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M. Bousder

Mohammed V University

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Author: LPHE-MS Laboratory, Department of physics, Faculty of Science, Mohammed V University in Rabat, Rabat, Morocco.

I. INTRODUCTION

An intriguing idea of D. Glavan and C. Lin [1] is to multiply the Gauss-Bonnet term by the factor $1/(D-4)$ and then took the limit $D \longrightarrow 4$, might be useful in determining a non-trivial 4-dimensional theory of gravity. However, it was shown in several papers that perhaps the idea of the limit $D \longrightarrow 4$ is not clearly defined, and several ideas have been proposed to remedy this in-consistency, as well as the absence of proper action [2, 3, 4, 5, 6]. It was explicitly confirmed by a direct product D-dimensional spacetime or by adding a counter term, before taking the limit $D \longrightarrow 4$, which can be seen as implied by a class of Horndeski theory [7] but with $2+1$-dofs. Although the EGB gravity is currently debatable, the spherically symmetric black hole solution is still meaningful and worthy of study [8]. In contrast, in [9] the black hole solution of EGB gravity considered as a theory of gravity by including the electric charge in AdS spaces. In [10, 11] it was shown the thermodynamics of this novel solution by studying the phase transition and microstructures. By focusing on the GB coupling constant, the validity of the weak cosmic censorship conjecture for a near-extremal black hole depends on the

Gauss–Bonnet coupling constant have also been studied [12]. In this basis, the shadow behaviors and photon sphere around such black hole [13]. In parallel, there have also been numerous proposals to directly modeling the black hole evaporation and calculations of the entanglement entropy [14, 15]. The ER = EPR [16] has given a new perspective on the gauge/gravity correspondence. Under this paradigm, a pair of entangled black holes are joined by an Einstein-Rosen (ER) bridge. This suggests that there is a relation between quantum entanglement and geometric spatial. In [17], we are interested in the entanglement between the particles near the event horizon and antiparticle near the Cauchy horizon of EGB black hole. Here we propose that the solution based on the work of Glavan & Lin, describes a 4D EGB black hole, but there is still a vast part of EGB gravity in $D \succ 4$, (**??**). In order to make the EGB combination dynamical in four dimensions, we can use the GB coupling as $\alpha \to \alpha/(D-4)$.

II. PARTICLE-ANTIPARTICLE IN EGB BLACK HOLE

Consider now the Einstein-Gauss-Bonnet-Maxwell theory in D-dimensions with a negative cosmological constant [10, 9]

$$I = \frac{1}{16\pi} \int d^D x \sqrt{-g} \left(R - 2\Lambda + \frac{\alpha}{D-4} \mathcal{G} - F_{\mu\nu} F^{\mu\nu} \right), \tag{2.1}$$

where α is a finite non-vanishing dimensionless Gauss-Bonnet coupling have dimensions of $[length]^2$, that represent ultraviolet (UV) corrections to Einstein theory, and $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet invariant with a negative cosmological constant $\Lambda = -(D-1)(D-2)/(2l^2)$, tere l is the AdS radius. $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell tensor of the real vector field A_μ of the form $A_\mu dx^\mu \equiv A_0 dt$, with the time component of the current density $j^\nu = \nabla_\mu F^{\mu\nu}$. In addition, we introduce a vector charge: $Q_{EGB} = Q_H + \frac{1}{4\pi} \int \sqrt{-g} j^0 dx^3$, where Q_H denotes the horizon charge. At infinity, the charge vanishes $Q_{EGB} = 0$. To solving the field equation we obtain the black hole solution $ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$. Taking the limit $D \longrightarrow 4$, we obtain the exact solution in closed form

$$f(r) \approx 1 + \frac{r^2}{2\alpha} \left(1 - \sqrt{1 + 4\alpha \left(2Mr^{-3} - Q_{EGB}^2 r^{-4} - l^{-2}\right)} \right). \tag{2.2}$$

