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A Topos-Theoretic Framework for Artistic-Oriented Knowledge Graphs

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ABSTRACT

This paper introduces a novel framework for Artistic-Oriented Knowledge Graphs that synthesizes the formal power of category theory with the philosophical insights of Alain Badiou's aesthetics. We propose a two-level arrangement where knowledge graphs serve as the foundational representation of artistic entities and their relationships, while a topos—a category of sheaves—provides the mathematical formalization of what Badiou calls the "logic of appearing." The formal structure of topos offers a rich language for modeling interpretation, symbolism, and context-dependent meaning in art. Accordingly, the framework implies that an artistic "event" is formalized as a change of topos, representing a fundamental shift in the interpretive logic through which art is perceived. Such a synthesis enables us to model the production of artistic truth and the subtleties of aesthetic judgment while respecting the complex, multi-dimensional nature of artistic expression. We demonstrate the framework by analyzing how different topoi can be constructed over the same knowledge graph, revealing how meaning is contingent on the prevailing logic of interpretation. An approach of this kind opens new possibilities for computational art history, digital aesthetics, and the formal study of cultural repertoires.

Index Terms: artistic knowledge graphs • cultural heritage • digital humanities • formal aesthetics • sheaf semantics • topos theory

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RESEARCH ARTICLE

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Abstract

This paper introduces a novel framework for Artistic-Oriented Knowledge Graphs that synthesizes the formal power of category theory with the philosophical insights of Alain Badiou's aesthetics. We propose a two-level arrangement where knowledge graphs serve as the foundational representation of artistic entities and their relationships, while a topos—a category of sheaves—provides the mathematical formalization of what Badiou calls the "logic of appearing." The formal structure of topos offers a rich language for modeling interpretation, symbolism, and context-dependent meaning in art. Accordingly, the framework implies that an artistic "event" is formalized as a change of topos, representing a fundamental shift in the interpretive logic through which art is perceived. Such a synthesis enables us to model the production of artistic truth and the subtleties of aesthetic judgment while respecting the complex, multi-dimensional nature of artistic expression. We demonstrate the framework by analyzing how different topoi can be constructed over the same knowledge graph, revealing how meaning is contingent on the prevailing logic of interpretation. An approach of this kind opens new possibilities for computational art history, digital aesthetics, and the formal study of cultural repertoires.

Keywords: *topos theory, artistic knowledge graphs, sheaf semantics, cultural heritage, formal aesthetics, digital humanities*

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1 Introduction

Knowledge graphs have emerged as powerful tools for organizing and connecting cultural data in the digital humanities [6]. These graph-based structures, consisting of entities (nodes) and relationships (edges), provide a flexible and expressive means of representing complex domains of experience. In the context of art and cultural heritage, knowledge graphs have been deployed to link artworks, artists, movements, techniques, and interpretive concepts, enabling new forms of computational analysis and discovery [12].

However, the application of knowledge graphs to the domain of art reveals a fundamental challenge: how can we represent not only what artworks are, but also how they mean? Art is characterized by multiple layers of interpretation, context-dependent symbolism, and the capacity to generate new truths that transcend existing categories. An artwork is not simply a collection of objective properties (medium, date, artist), but a site of meaning-making that varies across cultural contexts, historical periods, and interpretive frameworks. Standard knowledge graph models, while excellent at capturing factual relationships, often struggle to represent the interpretive richness and the dynamic, transformative nature of artistic significance [5, 6, 33].

This paper proposes a framework for Artistic-Oriented Knowledge Graphs (AOKGs) that addresses the aforementioned challenges through a synthesis of knowledge graph theory with category theory and the philosophical aesthetics of Alain Badiou. Our central insight is that artistic activity operates on two distinct but interconnected levels: the level of entities and relationships (what Badiou calls "being"), and the level of interpretation and meaning (what Badiou calls "appearing"). The distinction is formalized through a two-level framework [5, 6, 33]:

The Knowledge Graph Layer represents the network of artistic entities—artworks, artists, symbols, techniques, cultural contexts—and their relationships. It is the layer that captures the factual structure of the art world [5, 6, 33].

