Analysis of Fm/Fg/1 Retrial Queue with Bernoulli Schedule and Vacation using Hexagonal Fuzzy Numbers

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Classification: MSC 11B68, 94D05

Language: English

London Journals Press

LJP Copyright ID: 925661
Print ISSN: 2631-8490
Online ISSN: 2631-8504

London Journal of Research in Science: Natural and Formal

Volume 22 | Issue 4 | Compilation 1.0

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I. INTRODUCTION

Queueing systems with repeated attempts are characterized by the fact that a customer finding all the servers busy upon arrival must leave the service area and repeat his request for service after some random time. Between trials, the blocked customer joins a pool of unsatisfied customers called "orbit". Yang and Li[1] has investigated the M/G/1 retrial queue with the server subject to starting failures. Lie and Lee[2] Analysis of fuzzy queues computers an mathematics with applications. Ke et.al[3] studied On retrial queuing model with fuzzy parameter. Negi and Lee[4] investigated and simulation of fuzzy queue fuzzy sets and Systems. Dhurai and Karpagam[5] analyzed a new membership function on hexagonal fuzzy numbers. Zadeh[6] considered Fuzzy sets as a basis a theory of possibility. Rita and Robert[7] studied Application of fuzzy set theory to retrial queues. This paper is organized as follows. In division 2 describe deals the fuzzy queue model. In division 3 we discuss the mean number of customers in the orbit, the mean number of customers in the system and the mean waiting time in the system are studied in fuzzy environment. In division 4 includes numerical study about the performance measures. Finally, conclusion are gained.

London Journal of Research in Science: Natural and Formal

II. THE CRISP MODEL

Consisder an M/G/1 retrial queueing model with Bernoulli schedule and vacation. Customer arrive from outside the system according to a Poisson process with parameter λ. The server provides service rates common distribution with parameter μ_1 and μ_2. The server takes a Bernoulli vacation after each service completion, the server takes a vacation with probability q, and with probability p = 1-q, the vacation rates two moments ξ_1, ξ_2. Arrival rates, service rates, server vacation rates are assumed to be mutually independent.

The state of the system at time t can be described by the Markov process $\{N(t); t > 0\} = \{(C(t), Y(t), \xi_0(t), \xi_1(t), \xi_2(t) \geq 0\}$, where C(t) denotes the server state (0,1 or 2 depending if the server is free, busy or on vacation respectively) and X(t) corresponding to the number of customers in the orbit at time t. If C(t) = 0 and $X(t) > 0$, then $\xi_0(t)$, represents the elapsed retrial time, if C(t) = 1, then $\xi_1(t)$ corresponds to the elapsed time of the customer served, if C(t) = 2 and $Y(t) \geq 0$, then $\xi_2(t)$, represents the elapsed vacation time at time t.

(i) The mean number of customers in the orbit

$$\text{E(O)} = \frac{\lambda^2 p\xi_2}{2[\lambda\xi_1 + 1]} + \frac{\lambda^2\mu_2 + \lambda^2 q\xi_2 + 2\lambda^2 q\xi_1\mu_1}{2[1 - \lambda\mu_1 - \lambda q\xi_1]}$$

(ii) The mean number of customers in the system

$$\text{E(S)} = \lambda\mu_1 + \frac{2\lambda^2\mu_1\xi_1 + \lambda^2\xi_2 + \lambda^2\mu_2}{2[1 - \lambda(\mu_1 + \xi_1)]}$$

(iii) The mean waiting time in the system

$$\text{E(W)} = \mu_1 + \frac{2\lambda\mu_1\xi_1 + \lambda\xi_2 + \lambda\mu_2}{2[1 - \lambda(\mu_1 + \xi_1)]}$$

III. THE MODEL IN FUZZY ENVIRONMENT

In this section the arrival rate λ, service rate μ_1, μ_2, vacation rate ξ_1, ξ_2 are assumed to be fuzzy numbers respectively.

Now,

$$\bar{\lambda} = \left\{(b, \mu_{\bar{\lambda}}(b)); b \in s(\bar{\lambda})\right\}$$
$$\bar{\mu}_1 = \{(c_1, \mu_{\bar{\mu}_1}(c_1)); c_1 \in s(\bar{\mu}_1)\}$$
$$\bar{\mu}_2 = \{(c_2, \mu_{\bar{\mu}_2}(c_2)); c_2 \in s(\bar{\mu}_2)\}$$
$$\bar{\xi}_1 = \left\{(d_1, \mu_{\bar{\xi}_1}(d_1)); d_1 \in s(\bar{\xi}_1)\right\}$$

and

$$\bar{\xi}_2 = \left\{(d_2, \mu_{\bar{\xi}_2}(d_2)); d_2 \in s(\bar{\xi}_2)\right\}$$