This last solution could be obtained directly from the derivation done in [11]. In the limit $r \longrightarrow \infty$ with vanishing black hole charge, we asymptotically obtain the GR Schwarzschild solution. The event horizon in spacetime can be located by solving the metric equation: $f(r) = 0$ [12]. The extremal case correspond to $f(r_+) = 0$. In the limit $\alpha \longrightarrow 0$, we can recover the Reissner-Nordström-AdS solution. If $\alpha \prec 0$ the solution is still an AdS space, if $\alpha \succ 0$ the solution is a de Sitter space, [9]. The solutions show that the event horizon is located at:

$$r_\pm = M \pm \sqrt{M^2 - Q_{EGB}^2 - \alpha}, \tag{2.3}$$

where r_+ and r_- are the event horizon and the Cauchy horizon radius of the EGB black hole [1]. Next, we want to give a physical interpretation to the two horizons solutions. For an extremal black hole, one get $r_+ = r_-$ (degenerate solution) and $|Q_{EGB}| = \sqrt{M^2 - \alpha}$. First, we introduce the particle and antiparticle charges in the horizon [17]

$$q_\pm = \pm\sqrt{M^2 - \alpha}. \tag{2.4}$$

The charge of every particle (or antiparticle) near the horizon depends directly on the black hole mass and the Gauss-Bonnet coupling. Each particle of the charge $q_+ = +\sqrt{M^2 - \alpha}$, is entangled with another antiparticle of the charge $q_- = -\sqrt{M^2 - \alpha}$ by the entangled state $|\ \rangle$ in the basis $\{|+\rangle_L, |-\rangle_L\}$ of the Cauchy horizon $\mathcal{H}_L$, and $\{|+\rangle_R, |-\rangle_R\}$ a basis of the event horizon $\mathcal{H}_R$. The entangled state $|\ \rangle$ can be expressed as $|\ \rangle = \frac{1}{\sqrt{2}}(|+\rangle_L |-\rangle_R \pm |-\rangle_L |+\rangle_R)$. We can simplify the Eqs.(2.3,2.4) by

$$(r_\pm - M)^2 = q_\pm^2 - Q_{EGB}^2. \tag{2.5}$$

We notice that $q_\pm^2 \geq Q_{EGB}^2$. The charges q_+ are located on the $\mathcal{H}_R$, on the other hand, the charges of antiparticles q_- are located on the $\mathcal{H}_L$. The Eq.(2.5) behaves like virtual gravitational dipoles in the framework of the gravitational repulsion between matter and antimatter [19]. If we assume that there is a complete disappearance of an AdS black hole ($\alpha \prec 0$), we obtain the position of the two charges $r_\pm = q_\pm = \pm\sqrt{-\alpha}$. The problem of the negative radius $r_- = -\sqrt{-\alpha}$, indicates a disappearance of the antiparticles with the disappearance of the black hole singularity, on the other hand, the particles escape the singularity. This aspect is equivalent to the position of the two horizons; the horizon r_- exists on the singularity and r_+ is the edge of the black hole. Which agrees with the violation of CP symmetry between matter and antimatter [20]. This complete disappearance of the antiparticles looks like a scenario of the disappearance of antimatter after the big bang.

III. CHARGE OF EGB BLACK HOLE

We consider that the EGB black hole charge is the sum of the charges of the pairs by two different ways, given by $Q_{EGB} = q_+ + q_- + q_+ + ...$ or $Q_{EGB} = q_- + q_+ + q_- + ...$, or equivalently

$$Q_{EGB} = \pm q_+ \sum_{k=0}^{N-1} (-1)^k \, , \tag{3.1}$$

where N denotes the number of the particles and antiparticles.

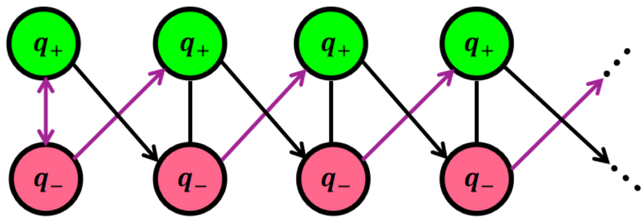

Figure 1: Two direction of the summation on the charges of N particles-antiparticles.