The Topos Layer A category of sheaves constructed over the knowledge graph, providing the "logic of a world" for interpretation, inspired by Badiou's use of topos theory in *Logics of Worlds* [3].

Such an approach allows us to model how the same set of artistic facts can yield radically different interpretations under different interpretive frameworks, and how artistic innovation can fundamentally transform the logic of aesthetic understanding. By grounding our framework in the rigorous mathematics of category theory and topos theory [8], we provide a formal language for reasoning about the production of artistic meaning and the nature of aesthetic truth.

The structure of this paper is as follows. In Section 2, we review the necessary background on knowledge graphs, category theory, and Badiou's philosophy. Section 3 presents the foundational layer of our framework: the knowledge graph representation of artistic entities. Section 4 introduces the topos-theoretic superstructure and explains how sheaves model the logic of appearing. Section 5 formalizes the concept of an artistic "event" as a change of topos. Section 6 explores the implications for aesthetics and the philosophy of art. Section 7 discusses implementation considerations and potential applications. Section 8 concludes with reflections on the broader significance of the present work for digital humanities and computational aesthetics [5, 6, 8, 33, 34].

2 Background and Theoretical Foundations

2.1 Knowledge Graphs: Structure and Semantics

A knowledge graph is a structured representation of experience consisting of entities, relationships, and semantic descriptions [5]. Formally, a knowledge graph can be defined as a tuple $K = (E, P, T)$, where namely E is a set of entities (nodes), P is a set of predicates (edge labels), and $T \subseteq E \times P \times E$ is a set of triples (e_1, p, e_2) representing relationships.

In the context of art, entities might include specific artworks (e.g., Guernica), artists (e.g., Picasso), movements (e.g., Cubism), symbols (e.g., the bull), and techniques (e.g., oil on canvas). Predicates capture relationships such as "created_by,""belongs_to_movement," "depicts," and "influenced_by.' [29, 32]

Knowledge graphs differ from simple databases in their emphasis on semantic richness and interconnectivity. They are often grounded in ontologies— formal specifications of the concepts and relationships in a domain—which provide a shared vocabulary and logical constraints. In the art domain, ontologies such as CIDOC-CRM and the ICON ontology [12] have been developed to capture the complexities of iconography, iconology, and art historical interpretation in the sense of Panofsky [10]. In the cultural-heritage ecosystem, such claims are often aligned with community models such as the ICON ontology [12], the CIDOC Conceptual Reference Model [13], and the Europeana Data Model [14], while controlled vocabularies such as the Getty Art & Architecture Thesaurus provide stable descriptors for media, techniques, and motifs [15].

2.2 Category theory and Topos theory

Category theory is a branch of mathematics that studies abstract structures and relationships between them [9]. A category C consists of a collection of objects Ob(C); for each pair of objects A, B, a collection of morphisms Hom(A, B); and a composition operation that is associative and has identities.

Category theory provides a powerful language for describing mathematical structures and their transformations. Of particular relevance to our work is the concept of a topos, a special kind of category that can serve as a "universe" for mathematical reasoning [8, 34].

A topos is a category that has certain properties making it resemble the category of sets, but with a richer internal structure. Specifically, a topos has finite limits and colimits, exponential objects (internal function spaces), and a subobject classifier (an object that classifies "truth values") [8, 34].

One of the most important examples of a topos is the category of sheaves on a topological space (or more generally, on a site). A sheaf is a mathematical object that assigns data to the open sets of a space in a way that respects the local-to-global principle: local data that agrees on overlaps can be glued together to form global data [8, 34].

The significance of topos theory for logic and philosophy lies in the fact that every topos has an internal logic—a system of reasoning that may differ from classical logic. In particular, the organizing principles of a topos are generally intuitionistic, meaning that the law of excluded middle (every statement is either true or false) may fail. Thus, topos theory becomes particularly well-suited for modeling context-dependent and constructive notions of truth [8, 34, 35].

