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Mannu Arya & Vipin Verma

Lovely Professional University

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Keywords: chebyshev polynomial, first kind, second kind, properties and applications.

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ABSTRACT

It develops a formula that explicitly expresses the general term of a linear recurrent sequence, allowing us to generalize J. McLaughlin's original finding on powers of 2 matrices to the case of a square matrix of size ≤ 2 matrix. The identities of Fibonacci and Stirling numbers, as well as a variety of combinatorial relations, are deduced. It uses two-variable Hermit polynomials and their operational laws to derive integral representations of Chebyshev polynomials. Most of the Chebyshev polynomial properties can be obtained using the Hermit polynomials $H_n(x, y)$ *definitions and formalism. They also show how to use these results to introduce valid generalizations of these polynomial groups and derive new identities and integral representations for them. For Chebyshev polynomials of the first and second kinds, its present new generating functions. A recurrence relation is an important mathematical concept. Recurrence relations are used in a variety of fields, including mathematics, economics, physics, and other sciences. It presents a significant finding on the convergence of recurrence relation sequences as a function of the recurrence relation coefficient.*

The major goal of this study is to use Girard and Waring's numerical and numerical adoption to the problem of Chebyshev polynomials' potential values and to identify certain structures that may be lost. As a specific application of our concept, we discovered two concurrent outcomes involving Fibonacci and Lucas numbers. The major goal of this work is to employ a few Chebyshev polynomial features to focus on the challenges of mixing the Sinx And Cosx energies, as well as to identify other interesting applications.

Keywords: chebyshev polynomial, first kind, second kind, properties and applications.

Author: Department of Mathematics, School of Chemical Engineering and Physical Sciences Lovely Professional University, Phagwara 144411, Punjab (INDIA).

I. INTRODUCTION

It is in the area of mathematical analysis that the orthogonal Polynomial and Dynamic functions have been established continuously and orthogonal analysis is not an exception. Orthogonal polynomials are the object of extensive employment, in particular and classical orthogonal polynomials have. Several problems apply to proven math, theory physics, chemistry, approximate principles, etc, with the accidentally of numbers of problems and other disciplines as well.

A significant subject in mathematics is a recurrent relationship. It is necessary for a recurrence relationship to the subject of mathematics. In both mathematics and economy, recurrent relations are used Physics and others in subject areas. Number theories are the principal field of study which this paper explores. "Learning to the popular theorist Carl Friedrich Gauss: "Mathematics is the queen of all science, and Numbers theory is the queen of Mathematical Studies. The study of numerology is the examination of the characteristics of integer and rational numbers, which exceed the habitual tricks of math." This in which nutritional philosophy will go into his history and not like "because of set" and

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Save in. Relationships between repeated subjects are implemented in both mathematics and economics. The key effects of the convergence of the series of recurrence depend on the recovery coefficient. We're talking about a few examples. We will use recurrence in network marketing in this document. Recurrence relationships are very useful subjects for mathematics that solve many real-life problems by repeat relations. In modern times, network marketing is a very well-known business practice for a lot of people. The use of a recurrence relationship method which results based on the recurrence coefficient of the network marketing enterprise. This approach is therefore very useful in evaluating the benefit of any network marketing provider. There are some major concepts developed for recurrence relations. These personalities are very valuable in seeking the terms in any series for the recurrent correlation. In this recurring method, to find out any terms all previous terms need to be found and that result is very necessary. A partnership between an individual has a significant property terms and root polynomial values of a recurrence relations for Second Order relationship, we gave the same recurrence relationship property of all Higher Order recurrence relations. We may eventually conclude that all recurrence relationship involve roots are different, this concept, being true, all order. We may therefore presume that the text of a joint relationship between a number of variables, the values, and roots of a polynomial relationship.

First Order Recurrence Relation

In the first order recurrence relation only one initial term is given. For example

$$a_{n+1} = a_n + 5, n \geq 1, a_0 = 0$$

we can find the terms

$$a_1 = 5, a_2 = 10, a_3 = 15$$

Second Order Recurrence Relation

In the second order recurrence relation new term depend on two previous terms and two initial terms are given.

For example

Third Order Recurrence Relation

In the third order recurrence relation new term depends on three previous terms with three initial terms are given.