Where $S(\bar{\lambda})$, $S(\bar{\mu_1})$, $S(\bar{\mu_2})$, $S(\bar{\xi_1})$, $S(\bar{\xi_2})$ are the universal sets of the arrival rate, service rate, vacation rate and reneging respectively. Define $f(b, c_1, c_2, d_1, d_2)$ as the system performance measure related to the above defined fuzzy queueing model, which depends on the fuzzy membership function $f(\bar{\lambda}, \bar{\mu_1}, \bar{\mu_2}, \bar{\xi_1}, \bar{\xi_2})$. Applying Zadeh's extension principle (1978) the membership function of the performance measure $f(\bar{\lambda}, \bar{\mu_1}, \bar{\mu_2}, \bar{\xi_1}, \bar{\xi_2})$ can be defined as

$$\mu_{\bar{f}(\bar{\lambda},\bar{\mu_1},\bar{\mu_2},\bar{\xi_1},\bar{\xi_2})}(H) = \sup_{\substack{b\in S(\bar{\lambda}) \\ c_1\in S(\bar{\mu_1}) \\ c_2\in S(\bar{\mu_2}) \\ d_1\in S(\bar{\xi_1}) \\ d_2\in S(\bar{\xi_2})}} \left\{min\left\{\mu_{\bar{\lambda}}(b), \mu_{\bar{\mu_1}}(c_1), \mu_{\bar{\mu_2}}(c_2), \mu_{\bar{\xi_1}}(d_1), \mu_{\bar{\xi_2}}(d_2)\right\}/H\right\} \quad (1)$$

where,

$$H = f(b, c_1, c_2, d_1, d_2)$$

If the α- cuts of $f(\bar{\lambda}, \bar{\mu_1}, \bar{\mu_2}, \bar{\xi_1}, \bar{\xi_2})$ degenerate to some fixed value, then the system performance is a crisp number, otherwise it is a fuzzy number.

We obtain the membership function some performance measures namely the mean number of customers in the orbit E(O), the mean number of customers in the system E(S), the mean waiting time in the system E(W). For the system in terms of this membership function are, as follows

$$\mu_{\overline{E[O]}}(I) = \sup_{\substack{b\in S(\bar{\lambda}) \\ c_1\in S(\bar{\mu_1}) \\ c_2\in S(\bar{\mu_2}) \\ d_1\in S(\bar{\xi_1}) \\ d_2\in S(\bar{\xi_2})}} \left\{min\left\{\mu_{\bar{\lambda}}(b), \mu_{\bar{\mu_1}}(c_1), \mu_{\bar{\mu_2}}(c_2), \mu_{\bar{\xi_1}}(d_1), \mu_{\bar{\xi_2}}(d_2)\right\}/I\right\} \quad (2)$$

where,

$$I = f(b, c_1, c_2, d_1, d_2)$$

where,

$$\mathrm{I} = \frac{b^2pd_2}{2[bpd_1 + 1]} + \frac{b^2c_2 + b^2qd_2 + 2b^2qd_1c_1}{2[1 - bc_1 - bqd_1]}$$

$$\mu_{\overline{E[S]}}(J) = \sup_{\substack{b\in S(\bar{\lambda}) \\ c_1\in S(\bar{\mu_1}) \\ c_2\in S(\bar{\mu_2}) \\ d_1\in S(\bar{\xi_1}) \\ d_2\in S(\bar{\xi_2})}} \left\{min\left\{\mu_{\bar{\lambda}}(b), \mu_{\bar{\mu_1}}(c_1), \mu_{\bar{\mu_2}}(c_2), \mu_{\bar{\xi_1}}(d_1), \mu_{\bar{\xi_2}}(d_2)\right\}/J\right\} \quad (3)$$

where,

$$J = f(b, c_1, c_2, d_1, d_2)$$

where,

$$J = bc_1 + \frac{2b^2c_1d_1 + b^2d_2 + b^2c_2}{2[1 - b(c_1 + d_1)]}$$

$$\mu_{\overline{E[W]}}(K) = \sup_{\substack{b \in S(\bar{\lambda}) \\ c_1 \in S(\bar{\mu_1}) \\ c_2 \in S(\bar{\mu_2}) \\ d_1 \in S(\bar{\xi_1}) \\ d_2 \in S(\bar{\xi_2})}} \left\{ min \left\{ \mu_{\bar{\lambda}}(b), \mu_{\bar{\mu_1}}(c_1), \mu_{\bar{\mu_2}}(c_2), \mu_{\bar{\xi_1}}(d_1), \mu_{\bar{\xi_2}}(d_2) \right\} / K \right\} \quad (4)$$

where,

$$K = f(b, c_1, c_2, d_1, d_2)$$

where,

$$K = c_1 + \frac{2bc_1d_1 + bd_2 + bc_2}{2[1 - b(c_1 + d_1)]}$$

Using the fuzzy analysis technique explain, we can find the membership of $\mu_{\overline{E[O]}}$, $\mu_{\overline{E[S]}}$, $\mu_{\overline{E[W]}}$ as a function of the parameter α. Thus the α-cut approach can be used to develop the membership function of $\mu_{\overline{E[O]}}$, $\mu_{\overline{E[S]}}$, $\mu_{\overline{E[W]}}$.