2.3 Badiou's Philosophy: Being, Event, and Truth

Alain Badiou's philosophical project, developed across works such as Being and Event [1] and Logics of Worlds [3], seeks to provide a rigorous account of novelty, truth, and subjectivity. Central to Badiou's philosophy is the distinction between two mathematical frameworks:

Set theory as the mathematics of being Set theory describes the raw existence of multiplicities. It is the ontology of what is, the "state of the situation.".

Category theory (specifically topos theory) as the mathematics of appearing Topos theory describes the logic of a world—the framework that governs how beings appear and are related to one another [4].

In Alain Badiou's philosophy, a situation is the structured presentation of being as multiplicity, constituted by the operation of the count-as-one that organizes what appears and excludes what remains unpresented prior to the event. As such, for Badiou, an event is a rupture in the state of the situation—something that occurs but it stands in tension with the established order. An event is not a part of being, but a supplement to it. Following an event, a truth procedure begins: a process of fidelity to the event that involves tracing its consequences and creating a new body of lived experience organized around it.

In Logics of Worlds, Badiou uses topos theory to formalize the concept of "appearing." The topos provides the logical structure of a world, and different worlds (different topoi) can have different logics. From the standpoint of Badiou's framework, an event can be understood as necessitating a change of topos—a fundamental shift in the logic of appearing [8, 34].

2.4 Badiou's Inaesthetics: Art as Truth Procedure

In his *Handbook of Inaesthetics* [2], Badiou develops a theory of art that he calls "inaesthetics." Unlike traditional aesthetics, which subordinates art to philosophical categories, inaesthetics treats art as an autonomous truth procedure. Art, for Badiou, is one of four conditions of philosophy (alongside science, politics, and love), meaning that art produces truths that philosophy can think but not create [7].

Key principles of Badiou's inaesthetics include that art is a truth procedure, generating truths through its own immanent operations; that artistic truths are singular and irreducible to pre-existing practices; and that philosophy's task is not to judge art, but to learn from the truths that art produces.

Such a perspective has profound implications for how we model artistic experience. If art is a truth procedure, then an adequate representation of art must capture not only factual information but also the mechanisms by which art generates new meanings and transforms our understanding of the world.

3 The knowledge Graph Layer: modeling Artistic Entities

3.1 Basic Structure of the Artistic knowledge Graph

The foundation of our framework is a knowledge graph $K = (E, P, T)$ that represents the factual structure of the art world. The set of entities E includes artworks (Individual works of art (paintings, sculptures, installations, etc.)), artists (Creators of artworks), movements (Artistic movements and styles (e.g., Impressionism, Surrealism)), symbols (Iconographic elements and motifs (e.g., the dove, the skull)), techniques (Methods and materials (e.g., oil painting, bronze casting)), concepts (Abstract ideas and themes (e.g., liberty, mortality, the sublime)), and cultural contexts (Historical periods, geographical locations, social conditions.) [5, 6, 33].

The set of predicates P captures various types of relationships, namely **creation**, i.e., created_by (artwork, artist) or created_in (artwork, year); **classification**, such as belongs_to_movement (artwork, movement) or has_technique (artwork, technique); **iconography**, including something that depicts (artwork, symbol) or contains (artwork, motif); **influence**, as something in influenced_by (artist, artist) or inspired_by (artwork, artwork); and **context**, captured by created_in_context (artwork, cultural_context).

The thus constructed knowledge graph records the structured presentation of an art-historical situation—the facts of who created what, when, and in what context. It is not a representation of Badiou's uncounted pure being: as a typed and relational construction, it is an operational formalization of a situation, shaped by prior selections and classifications. It is the material upon which interpretation will be built [5, 6, 33].

3.2 Extensions: Hypergraphs and Simplicial Complexes

While the basic triple-based knowledge graph is expressive, certain aspects of artistic practice are better captured by more sophisticated structures. We consider two extensions: hypergraphs and simplicial complexes [5, 6, 22, 25, 26, 28, 31, 33].

3.2.1 Hypergraphs.

A hypergraph is a generalization of a graph in which edges (called hyperedges) can connect any number of vertices. In the context of art, hyperedges can represent multi-entity relationships that resist decomposition into binary relations. For example, an iconographic constellation such as {Madonna, Child, Halo, Gold Background} forms a semantic unit that is more than the sum of its pairwise relationships. By representing it as a single hyperedge, we capture the holistic nature of the iconographic cluster [22, 25].