For example

$$a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3}, n \geq 3$$

With the initial terms $a_0 = 0, a_1 = 1, a_2 = 2$

$$a_n = a_{n-1} + 2a_{n-2}, n \geq 2$$

With the initial terms $a_0 = 0, a_1 = 1$

Third Order Recurrence Relation

In the 3rd order recurrence relation new term is depend on previous three terms. For example

$$a_n = a_{n-3} + 2a_{n-2} + a_{n-1}, n \geq 3$$

With the initials terms $a_0 = 0, a_1 = 1, a_2 = 2.$

The relation between primary numbers and perfect numbers, including composite numbers, exists.

The first term will be 2 × 2 = 4, next 2 × 3 = 6, 3 × 3 = 9, etc. in composite numbers by means of primaries; 2, 3,5,7,11,13,17,19,23,29,31 etc. and by means of a composite count number, in line with the formula for m and n composites, where m and n shall be both primarily numbers. The processing is such that there is no number between them, such as 3x3, and multiply 2 by 5, to get 10. However, it is laborious to achieve composite numbers that are far-reaching, so that the nth composite number (Cn) can be produced with the Wolfram Language Code as above. It is a code for the computer. A manual algorithm for extrapolating large composite numbers is exceedingly difficult to develop. But the generating function of Dirichlet is used for the characteristic function of the composite numbers. The relationship can also be dealt with as follows in ideal numbers; (Power of Two) × (Double that Power - 1). The prime number formula is given by (2n-1). To get the Perfect Number, the formula becomes (2n-1) × (2n-1). Getting the nth sequence of a perfect number is dependent on the equivalent nth sequence of prime numbers. The research finally concludes with the call for researchers to team up to make more explicit, the recurrence relation in composite numbers. The study focuses its intention on recurrence relation in perfect and composite numbers. It is hinged at analyzing the mathematical relationships between them, as well as asking new questions about them and to prove that these relationships are true.

Recurrence Relation of Network Marketing is the sector came into being, marketing has become one of the main data mining applications. The determination whether a specific individual is to advertise is usually based solely on its characteristics or those of the population segments to which they belong (direct marketing) (mass marketing). This also results in optimum targeting choices since the impact of a business consumers on each other's buying decisions is not taken into consideration. Customers are heavily affected by the views of their partners in many markets. Viral marketing does this to sell a commodity cheaply, mostly through marketing people with the highest consumer power.

II. RESEARCH METHODOLOGY

3.1 Introduction

Methodology is a "focused paradigm' for science, a consistent and rational approach centered on opinions, and principles that direct researchers and other users to choose. It contains theorical analyses of the community of methods and concepts linked to a branch of expertise, which differs according to their historical creation in the different disciplines. These methods, described in the methodology, define the means or modes of data collection or, sometimes, how a specific result is to be calculated. Methodology does not define specific methods, even though much attention is given to the nature and kinds of processes to be followed in a particular procedure or to attain an objective.

A methodology is a system of methods and principles for doing something, for example for teaching or for carrying out research. choosing a wholly suitable and sound method that is right for research project will give the path to help succeed. A methodology will give guidelines to make the project manageable, smooth, and effective.

Research methodology defines the pattern of performing research. The research opinion preferred for this chapter is based on the type of methodology selected. With the requirements and prerequisites that are defined by an individual, a specific methodology can be selected for testing the research questions and find results accordingly. A specific time limit is defined on every segment of the study and the desired target is achieved.

1. The proposed methodology of work will be phase wise as described in the following sequence: I. Development of techniques / recent advances for application in different fields.
2. Collection of literature regarding with the apropos research work.
3. Derivation of new results related with study of generalization of Fibonacci sequence.

There are two main directions in which the Fibonacci polynomials can be generalized either the recurrence relation can be generalized and extended, or the recurrence relation is preserved but the first two terms are replaced by arbitrary terms.

Research methodology refers to the steps, procedures and strategies used for gathering data during the research investigation . This chapter is concerned with methodology used in achieving the research aim. It also contains focus the of study and instrument of data collection used.

Recurrence relation is especially useful topic of mathematics. It is an equation that defines a sequence based on a method that gives the next term as relation of the previous terms. Recurrence relation is especially useful in mathematics as well as economics. This can calculate growth in economics by recurrence techniques. In recurrence relation for finding any term of sequence itneeds to find all previous terms of sequence but by using this theorem it can find direct any term of sequence. Recurrence relation is especially useful in real life problems. Should be most of the people in the network marketing are honest, trying to earn a living to provide a high-quality lifestyle for themselves and their families.