If N is even (1), one obtain $Q_{EGB} = 0$ (Schwarzschild black hole), which shows that the number of particles on $\mathcal{H}_R$ is equal to the number of antiparticles in the singularity of Schwarzschild black hole, which corresponds exactly with the proposition (2.4) of entangled pairs. *If N is odd*, one get $Q_{EGB} = \pm q_+$ and *if N is infinite*, and by using the analytic continuation of the Riemann zeta function of 0 ($\zeta(0) = 1/2$), one get $Q_{EGB}^{N \longrightarrow \infty} = \pm \frac{1}{2}\sqrt{M^2 - \alpha}$. The calculation has been already carried out in detail in [17]. This procedure can represent by the quantification of the total charge of the EGB black hole according to

$$Q_{EGB} = n\sqrt{M^2 - \alpha} \quad ; \quad n = \{-1, -1/2, 0, 1/2, 1\} \, , \tag{3.2}$$

where n is a quantum number of the particles if $n \geq 0$, or of antiparticle if $n \leq 0$. The charge Q_{EGB} is quantized in terms of M and α. This charge check the condition of existence of $\mathcal{H}_R$: $2Q_{EGB} \prec r_s$ and verify the condition Eq.(2.5). According to this last equations, even r_+, M and α are qauntified by the relation $(r_\pm - M)^2 = (1 - n^2)(M^2 - \alpha)$. For RN black hole one get $Q_{RN} = nM$. The EGB black hole is transformed into a Schwarzschild black hole if the particles have zero charge ($q_+ = 0$). The Bekenstein-Hawking (BH) formula with supplementary logarithmic term [9] is given by

$$S = \frac{A}{4} + 2\pi\alpha \log \frac{A}{A_0}, \tag{3.3}$$

where $A \equiv 4\pi r_+^2$ is the area of the event horizon of the black hole, whose can be expanded in two supplementary logarithmic term by the use of the charge above [17]. The prescription Eq.(3.3) is equivalent to the generalized entropy of the quantum extremal surface [18], if $S_{bulk} = 2\pi\alpha \log A/A_0$. The von Neumann entropy of the bulk fields is represented by the bulk entropy S_{bulk} in the boundary region.

IV. CP-SYMMETRY AND ER=EPR

To describes the behavior of some particles under the symmetry operation of charge conjugation, we introduce two operators $\mathcal{Q}$ and $\mathcal{C}$, the first represents charge operator and the second is the charge-conjugation operator. The operation $\mathcal{C}$ transforms a particle into antiparticle. Note that $\mathcal{C}$ and $\mathcal{Q}$ obeys anticomutation relation: $\mathcal{C}\mathcal{Q} + \mathcal{Q}\mathcal{C} = 0$, and therefore in general they do not possess the same eigenstate, where $\mathcal{C}\left|q\right\rangle = \left|\bar{q}\right\rangle$ and $\mathcal{Q}\left|q\right\rangle = q_+\left|q\right\rangle$ (which, in turns, requires $\mathcal{Q}\left|\bar{q}\right\rangle = q_-\left|\bar{q}\right\rangle$), which means that $\{\left|q\right\rangle, \left|\bar{q}\right\rangle\}$ are eigenstates of $\mathcal{Q}$. The parity operator satisfies $\mathcal{P}\left|q\right\rangle = -\left|q\right\rangle$ and $\mathcal{CP}\left|q\right\rangle = -\left|\bar{q}\right\rangle$. The vacuum state is invariant under the charge-conjugation operator $\mathcal{C}\left|0\right\rangle = \left|0\right\rangle$. Now, let $\left|q\right\rangle$ denote the quantum state of a system of N_+ particles with eigenvalue q_+. Similarly, $\left|\bar{q}\right\rangle$ is the quantum state of N_- antiparticles with eigenvalue q_-. Including the charge-conjugation operator that respects $\mathcal{C}^k\left|q\right\rangle = (-1)^k\left|q\right\rangle$. Note that $Q_{EGB} = nq_+$, which leads to $\pm n\left|q\right\rangle = \sum_{k=0}^{N-1}\mathcal{C}^k\left|q\right\rangle = N_+\left|q\right\rangle + N_-\left|\bar{q}\right\rangle$. Then, the corresponding states generated by