IV. PERFORMANCE OF MEASURE

The mean number of customers in the orbit

Based on Zadeh's extension principle $\mu_{\overline{E[O]}}(I)$ is the supremum of minimum over $\{\mu_{\bar{\lambda}}(b),\ \mu_{\bar{\mu_1}}(c_1),\ \mu_{\bar{\mu_2}}(c_2),\ \mu_{\bar{\xi_1}}(d_1),\ \mu_{\bar{\xi_2}}(d_1)/I = f(b, c_1, c_2, d_1, d_2)\}$

$$I = \frac{b^2pd_2}{2[bpd_1 + 1]} + \frac{b^2c_2 + b^2qd_2 + 2b^2qd_1c_1}{2[1 - bc_1 - bqd_1]}$$

to satisfying $\mu_{\overline{E[O]}}(I) = \alpha,\ 0 \le \alpha \le 1$.

We consider the following five cases:

Case(i): $\mu_{\bar{\lambda}}(b) = \alpha, \mu_{\bar{\mu_1}}(c_1) \ge \alpha, \mu_{\bar{\mu_2}}(c_2) \ge \alpha, \mu_{\bar{\xi_1}}(d_1) \ge \alpha, \mu_{\bar{\xi_2}}(d_2) \ge \alpha$,
Case(ii): $\mu_{\bar{\lambda}}(b) \ge \alpha, \mu_{\bar{\mu_1}}(c_1) = \alpha, \mu_{\bar{\mu_2}}(c_2) \ge \alpha, \mu_{\bar{\xi_1}}(d_1) \ge \alpha, \mu_{\bar{\xi_2}}(d_2) \ge \alpha$,
Case(iii): $\mu_{\bar{\lambda}}(b) \ge \alpha, \mu_{\bar{\mu_1}}(c_1) \ge \alpha, \mu_{\bar{\mu_2}}(c_2) = \alpha, \mu_{\bar{\xi_1}}(d_1) \ge \alpha, \mu_{\bar{\xi_2}}(d_2) \ge \alpha$,
Case(iv): $\mu_{\bar{\lambda}}(b) \ge \alpha, \mu_{\bar{\mu_1}}(c_1) \ge \alpha, \mu_{\bar{\mu_2}}(c_2) \ge \alpha, \mu_{\bar{\xi_1}}(d_1) = \alpha, \mu_{\bar{\xi_2}}(d_2) \ge \alpha$,
Case(v): $\mu_{\bar{\lambda}}(b) \ge \alpha, \mu_{\bar{\mu_1}}(c_1) \ge \alpha, \mu_{\bar{\mu_2}}(c_2) \ge \alpha, \mu_{\bar{\xi_1}}(d_1) \ge \alpha, \mu_{\bar{\xi_2}}(d_2) = \alpha$.

For case (i) the lower and upper bound of α- cuts of $\mu_{\overline{E[O]}}$ can be obtained through the corresponding parametric non-linear programs,

$$[E[O]]_{\alpha}^{L_1} = \min_{\Omega} \{[I]\} \text{ and } [E[O]]_{\alpha}^{U_1} = \max_{\Omega} \{[I]\}.$$

Similarly, we can calculate the lower and upper bounds of the α-cuts of $\mu_{\overline{E[O]}}$ for the all cases (ii),(iii),(iv) and (v). By considering all the cases simultaneously the lower and upper bounds of the α-cuts of $\overline{[E[O]]}$ can be written as

$$[E[O]]_{\alpha}^{L} = \min_{\Omega} \{[I]\} \quad \text{and} \quad [E[O]]_{\alpha}^{U} = \max_{\Omega} \{[I]\}$$

where,

$$\text{I} = \frac{b^2 p d_2}{2[bpd_1 + 1]} + \frac{b^2 c_2 + b^2 q d_2 + 2b^2 q d_1 c_1}{2[1 - bc_1 - bqd_1]}$$

such that

$$[b]_{\alpha}^{L} \leq \text{b} \leq [b]_{\alpha}^{U},\ [c_1]_{\alpha}^{L} \leq c_1 \leq [c_1]_{\alpha}^{U},\ [c_2]_{\alpha}^{L} \leq c_2 \leq [c_2]_{\alpha}^{U},$$
$$[d_1]_{\alpha}^{L} \leq d_1 \leq [d_1]_{\alpha}^{U} \text{ and } [d_2]_{\alpha}^{L} \leq d_2 \leq [d_2]_{\alpha}^{U}.$$