Number Theory is the study of positive numbers (1,2,3,4,5,6,7...) which scrutinize the properties of integers, the natural numbers which is common as -1,-2,0,1,2 and so forth. It is part theoretical and part experimental, as mathematics seeks to discover fascinating and even unexpected mathematical interactions. It is the study of subtle and far-reaching relationships of numbers. This far end reaching relationships have its application in computer algorithm as in Fibonacci numbers.

Since ancient times, people have separated the natural numbers into a variety of different types which includes but not restricted to odd numbers, cube numbers, prime numbers, composite numbers, 1(modulo 4) number, 3(modulo 4) numbers, triangular numbers, perfect numbers, Fibonacci numbers, etc.

3.2 Focus of the Study

1. The study focuses on the number and polynomial number recurrence relation. It is concerned with analyzing and asking new questions about the mathematical relationships between them and showing that these connections are valid.
2. Recurrences are an extremely useful tools (sometimes unique) to solve many counter problems, where the use of established combinatorial techniques makes it challenging (or impossible) to count items. Therefore, the RR and its solution complement the experience of combinatory and therefore in the philosophy of probability and statistics. In the well-known books of Combinatory, the subject of recurrences and their resolving takes place by chance.
3. The recurrence is causally linked to the recursion, which the names indicate. The RR is used less or more in recurrence for learners and is considered in several textbooks together. The recursive computing of n! the nth number of Fibonacci, etc. are classic examples of recurring functions in any programming textbook. These features are more naturally based on the respective recurrences (i.e., recursive definitions). They can then be used to explain recursion and the main steps it takes forward steps and backward steps in the simple case (below) of the recursion (optional in some recursions). For e.g., recursive calls are defined by the subsequent recurrence and the steps forward correspond to the extension of the recurrence terms as per the iteration process (but it happens automatically, by pushing stack frames in the program stack).

As a simple case of recursion, the original conditions are used. The number and polynomial measures are inductive for the computer – beginning from the initial words, when all terms are calculated up to the (n – 1) set, the nth term is calculated (the subject “recursion and iteration” is essential and comprehensive, so further consideration is needed). The Fibonacci number illustration is an inefficient recursion classical example. The best way to show and clarify that this recursion is inefficient is using the appropriate recursion tree. By it can infer for the utility of related function: "Let the recurrence of form (1) of order k > 1 be defined and its initial conditions. For each recurrence word, a recursive function that determines the nth term of this series with recursive calls is unsuccessful. Here are two established strategies to prevent the unsuccessful recursion: memorization (where a certain value is calculated, stored in an array that will be used any time a recurrence attempts to compute it) and repetition through iteration and usage of a stack if required (i.e., applying the bottom-up approach).The recurrences are used in the analysis of complexity of algorithms, mostly recursive. As it mentioned above, the recurrences and the recursion trees are appropriate, powerful, and unique tools for investigation the time-complexity of algorithms, based on a strategy "divide-and-conquer".

III. RESULT ANALYSIS

Analysis- Generalized Fibonacci sequences: Now, let us consider for q ≥ 1, the "multibonacci" sequence $(\phi_n^q)n \geq -q$ defined:

by

$$\begin{cases} \phi_{-q}^{(q)} = \ldots\ldots = \phi_{-2}^{(q)} = \phi_{-1}^{(q)} = 0, \\ \phi_0^{(q)} = 1, \\ \phi_n^{(q)} = \phi_{n-1}^{(q)} + \phi_{n-2}^{(q)} + \ldots + \phi_{n-q-1}^{(q)} \; for \; n \geq 1. \end{cases}$$

In[1] Belbachir and Bencherif showed that

$$\phi_n^{(q-1)} = \sum_{k_1+2k_2+\ldots\ldots+qk_q=n} \binom{k_1+k_2+\ldots\ldots+k_q}{k_1, k_2, \ldots\ldots, k_q},$$

And, for q ≥ 1,

$$\phi_n^{(q-1)} = \sum_{k=0}^{\lfloor n/(q+1) \rfloor} (-1)^k \frac{n-k(q-1)}{n-kq} \binom{n-kq}{k} 2^{n-1-k(q+1)},$$

Belongs to

$$\sum_{k_1+2k_2+\ldots\ldots+qk_q} \binom{k_1+k_2+\ldots\ldots+k_q}{k_1, k_2, \ldots\ldots, k_q} = \sum_{k=0}^{\lfloor n/(q+1) \rfloor} (-1)^k \frac{n-k(q-1)}{n-kq} \binom{n-kq}{k} 2^{n-1-k(q+1)}$$

Theorem 4.1.the identity is as following

$$\phi_n^{(q)} = \sum_{l=0}^{qm-r} \binom{n-l}{l}_q (4.1)$$

Where m is given for division via the extended euclidean algorithm: n = m(q +1)-r,0≤r≤q.