$$\left|q\right\rangle = \frac{-N_-}{N_+ - n}\left|\bar{q}\right\rangle \text{ or } \left|q\right\rangle = \frac{-N_-}{N_+ + n}\left|\bar{q}\right\rangle, \tag{4.1}$$

therefore, the state of a particle is formulated in terms of the state of antiparticle. The main reason is that this relation gives an aspect in of a wormhole between the two states. It was explicitly confirmed by ER bridges from ER = EPR [25]. We would like to point out that to obtain the normalizable states $\langle q\left|q\right\rangle = 1$ and $\langle\bar{q}\left|\bar{q}\right\rangle = 1$, one could take $n = \{(N_+ - N_-); N\}$ or $n = \{-N; (N_- - N_+)\}$, where $N = N_- + N_+$. Now, we consider the *two solutions* $n = -N$ and $n = N$. According to the condition Eq.(3.2) is impossible that $n \succ 1$, which means that for these two cases $N = 1$. For *the cases* $n = \pm(N_+ - N_-)$, and by using the same condition, we show that $N_\mp - 1 \leq N_\pm \leq N_\mp + 1$. This means that $n = 0$. Hence, following Eq.(4.1), we introduce

$$\left|q\right\rangle = -\left|\bar{q}\right\rangle \Longrightarrow n = \pm(N_+ - N_-) = 0, \tag{4.2}$$

$$\left|q\right\rangle = +\left|\bar{q}\right\rangle \Longrightarrow n = \pm N = \pm 1. \tag{4.3}$$

The case $\left|q\right\rangle = -\left|\bar{q}\right\rangle$ where N is even, i.e. $N_+ = N_-$, one could take one degenerate solution corresponding to $n = 0$, which describes the Schwarzschild black hole. Therefore, this solution is invariant under the CP-symmetry as $\mathcal{CP}\left|q\right\rangle = \left|q\right\rangle$. Next, let's consider *the case* $\left|q\right\rangle = \left|\bar{q}\right\rangle$ (Majorana fermion!), which corresponds exactly with an extremal black hole ($n = N = 1$) and one anti-extremal black hole ($n = -N = -1$). In this case, the Majorana fermion on the two horizons, lead to the violation of CP symmetry. Therefore, the EGB black hole behaves under the normalizable states as Schwarzschild or extremal black hole, also as anti-extremal black hole (i.e. a black hole bounded by antiparticles near the event horizon). If $r_- \succ r_+$ we get a anti-EGB black hole, i.e. there is a great abundance of antimatter. Therefore, the particles and antiparticles states are invariant under C-symmetry. Now, we consider the limit $N \longrightarrow \infty$, where $N = 2N_+ = N = 2N_-$, similarly to the case of $n = 0$, one obtain $|q\rangle = -|\bar{q}\rangle$, which requiring that $|q\rangle = (-1)^{1+\delta_N} |\bar{q}\rangle$, where $\delta_N (N = 1) = 1$ and $\delta_N (N \succ 1) = 0$. The next simplest means of verifying the existence of an Einstein-Rosen (ER) bridge between particle near the event horizon $\mathcal{H}_R (n \geq 0)$ and antiparticle near the Cauchy horizon $\mathcal{H}_L (n \leq 0)$. It is then reasonable to consider the two entangled horizons. A more direct physical reason for adopting this description is that in the situation indicated in 2