If both $[E[O]]_{\alpha}^{L}$ and $[E[O]]_{\alpha}^{U}$ are invertible with respect to α, the left and right shape function, $L(I) = [E[O]_{\alpha}^{L}]^{-1}$ and $R(I) = [E[O]_{\alpha}^{U}]^{-1}$ can be derived from which the membership function $\mu_{\overline{E[O]}}(I)$ can be constructed as

$$\mu_{\overline{E[O]}}(I) = \begin{cases} L(I), & [E[O]]_{\alpha=0}^{L} \leq I \leq [E[O]]_{\alpha=1}^{L} \\ 1, & [E[O]]_{\alpha=1}^{L} \leq I \leq [E[O]]_{\alpha=1}^{U} \\ R(I), & [E[O]]_{\alpha=1}^{U} \leq I \leq [E[O]]_{\alpha=0}^{U} \end{cases} \tag{5}$$

The mean number of customers in the system

We can calculate the lower and upper bounds of the α-cuts of$[E[S]]$ as, $\mu_{\overline{E[S]}}(J)$ is the supremum of minimum over

$$\{\mu_{\bar{\lambda}}(b),\ \mu_{\bar{\mu_1}}(c_1),\ \mu_{\bar{\mu_2}}(c_2),\ \mu_{\bar{\xi_1}}(d_1),\ \mu_{\bar{\xi_2}}(d_1)/J = f(b, c_1, c_2, d_1, d_2)\}$$

$$[E[S]]_{\alpha}^{L} = \min_{\Omega} \{[J]\} \quad \text{and} \quad [E[S]]_{\alpha}^{U} = \max_{\Omega} \{[J]\}$$

where,

$$\text{J} = bc_1 + \frac{2b^2 c_1 d_1 + b^2 d_2 + b^2 c_2}{2[1 - b(c_1 + d_1)]}$$

such that

$[b]_\alpha^L \leq \text{b} \leq [b]_\alpha^U,\ [c_1]_\alpha^L \leq c_1 \leq [c_1]_\alpha^U,\ [c_2]_\alpha^L \leq c_2 \leq [c_2]_\alpha^U,$
$[d_1]_\alpha^L \leq d_1 \leq [d_1]_\alpha^U$ and $[d_2]_\alpha^L \leq d_2 \leq [d_2]_\alpha^U$.

If both $[E[S]]_\alpha^L$ and $[E[S]]_\alpha^U$ are invertible with respect to α, the left and right shape function, $L(J) = [E[S]_\alpha^L]^{-1}$ and $R(J) = [E[S]_\alpha^U]^{-1}$ can be derived from which the membership function $\mu_{\overline{E[S]}}(J)$ can be constructed as,

$$\mu_{\overline{E[S]}}(J) = \begin{cases} L(J), & [E[S]]_{\alpha=0}^L \leq J \leq [E[S]]_{\alpha=1}^L \\ 1, & [E[S]]_{\alpha=1}^L \leq J \leq [E[S]]_{\alpha=1}^U \\ R(J), & [E[S]]_{\alpha=1}^U \leq J \leq [E[S]]_{\alpha=0}^U \end{cases} \tag{6}$$

The mean number of customers in the waiting time

We can calculate the lower and upper bounds of the α-cuts of$[E[W]]$ as, $\mu_{\overline{E[W]}}(K)$ is the supremum of minimum over
$\{\mu_{\bar{\lambda}}(b),\ \mu_{\bar{\mu_1}}(c_1),\ \mu_{\bar{\mu_2}}(c_2),\ \mu_{\bar{\xi_1}}(d_1),\ \mu_{\bar{\xi_2}}(d_1)/K = f(b, c_1, c_2, d_1, d_2)\}$

$$[E[W]]_\alpha^L = \min_\Omega \{[K]\} \text{ and } [E[W]]_\alpha^U = \max_\Omega \{[K]\}$$

where,

$$\text{K} = c_1 + \frac{2bc_1d_1 + bd_2 + bc_2}{2[1 - b(c_1 + d_1)]}$$

such that

$[b]_\alpha^L \leq \text{b} \leq [b]_\alpha^U,\ [c_1]_\alpha^L \leq c_1 \leq [c_1]_\alpha^U,\ [c_2]_\alpha^L \leq c_2 \leq [c_2]_\alpha^U,$
$[d_1]_\alpha^L \leq d_1 \leq [d_1]_\alpha^U$ and $[d_2]_\alpha^L \leq d_2 \leq [d_2]_\alpha^U$.

If both $[E[W]]_\alpha^L$ and $[E[W]]_\alpha^U$ are invertible with respect to α, the left and right shape function, $L(K) = [E[W]]_\alpha^L]^{-1}$ and $R(K) = [E[W]]_\alpha^U]^{-1}$ can be derived from which the membership function $\mu_{\overline{E[W]}}(K)$ can be constructed as,

$$\mu_{\overline{E[W]}}(K) = \begin{cases} L(K), & [E[W]]_{\alpha=0}^L \leq K \leq [E[W]]_{\alpha=1}^L \\ 1, & [E[W]]_{\alpha=1}^L \leq K \leq [E[W]]_{\alpha=1}^U \\ R(K), & [E[W]]_{\alpha=1}^U \leq K \leq [E[W]]_{\alpha=0}^U \end{cases} \tag{7}$$