Proof. It says that

$$\phi_n^{(q)}=\sum_{k_1+2k_2+......+qk_q=n}\binom{k_1+k_2+.....+k_{q+1}}{k_1,k_2,....,k_{q+1}}$$

$$=\sum_{L\geq 0}\sum_{k_1+2k_2+......+(q+1)k_{q+1}=n}\binom{L}{k_1,k_2,....,k_{q+1}}$$

$$=\sum_{L\geq 0}\sum_{k_1+2k_2+......+(q+1)k_{q+1}=n-L}\binom{L}{L-k_2-....-k_{q+1},k_2,....,k_{q+1}}$$

$$=\sum_{L\geq 0}\binom{L}{n-L}_q$$

$$=\sum_{L\geq\frac{n}{q+1}}^{n}\binom{L}{n-L}_q,$$

the fact that is using as $\binom{L}{a}_q = 0$ for a < 0 or a > qL

Consider the unique text of n given by the extended euclidean division algorithm

$$:n=m(q+1)-r, 0\leq r<q+1 \text{ then } \frac{n}{q+1}=m-\frac{r}{q+1}, \text{ which gives}$$

In[1]Belbachir and Bencherif showed that

$$\phi_n^{(q)}=\sum_{l=0}^{qm-r}\binom{m+k}{qm-r-k}_q=\sum_{l=0}^{qm-r}\binom{m+k}{(q+1)k+r}_q=\sum_{l=0}^{qm-r}\binom{n-l}{l}_q.$$

This is obtain the following identities as an immediate consequence of Theorem 4.1.

$$\phi_{(q+1)m}^{(q)}=\sum_{l=0}^{qm}\binom{(q+1)m-l}{l}_q=\sum_{k=0}^{qm}\binom{m+k}{(q+1)k}_q,$$

$$\phi_{(q+1)m-1}^{(q)}=\sum_{l=0}^{qm-1}\binom{(q+1)m-l-1}{l}_q=\sum_{l=0}^{qm}\binom{m+k}{(q+1)k+1}_q,$$

$$\phi_{(q+1)m-r}^{(q)}=\sum_{l=0}^{qm-1}\binom{(q+1)m-l-r}{l}_q=\sum_{l=0}^{qm}\binom{m+k}{(q+1)k+1}_q,$$

The classic sequence Fibonacci is found for q = 1:

$$F_{-1} = 0,\ F_0 = 1,\ F_{n+1} = F_n + F_{n-1}, \text{ for } n \geq 0.$$

Thus, it achieve the common identity $F_n=\sum_{l=0}^{\lfloor n/2\rfloor}\binom{n-l}{l}$.

Recently, Generalized Pascal triangle and sequences T_n.The following combinatorial interpretation occurs of the elements $\binom{n}{k}_s$ of the generalized Pascal triangle. The word $\binom{n}{k}_s$ assigns the number of ways of distributing uniform objects k to n boxes, which may have a maximum number of objects in each box. Clearly, $0 \le k \le sn$.In other words, $\binom{n}{k}_s = |\{f:\{0\ldots,n-1\}\to\{0\ldots,s\} \mid \sum_{i=0}^{n-1} f(i)=k\}|$.

For example, if s = 2 the triangle is achieved

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & 1 & 1 & & \\ & 1 & 2 & 3 & 2 & 1 & \\ 1 & 3 & 6 & 7 & 6 & 3 & 1 \end{array}$$

Where $\binom{n}{k}_2 = (\binom{n-1}{k-2}_2 + \binom{n-1}{k-1}_2 + \binom{n-1}{k}_2$, if the value is zero if v<0 or 2u<v, assumes that the u in $\binom{u}{v}_2$ is not negative. It will skip subscript 1 and write only for the normal binomial coefficients $\binom{n}{k}$ if s = 1. Now in further it is formulating a lemma for generalized binomial coefficients that is helpful in the theorem proof 4.2

Lemma 4.2. If s ≥ 2 then it will as

$$\binom{n}{k}_s = \sum_{k_1=\lceil\frac{k}{s}\rceil}^{min\{k,n\}} \binom{n}{k_1}\binom{k_1}{k-k_1}_{s-1} \qquad (4.2)$$

Proof of Lemma 4.2. If it chooses to distribute k elements, then it selects k1 boxes, with at most s – 1 element per box, and then distribute the other k–k1 elements among the k1 boxes specified. $\binom{u}{v}_s = 0$.