3.2.2 Simplicial Complexes.

A simplicial complex is a topological structure consisting of vertices, edges, triangles, and higher-dimensional simplices. In our framework, a simplicial complex can represent the "shape" of artistic knowledge: 0-simplices (vertices) represent individual entities; 1-simplices (edges) represent pairwise relationships; 2-simplices (triangles) represent ternary relationships or tight conceptual bonds among three entities; higher-dimensional simplices capture even more complex multi-entity structures [26, 28, 31].

The topological properties of the simplicial complex—such as its connected components, holes (cycles), and voids—reveal structural features of the artistic domain. Techniques from topological data analysis (TDA) can be applied to identify persistent features that are robust across different scales of observation [26, 28, 31].

3.3 Categorical Structure of knowledge Graphs

Following recent work on the categorical foundations of knowledge graphs [11], we can view the knowledge graph as generating a free category C(K) where objects are entities in E, morphisms are paths through the graph (compositions of relationships), and composition is concatenation of paths.

The categorical perspective provides a compositional structure for reasoning about connections in the knowledge graph. For example, if there is a path from artist A to artwork W via the "created_by" relation, and a path from artwork W to symbol S via the "depicts" relation, then there is a composite morphism from A to S representing the fact that artist A created something depicting symbol S [5, 6, 33].

The knowledge graph also induces various derived structures: Line graphs, which are graphs whose vertices are the triples of the original graph, with edges connecting triples that share entities. These capture higher-order patterns of co-occurrence. And Gaifman graphs defined as graphs whose vertices are entities, with edges connecting entities that co-occur in triples. These reveal which entities are closely related in the knowledge graph. These derived structures provide different perspectives on the same underlying data, each revealing different aspects of the artistic domain [5, 6, 33].

4 The Topos Layer: the Logic of Appearing

4.1 From Being to Appearing

The knowledge graph, as described in the previous section, records a typed and relational presentation of the art-historical situation rather than Badiouian being as such. It captures entities and relations that have already been selected, classified, and encoded. This structured presentation does not by itself capture the full richness of artistic meaning. The same configuration of entities and relations can be interpreted in radically different ways depending on the conceptual framework, cultural context, and historical moment. What an artwork means is not simply a function of what it is, but of how it appears within a particular interpretive world [5, 6, 33].

Here Badiou's distinction between being and appearing becomes crucial, but it requires a qualification. Set theory concerns being in Badiou's ontological vocabulary; the knowledge graph, by contrast, is not pure uncounted multiplicity but a typed and relational operational presentation of a situation. Topos theory then provides a formal vocabulary for appearing. In other words, a topos provides the "logic of a world"—the framework that governs how entities manifest, how they are related, and what can be said about them [5, 6, 8, 33, 34].

4.2 Constructing a Topos of Sheaves

We formalize the logic of appearing by constructing a topos of sheaves over the knowledge graph. To do so, we first need to define a site—a structured space over which sheaves can be defined [5, 6, 8, 33, 34].

4.2.1 The Site. A site consists of a category (or more generally, a space) equipped with a Grothendieck topology, which specifies which collections of morphisms should be considered as "covers." In our case, the site is derived from the knowledge graph (or its associated simplicial complex) [5, 6, 26, 28, 31, 33].

One natural choice is to take the category $\mathcal{C}(K)$ generated by the knowledge graph and equip it with a topology. For example, we might define a cover of an entity e to be a collection of morphisms $\{f_i : e_i \to e\}$ such that the entities e_i collectively "cover" e in some semantic sense (e.g., they are all the artworks in a movement, or all the symbols in an iconographic cluster) [5, 6, 33].

Alternatively, if we are working with a simplicial complex, we can use the topological structure directly: the open sets of the complex form a site, and we can define sheaves on the site [8, 26, 28, 31, 34].

4.2.2 Sheaves. A sheaf F on a site assigns to each object U (or open set) a set $F(U)$, subject to compatibility conditions. Intuitively, F(U) represents the possible ways a concept manifests in the region U of the knowledge graph [5, 6, 8, 33, 34].