V. NUMERICAL STUDY

The mean number of customers in the orbit

Suppose the fuzzy arrival rate λ, service rate μ_1, μ_2, vacation rate $\overline{\xi_1}$, $\overline{\xi_2}$ are assumed to be hexagonal fuzzy numbers described by
$\overline{\lambda} = [11, 12, 13, 14, 15, 16]$, $\overline{\mu_1} = [13, 14, 15, 16, 17, 18]$, $\overline{\mu_2} = [41, 42, 43, 44, 45, 46]$,
$\overline{\xi_1} = [91, 92, 93, 94, 95, 96]$ and $\overline{\xi_2} = [101, 102, 103, 104, 105, 106]$ per hour respectively.
Then,
$\lambda(\alpha) = \{ \min_{x \in s(\bar{\lambda})} \{x \in s(\bar{\lambda}), G(x) \geq \alpha\}, \max_{x \in s(\bar{\lambda})} \{x \in s(\bar{\lambda}), G(x) \geq \alpha\}\}$,
where,

$$G(x) = \begin{cases} \frac{1}{2}(\frac{x-a_1}{a_2-a_1}), & \text{for} \quad a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}(\frac{x-a_1}{a_3-a_2}), & \text{for} \quad a_2 \leq x \leq a_3 \\ 1, & \text{for} \quad a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}(\frac{x-a_4}{a_3-a_4}), & \text{for} \quad a_4 \leq x \leq a_5 \\ \frac{1}{2}(\frac{a_6-x}{a_6-a_5}), & \text{for} \quad a_5 \leq x \leq a_6 \\ 0, & \text{for} \quad \text{otherwise} \end{cases} \tag{8}$$

That is, $\lambda(\alpha) = [11 + \alpha, 16 - \alpha]$, $\mu_1(\alpha) = [13 + \alpha, 18 - \alpha]$, $\mu_2(\alpha) = [41 + \alpha, 46 - \alpha]$,
$\xi_1(\alpha) = [91 + \alpha, 96 - \alpha]$ and $\xi_2(\alpha) = [101 + \alpha, 106 - \alpha]$.

It is clear that, when $b = b_\alpha^U$, $c_1 = {c_1}_\alpha^U$, $c_2 = {c_2}_\alpha^U$, $d_1 = {d_1}_\alpha^U$ and $d_2 = {d_2}_\alpha^U$, I attains its maximum value and when $b = b_\alpha^L$, $c_1 = {c_1}_\alpha^L$, $c_2 = {c_2}_\alpha^L$, $d_1 = {d_1}_\alpha^L$ and $d_2 = {d_2}_\alpha^L$, I attains its minimum value.

From the generated for the given input values of $\overline{\lambda}$, $\overline{\mu}_1$, $\overline{\mu}_2$, $\overline{\xi}_1$, $\overline{\xi}_2$,

(i) For fixed values of b,c_1,c_2 and d_1 I decreases as d_2 increase.
(ii) For fixed values of c_1,c_2,d_1 and d_2 I decreases as b increase.
(iii)For fixed values of b,c_2,d_1 and d_2 I decreases as c_1 increase.
(iv) For fixed values of b,c_1,d_1 and d_2 I decreases as c_2 increase.
(v) For fixed value of b,c_1,c_2 and d_2 I decreases as d_1 increase.

The minimum value of occurs when x-takes its lower bound.
i.e)., $b = 11 + \alpha$ and c_1, c_2, d_1 and d_2 takes their upper bounds given by $c_1 = 18 - \alpha$, $c_2 = 46 - \alpha$, $d_1 = 96 - \alpha$, and $d_2 = 106 - \alpha$ respectively, and the maximum value of E[O] occurs when $b = 16 - \alpha$, $c_1 = 13 + \alpha$, $c_2 = 41 + \alpha$, $d_1 = 91 + \alpha$ and $d_2 = 101 + \alpha$. If both $[E[O]]_\alpha^L$ and $[E[O]]_\alpha^U$ are invertible with respect to 'α' then, the left shape function $L(I) = [E[O]_\alpha^L]^{-1}$ and right shape function $R(I) = [E[O]_\alpha^U]^{-1}$ can be obtained and from

which the membership function $\mu_{\overline{E[O]}}(I)$ can be constructed as

$$\mu_{\overline{E[O]}}(I) = \begin{cases} 0.5(x-1), & \text{for} \quad I_1 \leq I \leq I_2 \\ 0.5+0.5(x-1), & \text{for} \quad I_2 \leq I \leq I_3 \\ 1, & \text{for} \quad I_3 \leq I \leq I_4 \\ 1+0.5(x-4), & \text{for} \quad I_4 \leq I \leq I_5 \\ 0.5(6-x) & \text{for} \quad I_5 \leq I \leq I_6 \\ 0, & \text{for} \quad \text{otherwise} \end{cases} \tag{9}$$

