y_1), \mu_{\bar{\sigma}}(z), \mu_{\bar{\nu}}(u), \mu_{\bar{\delta}}(v), \mu_{\bar{\beta}}(r), \mu_{\bar{\gamma}}(s)$

$$P = \left(\frac{sy - x}{z + rsy_1} \right) - \left[\frac{(rsy_1)(z + rsy_1) + (rsy_1 - (sy - x))y_1(1 - r)s}{(y_1(1 - r)s)(z + rsy_1)} \right] A$$

to satisfying $\mu_{\overline{E[L_1]}}(P) = \alpha, 0 < \alpha \leq 1$.

We consider the following four cases:

$$\text{case}(i); \mu_{\bar{\lambda}}(x) = \alpha, \mu_{\bar{\eta}}(y) \geq \alpha, \mu_{\bar{\eta}_1}(y_1) \geq \alpha, \mu_{\bar{\sigma}}(z) \geq \alpha, \mu_{\bar{\nu}}(u) \geq \alpha, \mu_{\bar{\delta}}(v) \geq \alpha, \mu_{\bar{\beta}}(r) \geq \alpha, \mu_{\bar{\gamma}}(s) \geq \alpha$$

$$\text{case}(ii); \mu_{\bar{\lambda}}(x) \geq \alpha, \mu_{\bar{\eta}}(y) = \alpha, \mu_{\bar{\eta}_1}(y_1) \geq \alpha, \mu_{\bar{\sigma}}(z) \geq \alpha, \mu_{\bar{\nu}}(u) \geq \alpha, \mu_{\bar{\delta}}(v) \geq \alpha, \mu_{\bar{\beta}}(r) \geq \alpha, \mu_{\bar{\gamma}}(s) \geq \alpha$$

$$\text{case}(iii); \mu_{\bar{\lambda}}(x) \geq \alpha, \mu_{\bar{\eta}}(y) \geq \alpha, \mu_{\bar{\eta}_1}(y_1) = \alpha, \mu_{\bar{\sigma}}(z) \geq \alpha, \mu_{\bar{\nu}}(u) \geq \alpha, \mu_{\bar{\delta}}(v) \geq \alpha, \mu_{\bar{\beta}}(r) \geq \alpha, \mu_{\bar{\gamma}}(s) \geq \alpha$$

$$\text{case}(iv); \mu_{\bar{\lambda}}(x) \geq \alpha, \mu_{\bar{\eta}}(y) \geq \alpha, \mu_{\bar{\eta}_1}(y_1) \geq \alpha, \mu_{\bar{\sigma}}(z) = \alpha, \mu_{\bar{\nu}}(u) \geq \alpha, \mu_{\bar{\delta}}(v) \geq \alpha, \mu_{\bar{\beta}}(r) \geq \alpha, \mu_{\bar{\gamma}}(s) \geq \alpha$$

$$\text{case}(v); \mu_{\bar{\lambda}}(x) \geq \alpha, \mu_{\bar{\eta}}(y) \geq \alpha, \mu_{\bar{\eta}_1}(y_1) \geq \alpha, \mu_{\bar{\sigma}}(z) \geq \alpha, \mu_{\bar{\nu}}(u) = \alpha, \mu_{\bar{\delta}}(v) \geq \alpha, \mu_{\bar{\beta}}(r) \geq \alpha, \mu_{\bar{\gamma}}(s) \geq \alpha$$

$$\text{case}(vi); \mu_{\bar{\lambda}}(x) \geq \alpha, \mu_{\bar{\eta}}(y) \geq \alpha, \mu_{\bar{\eta}_1}(y_1) \geq \alpha, \mu_{\bar{\sigma}}(z) \geq \alpha, \mu_{\bar{\nu}}(u) \geq \alpha, \mu_{\bar{\delta}}(v) = \alpha, \mu_{\bar{\beta}}(r) \geq \alpha, \mu_{\bar{\gamma}}(s) \geq \alpha$$