Notice that the limit indication in sum (4.2) can be ignored by reminding the coefficient $\binom{u}{v}_s = 0$ for an unremarkable if the integer v is outside the range 0, ...,su.

The generalized Pascal triangle n k s , n ∈ N; 0 ≤ k ≤ sn is linked to the linear recurrence {Tn} given by (1) and (2) via the diagonal sum,

$$\sum_{k_1=0}^{\left\lfloor\frac{sn}{s+1}\right\rfloor} \binom{n-1}{k}_s = T_{n+s} \qquad (4.3)$$

The case s=1 returns the nice identify,

$$\sum_{k_1=0}^{\left\lfloor\frac{n}{2}\right\rfloor} \binom{n-k}{k}_s = F_{n+1}$$

A000045 number for Fibonacci, while s=2 is related to A000073 number for Tribonacci numbers. Generalized numbers associated with Fibonacci. More broadly, the unimodalities of all Pascal generalised rays are determined by showing that the sequence $w_k = \binom{n+\alpha k}{m+\beta}_q$ is log concave, then unimodal.

The Fibonaccci polynomials:Notes that if k is a real variable of x, then Fk;n = Fx;n is the polynomials defined by Fibonacci,

$$
Fn+1(x)=\begin{cases} 1 & \text{if } n=0 \\ x & \text{if } n=1 \\ xF_n(x)+F_{n-1}(x) & \text{if } n\geq 2 \end{cases}
$$

from where The first polynoms of Fibonacci are

$$F_1(x)=1$$

$$F_2(x)=x$$

$$F_3(x)=x^2+1$$

$$F_4(x)=x^3+2x$$

$$F_5(x)=x^5+4x^3+3x$$

And more it can write k-Fibonacci numbers from these expressions:

$$Fn+1(x)=\sum_{l=0}^{\lfloor n/2\rfloor}\binom{n-i}{l}x^{n-2i} \quad \text{for } n\geq 0$$

Notice is that F2n(0) = 0 and x = 0is the only real root, while F2n+1(0)=1 to with no real roots.

Also for x = k2N The k-Fibonacci sequence elements are obtained.

Analysis: Chebyshev polynomials show integral representations of the hermit polynomials and the generation process will add the new representations of Chebyshev polynomials. Chebyshev polynomials after the second kind $U_n(x)$

$$U_n(x) = \sum_{k=0}^{\lfloor n/2\rfloor}\frac{(-1)^k(n-k)!(2x)^{n-2k}}{k!(n-2k)!}$$

Proposition 1. The two polynomials of Chebyshev satisfy the following integral characterization:

$$U_n(x) =\frac{1}{n!}\int_0^{+\infty} e^{-t}t^n H_n(2x,-\frac{1}{t})dt$$

Proof: By taking note of this

$$n!=\int_0^{\infty} e^{-t}t^n dt$$

It can write as

$$(n-k)! = \int_0^\infty e^{-t}t^{n-k}dt$$

The explicit form of $U_n(x)$ the Chebyshev polynomials, and the standard two-variable Hermit polynomials:

We know that

$$H_n(x,y) = n!\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(y)^k(x)^{n-2k}}{k!\,(n-2k)!} \qquad (2.1)$$

$$U_n(x) = \sum_{k=0}^{[\frac{n}{2}]}\frac{(-1)^k\ (n-k)!\,(2x)^{n-2k}}{k!\,(n-2k)!} \qquad (2.2)$$

$$In\ (2.1)\ x\ repalce\ by\ 2x\ and\ y\ replace\ by\ -\frac{1}{t}\ \ we\ will\ get$$

$$H_n\left(2x,-\frac{1}{t}\right) = n!\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k(2x)^{n-2k}t^{-k}}{k!\,(n-2k)!} \qquad (2.3)$$

Now (2.3) multiplying both side by $e^{-t}t^n$ $and\ integrating\ limit\ 0\ to\ \infty$