For example, consider a sheaf of "heroism": F(Greek Art) might be the set of ways heroism is depicted in Greek art (e.g., through muscular male figures, battle scenes, gods). F(Romantic Art) might be the set of ways heroism is depicted in Romantic art (e.g., through solitary figures in sublime landscapes, revolutionary acts). The sheaf condition ensures that if two regions overlap, the depictions of heroism in the overlap are compatible with the depictions in each region [8, 34].

Sheaves capture the idea that meaning is locally variable but globally coherent. A concept like "heroism" does not have a single, fixed definition, but rather manifests differently in different contexts. The sheaf provides a formal way to track these variations while ensuring consistency [8, 34].

4.3 The Topos of Sheaves

The collection of all sheaves on a site forms a category, denoted Sh($\mathcal{C}(K)$) or Sh(X) (where X is the simplicial complex), which is a topos [8]. As a topos, it has the following elements: all finite limits and colimits (allowing us to construct new sheaves from old ones), exponential objects (allowing us to define "function sheaves"), and a subobject classifier Ω (a sheaf that classifies "truth values").

The subobject classifier is particularly important. In the topos of sets the subobject classifier is the two-element set {0, 1}, corresponding to "false" and "true." In a general topos, the subobject classifier can be much more complex, reflecting the fact that truth is context-dependent [8, 34].

4.4 The Internal Logic of the Topos

Every topos has an internal logic—a reasoning apparatus that can be used to make statements about the objects in the topos. The internal logic of a topos is generally intuitionistic, meaning that: the law of excluded middle ($P \vee \neg P$) may fail; to prove a disjunction ($P \vee Q$) , one must provide evidence for either P or Q; to prove an existential statement ($\exists x.P(x)$) , one must provide a witness [8, 34, 35].

The intuitionistic character is well-suited to the context-dependent nature of aesthetic judgment. A statement like "This artwork represents liberty" is not absolutely true or false, but true or false relative to a particular interpretive framework (sheaf). In one topos (e.g., the topos of French Revolutionary art), the statement might be true; in another topos (e.g., the topos of Medieval religious art), it might be false or meaningless [8, 34, 35].

The internal logic allows us to reason formally about these contextdependent truths. We can ask questions like namely In which regions of the knowledge graph is a particular interpretation valid?, How do interpretations in different regions relate to one another?, and What are the logical consequences of a particular interpretive stance [5, 6, 33, 34, 35]?

4.5 Examples of Sheaves in the Artistic Context

To make these abstract ideas concrete, in what follows, we provide several examples of sheaves that might be defined over an artistic knowledge graph [5, 6, 8, 33, 34].

4.5.1 The Sheaf of Iconographic Meanings. For each region U of the knowledge graph (e.g., a cluster of artworks from a particular period or movement), the sheaf T(U) assigns the set of iconographic meanings that are operative in that region. For example, Ī(Renaissance Italy) might include meanings like "the dove represents the Holy Spirit" or "the skull represents mortality"; Z(Surrealism) might include meanings like "the melting clock represents the fluidity of time" or "the elephant on stilts represents the fragility of power." The sheaf condition ensures that if two regions overlap (e.g., an artist who bridges Renaissance and Mannerist styles), the iconographic meanings in the overlap are compatible with those in each region [5, 6, 8, 33, 34].

4.5.2 The Sheaf of Stylistic Categories. For each region U, the sheaf S(U) assigns the set of stylistic categories that are applicable in that region. For example, S(19th Century Europe) might include categories like "Realism,""Impressionism" or "Romanticism"; S(20th Century Avant-Garde) might include categories like "Cubism," "Surrealism" or "Abstract Expressionism." Again, the sheaf condition ensures consistency across overlapping regions [8, 29, 32, 34].

4.5.3 The Sheaf of Aesthetic Values. For each region U, the sheaf A(U) assigns the set of aesthetic values that are operative in that region. For example, A(Classical Art) might include values like "harmony,""proportion" or "idealization"; A(Romantic Art) might include values like "sublimity," "emotional intensity" or "individuality."