$$\text{case}(vii); \mu_{\bar{\lambda}}(x) \geq \alpha, \mu_{\bar{\eta}}(y) \geq \alpha, \mu_{\bar{\eta}_1}(y_1) \geq \alpha, \mu_{\bar{\sigma}}(z) \geq \alpha, \mu_{\bar{\nu}}(u) \geq \alpha, \mu_{\bar{\delta}}(v) \geq \alpha, \mu_{\bar{\beta}}(r) = \alpha, \mu_{\bar{\gamma}}(s) \geq \alpha$$

$$\text{case}(viii); \mu_{\bar{\lambda}}(x) \geq \alpha, \mu_{\bar{\eta}}(y) \geq \alpha, \mu_{\bar{\eta}_1}(y_1) \geq \alpha, \mu_{\bar{\sigma}}(z) \geq \alpha, \mu_{\bar{\nu}}(u) \geq \alpha, \mu_{\bar{\delta}}(v) \geq \alpha, \mu_{\bar{\beta}}(r) \geq \alpha, \mu_{\bar{\gamma}}(s) = \alpha$$

For case (i) the lower and upper bound of α - cuts of $\mu_{\overline{E[L_1]}}$ can be acquire through the corresponding parametric non-linear programs,

$$[E[L_1]]_{\alpha}^{L_1} = \min_{\Omega} \{[P]\} \text{ and}$$

$$[E[L_1]]_{\alpha}^{U_1} = \max_{\Omega} \{[P]\}$$

Similarly, we can calculate the lower and upper bounds of the α -cuts of $[E[L_1]]$ for the case (ii), (iii), (iv)(v), (vi) and (vii). By considering all the cases simultaneously the lower and upper bounds of the α -cuts of $[E[L_1]]$ can be written as

$$P = \left(\frac{sy - x}{z + rsy_1} \right) - \left[\frac{(rsy_1)(z + rsy_1) + (rsy_1 - (sy - x))y_1(1 - r)s}{(y_1(1 - r)s)(z + rsy_1)} \right] A$$

$$[E[L_1]]_\alpha^L = \min \{[P]\} \quad \text{and} \quad [E[L_1]]_\alpha^U = \max \{[P]\}.$$

such that

$$x_\alpha^L \leq x \leq x_\alpha^U, \quad y_\alpha^L \leq y \leq y_\alpha^U, \quad y_{1\alpha}^L \leq y_1 \leq y_{1\alpha}^U, \quad z_\alpha^L \leq z \leq z_\alpha^U,$$

$$u_\alpha^L \leq u \leq u_\alpha^U, \quad v_\alpha^L \leq v \leq v_\alpha^U, \quad r_\alpha^L \leq r \leq r_\alpha^U, \quad s_\alpha^L \leq s \leq s_\alpha^U.$$

If both $[E[L_1]]_\alpha^L$ and $[E[L_1]]_\alpha^U$ are invertible with respect to α , the left and right shape function, $L(M) = [E[L_1]]_\alpha^L^{-1}$ and $R(M) = [E[L_1]]_\alpha^U^{-1}$ can be derived from which the membership function $\mu_{\overline{E[L_1]}}(M)$ can be constructed as

$$\mu_{\overline{E[L_1]}}(P) = \begin{cases} L(P), & [E[L_1]]_{\alpha=0}^L \leq P \leq [E[L_1]]_{\alpha=0}^U \\ 1, & [E[L_1]]_{\alpha=1}^L \leq P \leq [E[L_1]]_{\alpha=1}^U \\ R(P), & [E[L_1]]_{\alpha=1}^L \leq P \leq [E[L_1]]_{\alpha=0}^U \end{cases} \quad (4)$$

In the same way we get the following results.