For the given set of input values, the values of I_1, I_2, I_3, I_4, I_5 and I_6 evaluated c programme given above are 0.000, 8.717, 9.264, 9.154, 8.826 and 0.000 respectively and

$$\mu_{\overline{E[O]}}(I) = \begin{cases} 0.5(x-1), & \text{for} \quad 0.000 \leq I \leq 8.717 \\ 0.5+0.5(x-1), & \text{for} \quad 8.717 \leq I \leq 9.264 \\ 1, & \text{for} \quad 9.264 \leq I \leq 9.154 \\ 1+0.5(x-4), & \text{for} \quad 9.154 \leq I \leq 8.826 \\ 0.5(6-x) & \text{for} \quad 8.826 \leq I \leq 0.000 \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \tag{10}$$

The graphs of the shape function L(I) and R(I) are given in the figure 1

The mean number of customers in the system

The minimum value of occurs when x-takes its lower bound. i.e)., $b = 11 + \alpha$ and c_1, c_2, d_1 and d_2 takes their upper bounds given by $c_1 = 18 - \alpha$, $c_2 = 46 - \alpha$, $d_1 = 96 - \alpha$ and $d_2 = 106 - \alpha$ respectively and the maximum value of E[S] occurs when $b = 16 - \alpha$, $c_1 = 13 + \alpha$, $c_2 = 41 + \alpha$, $d_1 = 91 + \alpha$ and $d_2 = 101 + \alpha$. If both $[E[S]]_\alpha^L$ and $[E[S]]_\alpha^U$ are invertible with respect to 'α' then, the left shape function $L(J) = [E[S]_\alpha^L]^{-1}$ and right shape function $R(J) = [E[S]_\alpha^U]^{-1}$ can be obtained and from which the membership function $\mu_{\overline{E[S]}}(J)$ can be constructed as:

$$\mu_{\overline{E[S]}}(J) = \begin{cases} 0.5(x-1), & \text{for} \quad J_1 \leq J \leq J_2 \\ 0.5+0.5(x-1), & \text{for} \quad J_2 \leq J \leq J_3 \\ 1, & \text{for} \quad J_3 \leq J \leq J_4 \\ 1+0.5(x-4), & \text{for} \quad J_4 \leq J \leq J_5 \\ 0.5(6-x) & \text{for} \quad J_5 \leq J \leq J_6 \\ 0, & \text{for} \quad \text{otherwise} \end{cases} \tag{11}$$

For the given set of input values, the values of J_1, J_2, J_3, J_4, J_5 and J_6 evaluated c programme given above are 0.8462, 1.9776, 2.0829, 2.0847, 1.9793 and 0.8710 respectively and

$$\mu_{\overline{E[S]}}(J) = \begin{cases} 0.5(x-1), & \text{for} \quad 0.8462 \leq J \leq 1.9776 \\ 0.5+0.5(x-1), & \text{for} \quad 1.9776 \leq J \leq 2.0829 \\ 1, & \text{for} \quad 2.0829 \leq J \leq 2.0847 \\ 1+0.5(x-4), & \text{for} \quad 2.0847 \leq J \leq 1.9793 \\ 0.5(6-x) & \text{for} \quad 1.9793 \leq J \leq 0.8710 \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \tag{12}$$

The graphs of the shape function L(J) and R(J) are given in the figure 2

The mean number of customers in the waiting time

The minimum value of occurs when x-takes its lower bound.
i.e)., $b = 11 + \alpha$ and c_1, c_2, d_1 and d_2 takes their upper bounds given by $c_1 = 18 - \alpha$, $c_2 = 46 - \alpha$, $d_1 = 96 - \alpha$ and $d_2 = 106 - \alpha$ respectively and the maximum value of E[W] occurs when $b = 16 - \alpha$, $c_1 = 13 + \alpha$, $c_2 = 41 + \alpha$, $d_1 = 91 + \alpha$ and $d_2 = 101 + \alpha$. If both $[E[W]]_\alpha^L$ and $[E[W]]_\alpha^U$ are invertible with respect to 'α' then, the left shape function $L(K) = [E[W]_\alpha^L]^{-1}$ and right shape function $R(K) = [E[W]_\alpha^U]^{-1}$ can be obtained and from which the membership function $\mu_{\overline{E[W]}}(K)$ can be constructed as:

$$\mu_{\overline{E[W]}}(K) = \begin{cases} 0.5(x-1), & \text{for} \quad K_1 \le K \le K_2 \\ 0.5+0.5(x-1), & \text{for} \quad K_2 \le K \le K_3 \\ 1, & \text{for} \quad K_3 \le K \le K_4 \\ 1+0.5(x-4), & \text{for} \quad K_4 \le K \le K_5 \\ 0.5(6-x) & \text{for} \quad K_5 \le K \le K_6 \\ 0, & \text{for} \quad \text{otherwise} \end{cases} \tag{13}$$