These sheaves capture the idea that aesthetic judgment is not universal but varies across cultural and historical contexts [8, 34].

5 Artistic Events and Changes of Topos

5.1 The Badiouian Event

In Badiou's philosophy, an event is a rupture in the established order something that occurs but has no place in the current state of affairs [1]. An event is not a part of being (it is not simply a new element added to the situation), but a supplement to it. An event forces a reconfiguration of the situation, opening up new possibilities that were previously invisible or unthinkable.

In the context of art, an event is a work or movement that cannot be adequately understood within the existing interpretive framework. It is an artwork that resists classification, that challenges the prevailing aesthetic values, that forces us to rethink what art is and what it can do.

5.2 Formalizing the Event as a Change of Topos

We model an artistic event through a contrast between topoi of appearing. The underlying knowledge graph presentation of the situation may remain largely unchanged-the recorded facts of who painted what and when remain available-while the interpretive conditions under which those entities appear may be reconfigured [5, 6, 8, 33, 34].

In this framework, however, a change of topos' functions as a conceptual schema for an interpretive rupture in the logic of appearing, not as the assertion of an internal mathematical morphism between topoi. A categorically precise transition would require explicitly specified sites and an appropriate geometric morphism, or another precisely stated categorical relation. No such morphism is constructed here. Rather, the contrast between the old and new topoi clarifies how the conditions under which an artwork can appear intelligibly may be reconfigured.

More precisely, an artistic event might occur in circumstances like the following: (i) When a new artwork W is added to the knowledge graph. (ii) When the existing topos τ_{old} is inadequate for interpreting W, meaning that there is no sheaf in $\mathcal{T}_{\text{old}}$ that can assign a coherent meaning to W. (iii) When the resulting interpretive tension may be represented by introducing a new topos $\mathcal{T}_{\text{new}}$, with new sheaves capable of accommodating W and, in the process, reinterpreting existing artworks [5, 6, 8, 33, 34].

The change from $\mathcal{T}_{\text{old}}$ to $\mathcal{T}_{\text{new}}$ is not simply an addition of new sheaves, but a fundamental shift in the underlying logic of the topos. Different topoi can have different logics, and what is true in one topos may be false or meaningless in another [8, 34].

5.3 Example: the Emergence of Non-Objective Art

To illustrate this concept, consider the emergence of non-objective art in the early 20th century, exemplified by works such as Kazimir Malevich's Black Square (1915) [36].

5.3.1 The Old Topos: Mimesis. Before the advent of non-objective art, the dominant interpretive framework was the topos of mimesis (representation), in which the sheaf of "meaning" was tied to what an artwork depicts. An artwork's significance came from its representation of objects, scenes, or narratives from the world. The conceptual structure of the considered topos included principles such as: (i) An artwork is meaningful if and only if it represents something. (ii) The quality of an artwork is related to the skill with which it represents its subject. (iii) Interpretation involves identifying what is represented and understanding its symbolic significance [8, 30, 34, 36].

Within such a topos, Malevich's Black Square—a black square on a white background, with no representational content—is nonsensical. There is no sheaf in the topos of mimesis that can assign a coherent meaning to it. The artwork is, in Badiou's terms, an event, i.e., it has no place in the existing order [8, 34, 36].

5.3.2 The Event. The appearance of Black Square forces a crisis in the logic of appearing. The existing topos cannot accommodate it. Viewers and critics are faced with a choice to either dismiss the work as meaningless (as many initially did) or construct a new topos in which it can be interpreted [8, 34].

5.3.3 The New Topos: Abstraction. The new topos that emerges is the topos of abstraction, where the sheaf of "meaning" is no longer tied to representation, but to pure form, color, and emotional or spiritual resonance. Here, the internal structure of this topos includes principles such as: (i) An artwork is meaningful through its formal properties, independent of representation. (ii) The quality of an artwork is related to its ability to evoke feeling or thought through non-representational means. (iii) Interpretation involves engaging with the artwork's formal structure and its effects on the viewer. Within the new topos, Black Square becomes not only interpretable but paradigmatic. It is the "zero point" of painting, the pure essence of form. The event has forced the construction of a new logic of appearing [8, 34].