The mean number of customers in the system during normal busy period

$$\mu_{\overline{E[L_2]}}(Q) = \begin{cases} L(Q), & E[L_2]_{\alpha=0}^L \leq Q \leq E[L_2]_{\alpha=0}^U \\ 1, & E[L_2]_{\alpha=1}^L \leq Q \leq E[L_2]_{\alpha=1}^U \\ R(Q), & E[L_2]_{\alpha=1}^L \leq Q \leq E[L_2]_{\alpha=0}^U \end{cases} \quad (5)$$

V. NUMERICAL STUDY

The mean number of customers in the system during working vacation period

Suppose the fuzzy arrival rate $\bar{\lambda}$, service rate for normal busy period $\bar{\eta}$, service rate for working vacation period $\bar{\eta}_1$, working vacation time $\bar{\sigma}$, impatient of customers $\bar{\nu}$, abandon the queue $\bar{\delta}$, leave the system after completion of each service $\bar{\beta}$, server resumes a regular busy period $\bar{\gamma}$, are assumed to be pentagonal fuzzy numbers described by:

$$\bar{\lambda} = [1, 2, 3, 4, 5], \quad \bar{\eta} = [6, 7, 8, 9, 10], \quad \bar{\eta}_1 = [11, 12, 13, 14, 15] \quad \bar{\sigma} = [21, 22, 23, 24, 25],$$

$$\bar{\nu} = [31, 32, 33, 34, 35], \quad \bar{\delta} = [26, 27, 28, 29, 30], \quad \bar{\beta} = [41, 42, 43, 44, 45], \quad \bar{\gamma} = [36, 37, 8, 39, 40]$$

per hour respectively then

$$\lambda(\alpha) = \min_{x \in s(\bar{\lambda})} \{x \in s(\bar{\lambda}), G(x) \geq \alpha\}, \quad \max_{x \in s(\bar{\lambda})} \{x \in s(\bar{\lambda}), G(x) \geq \alpha\},$$

where

$$G(x) = \begin{cases} 0 & , \text{if } x \leq a_1 \\ 1 - (1 - r) \frac{x - a_2}{a_3 - a_2} & , \text{if } a_2 \leq x \leq a_3 \\ 1 & \text{if } x = a_3 \\ 1 - (1 - r) \frac{a_4 - x}{a_4 - a_3} & , \text{if } a_3 \leq x \leq a_4 \\ r \frac{a_5 - x}{a_5 - a_4} & , \text{if } a_4 \leq x \leq a_5 \\ 0 & , \text{if } x \geq a_5 \end{cases}$$

That is, $\lambda(\alpha) = [1 + \alpha, 5 - \alpha]$, $\eta(\alpha) = [6 + \alpha, 10 - \alpha]$, $\eta_1(\alpha) = [11 + \alpha, 15 - \alpha]$, $\sigma(\alpha) = [21 + \alpha, 25 - \alpha]$, $\nu(\alpha) = [31 + \alpha, 35 - \alpha]$, $\delta(\alpha) = [26 + \alpha, 30 - \alpha]$, $\beta(\alpha) = [41 + \alpha, 45 - \alpha]$, $\gamma(\alpha) = [36 + \alpha, 40 - \alpha]$

It is clear that, when $x = x_\alpha^U$, $y = y_\alpha^U$, $y_1 = (y_1)_\alpha^U$, $z = z_\alpha^U$, $u = u_\alpha^U$, $v = v_\alpha^U$, $r = r_\alpha^U$ and $s = s_\alpha^U$ P attains its maximum value and when $x = x_\alpha^L$, $y = y_\alpha^L$, $y_1 = (y_1)_\alpha^L$, $z = z_\alpha^L$, $u = u_\alpha^L$, $v = v_\alpha^L$, $r = r_\alpha^L$ and $s = s_\alpha^L$ P attains its minimum value.

From the generated for the given input values of $\bar{\lambda}$, $\bar{\eta}$, $\bar{\eta}_1$, $\bar{\sigma}$, $\bar{\nu}$, $\bar{\delta}$, $\bar{\beta}$, $\bar{\gamma}$,

- i) For fixed values of x, y, y_1, z, u, v and r , P decreases as s increase.
- ii) For fixed values of y, y_1, z, u, v, r and s , P decreases as x increase.
- iii) For fixed values of y_1, z, u, v, r, s and x , P decreases as y increase.
- iv) For fixed values of z, u, v, r, s, x and y , P decreases as y_1 increase.
- v) For fixed values of u, v, r, s, x, y and y_1 , P decreases as z increase.
- vi) For fixed values of v, r, s, x, y, y_1 and z , P decreases as u increase.
- vii) For fixed values of r, s, x, y, y_1, z and u , P decreases as v increase.
- viii) For fixed values of s, x, y, y_1, z, u and v , P decreases as r increase.