We will get

$$\int_0^\infty e^{-t}t^n H_n\left(2x,-\frac{1}{t}\right)dt = n!\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k(2x)^{n-2k}}{k!\,(n-2k)!}\int_0^\infty e^{-t}\,t^{n-k}dt \qquad (2.4)$$

$$using\ (n-k)! = \int_0^\infty e^{-t}t^{n-k}dt\ \ in\ (2.4)\ \ we\ will\ get$$

$$\int_0^{\infty} e^{-t} t^n H_n\left(2x, -\frac{1}{t}\right) dt = n! \sum_{k=0}^{\left[\frac{n}{2}\right]} \frac{(-1)^k (2x)^{n-2k} (n-k)!}{k!\,(n-2k)!}$$

So we have

$$U_n(x) = \frac{1}{n!} \int_0^{+\infty} e^{-t} t^n H_n(2x, -\tfrac{1}{t}) dt$$

and then the study.

Theorem 2: The Chebyshev polynomials $T_n(x)$ and $U_n(x)$ satisfy the following recurrence relations:

$$\frac{d}{dx} U_n(x) = nW_{n-1}(x)$$

$$U_{n+1}(x) = xW_n(x) - \frac{n}{n+1} W_{n-1}(x)$$

Where,

$$W_n(x) = \frac{2}{(n+1)!} \int_0^{+\infty} e^{-t} t^{n+1} H_n(2x, -\tfrac{1}{t}) dt.$$

Proof- In the above section the recurring relations of the standard hermit polynomials $H_n(x, y)$ can be costumed as follows.

First we will prove identity (2.5) and (2.6)

$$\left[2x + \frac{1}{-t}\frac{\partial}{\partial x}\right] H_n(2x, \tfrac{1}{-t}) = H_{n+1}(2x, -\tfrac{1}{t}) \qquad (2.5)$$

$$\frac{1}{2}\frac{\partial}{\partial x} H_n(2x, -\tfrac{1}{t}) = n\, H_{n-1}(2x, -\tfrac{1}{t}) \qquad (2.6)$$

Consider

$$\frac{\partial H_n(2x, -\frac{1}{t})}{\partial x} = n! \sum_{k=0}^{\left[\frac{n-1}{2}\right]} \frac{2(-1)^k (2x)^{n-2k-1} (n-2k) t^{-k}}{k!(n-2k)!}$$

$$\frac{\partial H_n(2x, -\frac{1}{t})}{2\partial x} = n! \sum_{k=0}^{\left[\frac{n-1}{2}\right]} \frac{(-1)^k (2x)^{n-2k-1} t^{-k}}{k!(n-2k-1)!}$$

So we have

$$\frac{1}{2}\frac{\partial}{\partial x} H_n(2x, -\tfrac{1}{t}) = n\, H_{n-1}(2x, -\tfrac{1}{t})$$

So (2.6) has proved some process we can prove (2.5)

By above theorem we have

$$U_n(x) = \frac{1}{n!}\int_0^{+\infty} e^{-t}t^n H_n(2x,-\frac{1}{t})dt$$

Differentiation both side with respect to we get

$$\frac{d}{dx}U_n(x)=\frac{1}{n!}\int_0^{+\infty} e^{-t}t^n \frac{\partial}{\partial x}H_n(2\text{x},-\frac{1}{t})\,dt$$

Now using identity (2.5) we will get

$$\frac{d}{dx}U_n(x)=\frac{2n}{n!}\int_0^{+\infty} e^{-t}t^n H_{n-1}(2\text{x},-\frac{1}{t})\,dt \qquad (2.7)$$

The relation above provides a link between polynomials $T_n(x)$ and $U_n(x)$ however, as:

$$U_{n-1}(\text{x})=\frac{1}{(n-1)!}\int_0^{+\infty} e^{-t}t^{n-1}H_{n-1}(2x,-\frac{1}{t})dt \qquad (2.8)$$

Using (2.8) in (2.7) immediately get:

$$\frac{d}{dx}T_n(\text{x})=\text{n}U_{n-1}(\text{x}).$$

By using Second kind of Chebyshev polynomials in the first identity

$$U_{n+1}(x) =\frac{1}{(n+1)!}\int_0^{+\infty} e^{-t}t^{n+1}H_{n+1}(2x,-\frac{1}{t})dt$$