For the given set of input values, the values of K_1, K_2, K_3, K_4, K_5 and K_6 evaluated c programme given above are 0.0000, 13.5398, 14.1564, 14.4615, 13.8491 and 0.0000 respectively and

$$\mu_{\overline{E[W]}}(K) = \begin{cases} 0.5(x-1), & \text{for} \quad 0.0000 \le K \le 13.5398 \\ 0.5+0.5(x-1), & \text{for} \quad 13.5398 \le K \le 14.1564 \\ 1, & \text{for} \quad 14.1564 \le K \le 14.4615 \\ 1+0.5(x-4), & \text{for} \quad 14.4615 \le K \le 13.8491 \\ 0.5(6-x) & \text{for} \quad 13.8491 \le K \le 0.0000 \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \tag{14}$$

The graphs of the shape function L(K) and R(K) are given in the figure 3

In this paper we fix the service rate $\bar{\mu_2}$ by crisp value 43.5 and taking arrival rate $\overline{\lambda} = [11, 12, 13, 14, 15, 16]$, service rate $\overline{\mu_1} = [13, 14, 15, 16, 17, 18]$ both hexagonal fuzzy numbers and the values of the mean number of customer in the orbit were generated and from the figure 6.1. It is observed that as $\overline{\lambda}$ increases the mean number of customer in the orbit increases for the fixed value of the service rate. Whereas for fixed value of arrival rate, the mean number of customer in the orbit decreases as service rate increases. It is also observed from the data generated that, the membership value of the mean number of customer in the orbit is 9.4, when the ranges of arrival rate, service rate lie in the intervals $(13, 14.4)$, $(15.5, 16.5)$ respectively.

For fixed value of $\xi_1 = 93$ and 93.3 the graph of the mean number of customer in the system drawn in figure 6.2 shows that, as arrival rate increases the mean number of customer in the system also increases, while the vacation time decreases as the mean number of customer in the system increases. It is also observed from the data generated that, the membership value of the mean number of customer in the system is 2.1, when the ranges of arrival rate, service rate, and the working vacation rate lie in the intervals $(13.5, 14.3)$, $(93, 94.3)$ respectively.

Again for fixed values of $\xi_2 = 102.5$ and 103.7 the graph of the mean reneging rate of the system are drawn in figure 6.3 respectively. This figure shows that as arrival rate increases, also the mean number of customer in the waiting time increases, while the the mean number of customer in the waiting time decreases as the service rate increases. It is also observed from the data generated, that the membership value of the the mean number of customer in the waiting time is 15.06, when the ranges of arrival rate, service rate, and the vacation rate lie the intervals $(12.5, 13.8)$, $(102, 104.6)$ respectively.

The following three graphs are represent the performance mea- sures

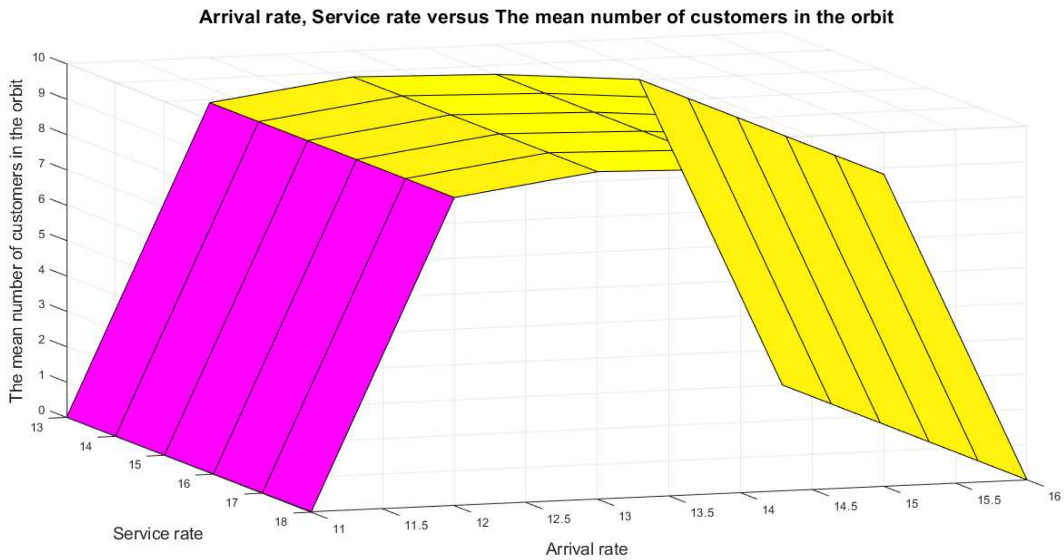

Figure 1: Arrival Rate, Service Rate Versus the Mean Number of Customer in the Orbit