5.4 The Truth Procedure

Following an event, a truth procedure begins [1] as a process of exploring the world through the lens of the new topos. It involves the following phases: constructing new sheaves within the new topos; re-evaluating existing artworks according to a new internal logic; discovering which aspects of the old world were invisible in the old topos but become visible in the new one.

In the case of the emergence of abstraction, the truth procedure involved such stages as the following: the creation of new abstract artworks that explored the possibilities opened up by the event; the reinterpretation of earlier artworks (e.g., late works of Cézanne, certain Byzantine icons) as proto-abstract, revealing aspects that were invisible in the topos of mimesis; the development of new critical and theoretical discourses (e.g., formalism, theories of the sublime) that articulated the principles of the new topos [8, 34].

A truth procedure is a process of fidelity to the event, tracing its consequences and building a newly constituted experiential framework centered on it. In our framework, this is formalized as the construction and exploration of the new topos [8, 34].

6 Aesthetics in the Topos-Theoretic framework

6.1 Aesthetic Judgment as Internal Logic

One of the key advantages of our framework is that it provides a formal language for aesthetic judgment. An aesthetic judgment is a statement in the internal logic of a particular topos. Its truth is not absolute but relative to the interpretive world defined by the topos [8, 34, 35].

For example, the statement "Malevich's Black Square is a masterpiece" is true in the topos of abstraction (where it is a paradigmatic work) and false or meaningless in the topos of mimesis (where it fails to represent anything) [8, 34, 36].

The previous account captures the intuition that aesthetic judgments are both subjective (context-dependent) and objective (they can be right or wrong within a given framework). The framework avoids both naive realism (the idea that there is a single correct interpretation) and radical relativism (the idea that all interpretations are equally valid). Instead, it recognizes that interpretations are relative to a topos, but within a topos, there are objective standards of truth [8, 34].

6.2 Beauty and the Sublime

We can use the topos-theoretic framework to distinguish between two fundamental aesthetic categories: beauty and the sublime [8, 34].

6.2.1 Beauty. An artwork is beautiful if it is a perfect exemplar of the logic of the current topos. It fully realizes the possibilities of the existing sheaves of meaning. In the topos of classical art, a beautiful work is one that achieves perfect harmony, proportion, and idealization. In the topos of Romanticism, a beautiful work is one that achieves emotional intensity and expressive power. Beauty, in this sense, is conservative, meaning that it affirms and perfects the established structure. A beautiful artwork does not challenge the topos but fulfills it [8, 34].

6.2.2 The Sublime. An artwork is *sublime* if it gestures towards the limits of the current topos, hinting at the inadequacy of the existing logic and the necessity of a new one. The sublime artwork is the harbinger of an event. It is an artwork that cannot be fully accommodated by the existing sheaves, that strains against the boundaries of the current interpretive framework [8, 34].

In Kant's aesthetics, the sublime is associated with experiences that exceed our capacity for comprehension—the vastness of nature, the power of storms, the infinity of the cosmos [16, 17]. In our framework, the sublime is associated with artworks that exceed the capacity of the current topos. They point towards a new logic of appearing that has not yet been fully articulated.

Malevich's *Black Square*, in the context of the topos of mimesis, is sublime. In other words, it is an artwork that cannot be comprehended within the existing framework, that forces us to confront the limits of our interpretive categories [8, 34, 36].

6.3 Form and Content

The topos-theoretic framework also illuminates the relationship between form and content in art. The knowledge graph represents the "form" of artistic practice—the structural relationships between entities. The sheaves represent the "content"—the distribution of concepts and meanings across the underlying structure [5, 6, 8, 33, 34].

Crucially, form and content are inseparable. The content (the sheaves) is always content of a form (the knowledge graph), and the form is always form for a content. A change in the knowledge graph (e.g., the addition of new artworks) can necessitate a change in the topos (new sheaves, new logic). Conversely, a change in the topos (a new interpretive framework) can reveal new structures in the knowledge graph that were previously invisible. The inseparability of form and content is a central insight of modern aesthetics, and the topos-theoretic framework provides a rigorous mathematical expression of it [5, 6, 8, 33, 34].