The smallest value of occurs when x -takes its lower bound. i.e.), $x = 16 + \alpha$ and y, y_1, z, u, v, r and s take their upper bounds given by $y = 10 - \alpha$, $y_1 = 15 - \alpha$, $z = 25 - \alpha$, $u = 35 - \alpha$, $v = 30 - \alpha$, $r = 45 - \alpha$ and $s = 40 - \alpha$ respectively. And maximum value of $E[L_1]$ occurs when $x = 5 - \alpha$, $y = 6 + \alpha$, $y_1 = 11 + \alpha$, $z = 21 + \alpha$, $u = 31 + \alpha$, $v = 26 + \alpha$, $r = 41 + \alpha$, and $s = 36 + \alpha$. If both $[E[L_1]]_\alpha^L$ & $[E[L_1]]_\alpha^U$ are invertible with respect to ' α ' then, the left shape function $L(P) = [E[L_1]]_\alpha^L$ and right shape function $R(P) = [E[L_1]]_\alpha^U$ can be acquired and from which the membership function $\mu_{\overline{E[L_1]}}(P)$ can be constructed as:

$$\mu_{\overline{E[L_1]}}(P) = \begin{cases} 0, & \text{if } P \leq P_1 \\ 0.4(x - 2), & \text{if } P_1 \leq P \leq P_2, \\ 0.4(4 - x), & \text{if } P_2 \leq P \leq P_3, \\ 0.6(5 - x), & \text{if } P_3 \leq P \leq P_4, \\ 0, & \text{if } P \leq P_5 \end{cases} \quad (6)$$

The values of P_1, P_2, P_3, P_4 and P_5 as acquired from (8) are:

$$\mu_{\overline{E[L_1]}}(P) = \begin{cases} 0, & \text{if } P \leq 0.0000 \\ 0.4(x - 2), & \text{if } 0.0000 \leq P \leq 0.01223, \\ 1, & \text{if } x = 1 \\ 0.4(4 - x), & \text{if } 0.01223 \leq P \leq 0.05788, \\ 0.6(5 - x), & \text{if } 0.05788 \leq P \leq 0.02245, \\ 0, & \text{if } P \geq 0.0000 \end{cases}$$

In the same way we get the following results.

The mean number of customers in the system during normal busy period

$$\mu_{\overline{E[L_2]}}(Q) = \begin{cases} 0, & \text{if } Q \leq Q_1 \\ 0.5(x - 2), & \text{if } Q_1 \leq Q \leq Q_2, \\ 0.5(4 - x), & \text{if } Q_2 \leq Q \leq Q_3, \\ 0.5(5 - x), & \text{if } Q_3 \leq Q \leq Q_4, \\ 0, & \text{if } Q \geq Q_5 \end{cases} \tag{7}$$

The values of Q_1, Q_2, Q_3, Q_4 and Q_5 as acquired from (9) are:

$$\mu_{\overline{E[L_2]}}(Q) = \begin{cases} 0, & \text{if } Q \leq 0.0000 \\ 0.5(x - 2), & \text{if } 0.0000 \leq Q \leq 0.6331, \\ 0.5(4 - x), & \text{if } 0.6331 \leq Q \leq 1.1266, \\ 0.5(5 - x), & \text{if } 1.1266 \leq Q \leq 0.8810, \\ 0, & \text{if } Q \geq 0.0000 \end{cases}$$

VI. CONCLUSION

In this paper we have studied the analysis of $M/M/1$ queue with renegeing and retention of renegeed customers using pentagonal fuzzy numbers. we have obtained the performance measure such as the mean number of customers in the system during working vacation period, the mean number of customers in the system during normal busy period. we have obtained the numerical result to all the performance measures for this fuzzy queues. In this queueing model is very important in system performance and solve various problems in many complex system, such as communication systems, computer systems, call centers and flexible manufacturing systems.

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