Using $\left[2x+\frac{1}{-t}\frac{\partial}{\partial x}\right]H_n(2x,\frac{1}{-t}) = H_{n+1}(2\text{x},-\frac{1}{t})$ in (2.9)

$$U_{n+1}(x) =\frac{1}{(n+1)!}\int_0^{+\infty} e^{-t}t^{n+1}\left[2x+\frac{1}{-t}\frac{\partial}{\partial x}\right]H_n(2x,-\frac{1}{t})dt$$

That is

$$U_{n+1}(\text{x})=\text{x}\frac{2}{(n+1)!}\int_0^{+\infty} e^{-t}t^{n+1}H_n(2x,-\frac{1}{t})dt-\frac{1}{(n+1)!}\int_0^{+\infty} e^{-t}t^n\frac{\partial}{\partial x}H_n(2\text{x},-\frac{1}{t})dt \qquad (2.10)$$

Using $\frac{1}{2}\frac{\partial}{\partial x}H_n(2\text{x},-\frac{1}{t})=\text{n}\ H_{n-1}(2\text{x},-\frac{1}{t})$ in (2.10)

We have

$$U_{n+1}(\text{x})=\text{x}\frac{2}{(n+1)!}\int_0^{+\infty} e^{-t}t^{n+1}H_n(2x,-\frac{1}{t})dt-\frac{2n}{(n+1)!}\int_0^{+\infty} e^{-t}t^n\,H_{n-1}(2\text{x},-\frac{1}{t})\,\text{dt} \qquad (2.11)$$

Using $W_n(x)=\frac{2}{(n+1)!}\int_0^{+\infty} e^{-t}t^{n+1}H_n(2x,-\frac{1}{t})dt$ in (2.11) we have

$$U_{n+1}(x) = xW_n(x) - \frac{n}{n+1} W_{n-1}(x)$$

Also we have

$$\frac{d}{dx} U_n(x) = \frac{2n}{n!} \int_0^{+\infty} e^{-t} t^n H_{n-1}(2\text{x}, -\frac{1}{t})\, dt$$

And

$$W_n(x) = \frac{2}{(n+1)!} \int_0^{+\infty} e^{-t} t^{n+1} H_n(2x, -\frac{1}{t}) dt$$

$$Replace\ n + 1\ by\ n$$

We get

$$W_{n-1}(x) = \frac{2}{(n)!} \int_0^{+\infty} e^{-t} t^n H_{n-1}(2x, -\frac{1}{t}) dt$$

So we have

$$\frac{d}{dx} U_n(x) = \text{n } W_{n-1}(x)$$

Generation of functions: The first and second type of Chebyshev polynomials can draw a slightly different links from these polynomials and their generative functions using the integrated representations in the previous section and related recurrence relations.

For the $U_n(x)$ Polynomials of Chebyshev, we notice that both sides of the equation by $\xi^n\,|\xi| < 1$ and it follow by sum marizing over n

$$\sum_{n=0}^{+\infty} \xi^n\, U_n(x) = \int_0^{+\infty} e^{-t} \sum_{n=0}^{+\infty} \frac{(t\xi)^n}{n!} H_n(2x, -\frac{1}{t}) dt$$

By remembering the polynomials of the $\sum_{n=0}^{+\infty} \frac{t^n}{n!} H_n(x, y) = e^{(xt+yt^2)}$ in the above relation generation and t integration, we end.

$$\sum_{n=0}^{+\infty} \xi^n\, U_n(x) = \frac{1}{1-2\xi x+\xi^2}.$$

By using the results seen in the previous theorem, we can now state the respective generation function for the first Chebyshev polynomial $T_n(x)$ and $U_n(x)$

IV. APPLICATIONS OF RECURRENCE RELATIONS

- *Biology*

Some of the most recognised variability calculations arise in attempts to shape population dynamics. The Fibonacci numbers, for examples, were once utilised as an example of growing the population of rabbit.Integrodifference equations are a significant type of recurrent relationship to spatial ecology. This and other variability computations are especially suitable for modelling in univoltine populations.

- *Computer Science*

Recurrence relation is also an important value in algorithm analysis. If an algorithm is intended to break a problem into more low (the divide and conquer) problems it run at times will be described by a relationship repetition.Simpler instances are that it takes the time and find an item with n elements in the order vector (in case no components can take such a density).One component at a time, from left to right, will be sought with a native algorithm. The worst case would be where the desired substance is the last element, so the range of contrasts is n.Binary search is the better algorithm. It takes a single, ordered vectors. That checks whether the element is in centre of the vector first. If not, then whether the centre component is greater than or less than the desired component is monitored.