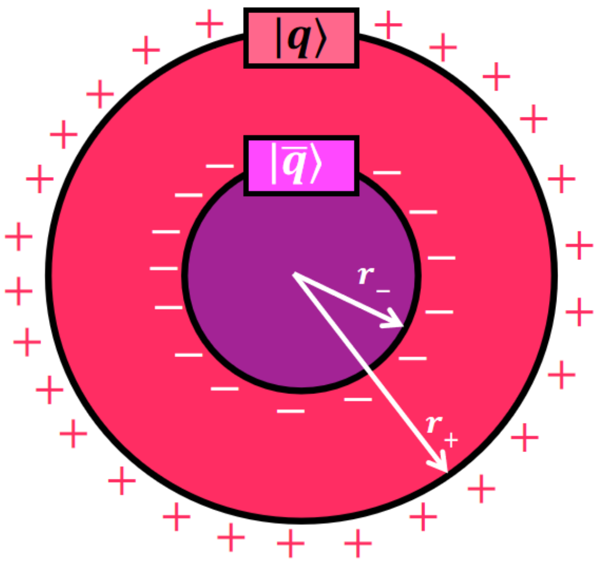

Figure 2: Distribution of charges and anti-charges on the two horizons $\mathcal{H}_L$ and $\mathcal{H}_R$.

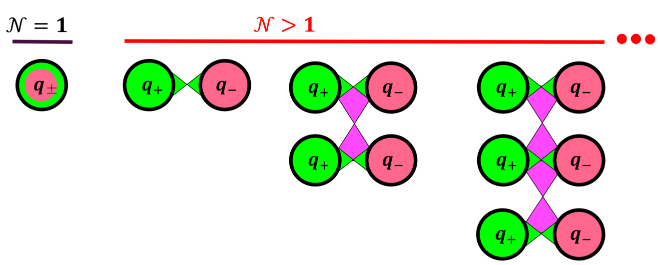

Figure 3: For $N = 1$, the particle can be both on $\mathcal{H}_L$ and $\mathcal{H}_R$. For $N \succ 1$, every particle on $\mathcal{H}_R$ is entangled with antiparticle on $\mathcal{H}_L$.

We notice that we can write Eq.(4.2) as follows $|q\rangle_{RL} = (-1)^{1+n}\,|\bar{q}\rangle_{RL}$. We introduce the state $|\Psi\rangle_N$ of a black hole according to two possibilities $\{N = 1, N \succ 1\}$. We thus take any two horizons $\mathcal{H}_L$ and $\mathcal{H}_R$ that are entangled to be in the thermofield double state [22, 24]

$$|TFD(t)\rangle = \sum_n e^{-\left(\frac{\beta}{2}+2it\right)E_n}\,|n\rangle_L \otimes |n\rangle_R\,, \tag{4.4}$$

where $\beta = 1/T$ and $|n\rangle_{LR}$ defined in the microscopic UV-complete theories, they create CFT states with energy [21]. The eigenstate $|n\rangle_L$ (or $|n\rangle_R$) of the CFT_L (or CFT_R) corresponding to the degrees of freedom in $\mathcal{H}_L$ (or $\mathcal{H}_R$), and $|n\rangle_L$ is CPT conjugate of state $|n\rangle_R$. The TFD is a maximally entangled state, which represents the formal purification of the thermal mixed state of a one horizon ($\mathcal{H}_L$ or $\mathcal{H}_R$) [23], with the reduced density matrix within AdS/CFT via the Ryu-Takayanagi formula [15]. The mixed state given by the incoherent sum over all generalized TFD states $\rho_{TMD} = (1/N)\sum_k |TFD\rangle_k\ {}_k\langle TFD|$ call the the thermo-mixed double, where N the total number of basis states. The corresponding bulk geometry of the wormhole is formed by $tr\,(\rho^2_{TMD}) = \sum_k e^{-2\beta E_k}/Z^2$, where $Z = \sum_k e^{-\beta E_k}$ is partition function, the square number 2 corresponds to the two horizons. The generalized TFD states are orthogonal $\langle TFD\,|TFD\rangle \sim \delta_N$ [23] in Eq.(**??**) give $\langle q\,|\bar{q}\rangle = (-1)^{1+\langle TFD\,|TFD\rangle}$, which corresponds to the result found by [27]. Now, similarly to ER bridges connecting two black holes [16], considering ER bridge connecting the two horizons $\mathcal{H}_L$ and $\mathcal{H}_R$. In this view, the EGB black hole can be regarded as the one constructed by suitably gluing two black holes $\mathcal{A}$ and $\mathcal{B}$. The black hole $\mathcal{A}$ bounded by the horizon $\mathcal{H}_R$ (the quantum extremal surface), which is entangled with the anti-black hole $\mathcal{B}$ bounded by the horizon $\mathcal{H}_L$ 4.