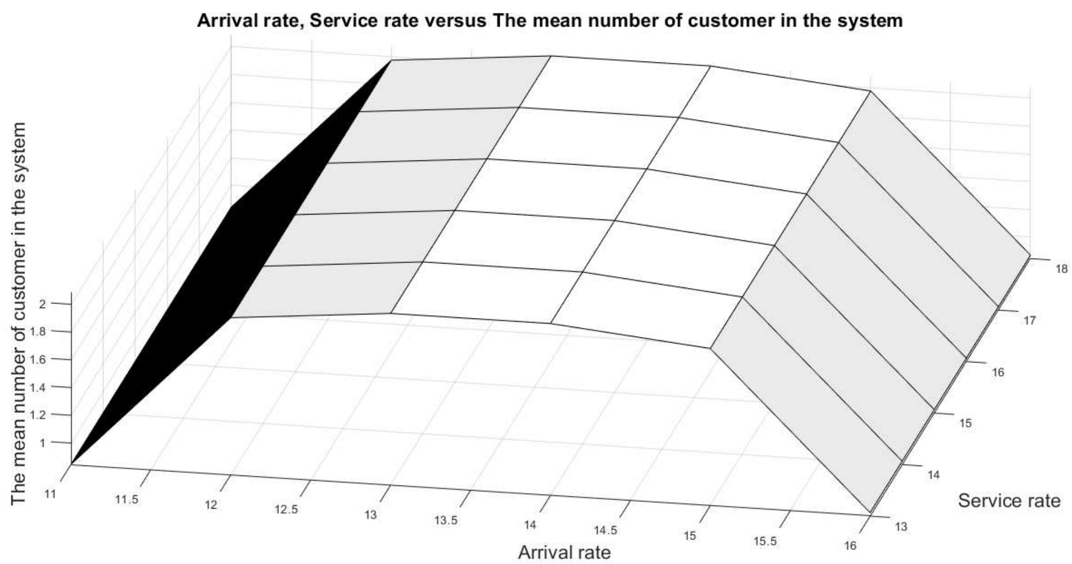

Figure 2: Arrival Rate, Service Rate Versus the Mean Number of Customer in the System

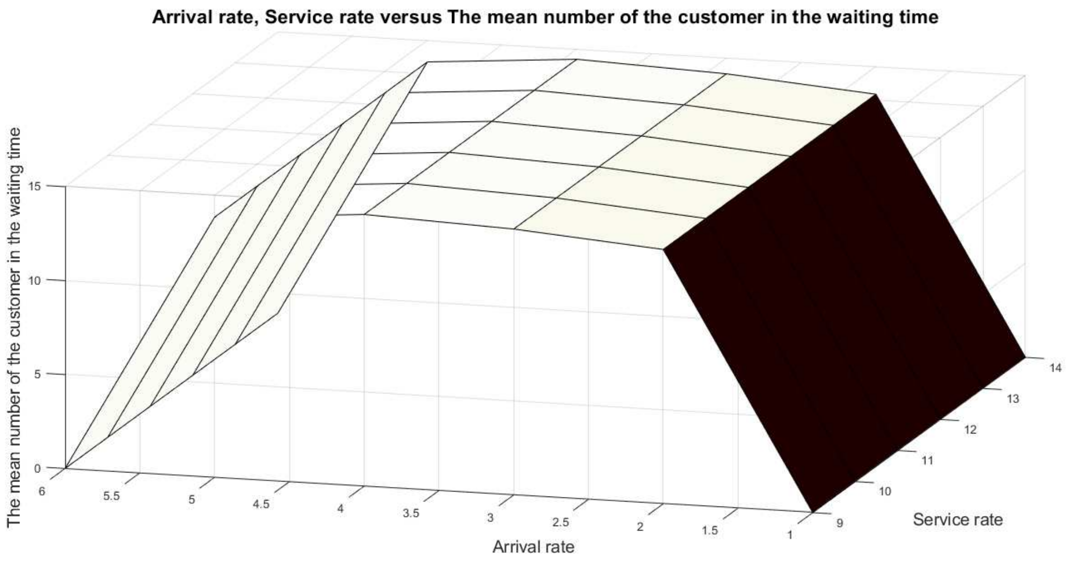

Figure 3: Arrival Rate, Service Rate Versus the Mean Number of Customer in the Waiting Time

VI. CONCLUSION

In this paper we have studied analysis of FM/FG/1 retrial queue with Bernoulli schedule and vacation using hexagonal fuzzy numbers. Based on Zadeh's extension principle, system performances of interest for the mean number of customers in the orbit, the mean number of customers in the system and the mean number in the waiting time. We have obtained numerical results to all the performances and graphs to the corresponding measures are obtained. Basically, queue formation is a phenomenon that often occurs when the current demand for a service excees the current capacity to offer that service. This fuzzy retrial queueing systems are useful everywhere in the society. The capablity of these systems can have an impact result on the quality of human lives and productivity of the process. Fuzzy retrial queueing systems were applied to procure the performance analysis of different system like telephone switching systems, computers competing to gain service from a central processing unit etc., Fuzzy retrial queues are more realistic when compared to crisp queues.

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