- *Digital Signal Processing*

Digital signal processors) is the digital processing method for a wide variety of signal processing activities, as on the part of computer machines or more specialised digital signal processors. The digit this way is a set of numbers that is a series of digits forming samples of a continuous variable in a particular domain such as time, area, space, or frequency in this order. In digital signal processing, repetition links may impact feedback in a process where outputs, at the same time, become products. Thus, they occur in digital filters of infinite impulse response (IIR).Multi-use improvements can enhance the cryptographic safety of digital signals the encrypted and decrypted algorithms.

- *Economics*

In both theoretical and empirical economies, recurrent relationships, particularly straight recurrence interactions, are widely used. On a macroeconomic stage in particular, a model regions of big economy (the financial sector, the commodity area and the labour market etc.) might be developed based in that some actors' action depends on lagging trend modalities. You would be willing to solve the model to fix main variables' current value with regard to past and actual values (interest rate, true GDP, etc.) in other variables.23 .

- *Network Marketing*

Network business is a business method or activity in which people are compensated for not only work created by themselves, but for the work also produced by others. "The network business model is referred to as "down line model," since it is developed by distributors with multiple levels of compensation and "several" levels in.

All sorts of network enterprise platforms are there. Employees directly sell products to customers through link referrals and word of voice marketing in most network business platforms. Many grid marketing organizations aimed to build opportunities for individuals who would not otherwise have them, including those who. Less privacy in running their own company, Do not have limited cash, Compatible with their present labor level, Has been unwise with own companies failed to work. Recurrence is a very helpful subject in mathematics which resolves many issues of real lives many through repeated interactions.

A lot of individuals are involved in network marketing companies in modern time Network Marketing is very renowned company direction·

V. CONCLUSION

Recurrence relation is very useful topic of mathematics many problems of real life many be solved by recurrence relations but in recurrence relation there is a major difficulty in the recurrence relation if we want find 100th term of sequence then we need to find all previous 99 terms of given sequence then we can get 100th term of sequence but above theorem is very useful if coefficients of recurrence relation of given sequence are satisfied the condition of the above theorem then we can apply above theorem and we can find direct any term of sequence without find all previous terms. There is important property of a relation between coefficients of recurrence relation terms and roots of a polynomial for second order relation but in this paper, we gave this same property of recurrence relation of all higher order recurrence relation. So finally, we can say that this theorem is valid all order of recurrence relation only condition that roots are distinct. So, we can say that this paper is generalization of property of a relation between coefficients of recurrence relation terms and roots of a polynomial.

The relation between primary numbers and perfect numbers, including composite numbers, exists.

The first term will be $2 \times 2 = 4$, next $2 \times 3 = 6$, $3 \times 3 = 9$, etc. in composite numbers by means of primaries; 2, 3,5,7,11,13,17,19,23,29,31 etc. and by means of a composite count number, in line with the formula for m and n composites, where m and n shall be both primarily numbers. The processing is such that there is no number between them, such as 3x3, and multiply 2 by 5, to get 10. However, it is laborious to achieve composite numbers that are far-reaching, so that the nth composite number (Cn) can be produced with the Wolfram Language Code as above. It is a code for the computer. A manual algorithm for extrapolating large composite numbers is exceedingly difficult to develop. But the generating function of Dirichlet is used for the characteristic function of the composite numbers. The relationship can also be dealt with as follows in ideal numbers; (Power of Two) × (Double that Power - 1). The prime number formula is given by (2n-1). To get the Perfect Number, the formula becomes (2n-1) × (2n-1). Getting the nth sequence of a perfect number is dependent on the equivalent nth sequence of prime numbers. The research finally concludes with the call for researchers to team up to make more explicit, the recurrence relation in composite numbers. The study focuses its intention on recurrence relation in perfect and composite numbers. It is hinged at analyzing the mathematical relationships between them, as well as asking new questions about them and to prove that these relationships are true.

Recurrence Relation of Network Marketing is the sector came into being, marketing has become one of the main data mining applications. The determination whether a specific individual is to advertise is usually based solely on its characteristics or those of the population segments to which they belong (direct marketing) (mass marketing). This also results in optimum targeting choices since the impact of a business consumers on each other's buying decisions is not taken into consideration. Customers are heavily affected by the views of their partners in many markets. Viral marketing does this to sell a commodity cheaply, mostly through marketing people with the highest consumer power.

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