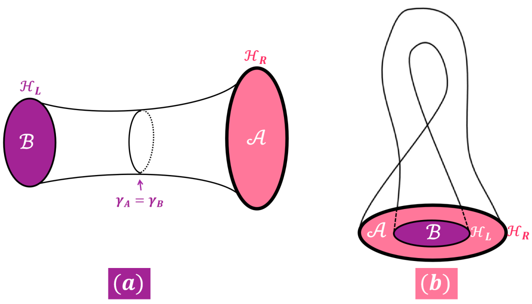

Figure 4: This figure represents two possibilities to connecting the two horizons. (a): EGB black hole behaves as black hole $\mathcal{A}$ bounded by $\mathcal{H}_R$ and anti-black hole $\mathcal{B}$ bounded by $\mathcal{H}_L$. The two horizons are connected between them by an ER bridge. (b): the possibility to have the two entangled horizons on the same plane, is to use an ER bridge in the form of Klein bottle.

If we consider that the black hole absorbs matter, the anti-black hole absorbs the antimatter. Moreover, the two horizons $\mathcal{H}_L$ and $\mathcal{H}_R$ posits that this purification describes two black holes with an ER bridg pin the extended space-time. The $|TFD\rangle$ states evolved over the corresponding time. Moreover, the information transferred from $\mathcal{H}_R$ to $\mathcal{H}_L$ by the pure state of $L \otimes R$. Since this TFD state is pure on $\mathcal{A} \cup \mathcal{B}$, it is instructive to write in terms of the minimum area. The entanglement entropy (EE) [15] of the blak hole $\mathcal{A}$ equal to the entanglement entropy of black hole $\mathcal{B}$: $S_E(\mathcal{A}) = S_E(\mathcal{B})$ (because $\gamma_{\mathcal{A}} = \gamma_{\mathcal{B}}$) 4. This entropy is closely related to the holographic EE in asymptotically AdS spaces [15]. The EE satisfies the subadditivity: $S_E(\mathcal{A} \cup \mathcal{B}) \leq S_E(\mathcal{A}) + S_E(\mathcal{B})$, the validity of this inequality shows the presence of ER = EPR [25].

V. CONCLUSION

This Letter, based on the GB coupling which has been rescaled as $\alpha/(D-4)$. We investigated the relationship between the event horizon (with particles) and the Cauchy horizon (with antiparticles) of the EGB black hole. Our results clearly demonstrate that these two horizons are placed in two different black holes. The first black hole $\mathcal{A}$ is the standard which is bounded by particles and the second limited by antimatter (anti-black hole $\mathcal{B}$). The choice of the value of the number of particles $\{N = 1, N \succ 1\}$, allowed to include a wormhole or an ER bridge between the two horizons $\mathcal{H}_L$ and $\mathcal{H}_R$. In this basis, the state of the charge is formulated in terms of the anticharge state. The solution $|q\rangle = -|\bar{q}\rangle$ is invariant under the CP-symmetry, on the other hand, the case $|q\rangle = |\bar{q}\rangle$, corresponds to the violation of CP. We have already established the invariance of the charge state under the CP-symmetry, whose general can be expanded for anticharge

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