6.4 Inaesthetics: Art as a Truth Procedure

Badiou's concept of *inaesthetics* holds that art is itself a truth procedure—a process that generates truths that philosophy can think but not create [2]. In our framework, the knowledge graph captures the historical facts of art; the topos captures the logic through which art appears; the event and its truth procedure represent art's capacity to produce new truths by forcing the construction of new logics.

Philosophy does not impose its truths on art but learns from the truths that art produces through its own procedures. The role of the philosopher (or the art historian, or the computational aesthetician) is to formalize and articulate the logics that art has already discovered.

In such terms, our framework is not a prescription for how art should be interpreted, but a description of how art actually functions as a truthproducing activity. It is a tool for understanding the mechanisms by which art generates meaning and transforms our understanding of the world.

7 Implementation and applications

7.1 Constructing the AOKG

The practical implementation of the proposed framework involves several steps, outlined below.

In practical settings, the step from raw curatorial notes or scholarly prose to a graph often proceeds via established information-extraction routines: named-entity recognition to detect artists, works, places, and institutions, and relation extraction to propose candidate links (for instance, createdBy, exhibitedAt, or influencedBy). Both steps are well studied in natural language processing, and their outputs can be validated against controlled vocabularies before being committed to the AOKG [38, 41].

7.1.1 Building the knowledge Graph. The first step is to construct the knowledge graph by integrating data from various sources. For instance, art historical databases (e.g., museum collections, auction records) or scholarly publications (e.g., monographs, exhibition catalogs) or ontologies and controlled vocabularies (e.g., CIDOC-CRM, Getty vocabularies) [5, 6, 13, 15, 33].

The process now involves entity extraction, relationship extraction, and ontology alignment. Natural language processing (NLP) techniques can be used to extract entities and relationships from unstructured text, while manual curation ensures accuracy and completeness.

7.1.2 Defining the Site. The next step is to define a site over the knowledge graph, which involves choosing a topology (e.g., based on similarity metrics, co-occurrence patterns, or expert-defined clusters) and defining what constitutes a "cover" (e.g., a collection of artworks that collectively represent a movement). Remarkably, similarity metrics and co-occurrence patterns may suggest candidate coverings, but they do not themselves define a Grothendieck topology; the coverage relation must be explicitly specified and mathematically checked. The choice of topology is not unique and will depend on the specific research questions and interpretive goals. Different topologies will yield different topoi and different internal logics [5, 6, 8, 33, 34].

7.1.3 Constructing Sheaves. The third step is to define sheaves that capture key interpretive concepts, something that can be done in several ways. In a **manual definition**, experts define the sheaves based on art historical practice. In an **exploratory data-assisted approach**, machine-learning techniques can identify candidate themes or stylistic families (for example, through topic modeling or clustering). Such statistical outputs do not, by themselves, induce a

Grothendieck topology or establish sheaf compatibility: a topology requires an explicitly specified coverage relation, while sheaf compatibility requires explicit restriction maps and verification that locally compatible sections glue. Data-driven methods can therefore inform, but cannot replace, the mathematical specification and art-historical judgment required to construct a site and its sheaves. **Hybrid approaches** combine expert knowledge with these data-assisted suggestions. Each sheaf assigns to each region of the knowledge graph a set of possible meanings or interpretations. The sheaf condition ensures that these local meanings are globally coherent [5, 6, 8, 23, 33, 34].

7.1.4 Computing the Internal Logic. The final step is to use the sheaf-theoretic structure to reason about the truth of aesthetic statements within different interpretive frameworks. Thus, the following tasks are involved: formulating statements in the internal logic of the topos; using the subobject classifier to determine the truth values of these statements in different regions; exploring the logical consequences of different interpretive stances, something which can be implemented using proof assistants or automated theorem provers that support intuitionistic logic [8, 34, 35].

7.2 Applications

The AOKG framework enables several novel applications in digital humanities and computational aesthetics. Here are some examples:

On the technical side, once an AOKG is represented in an RDF-compatible form, standard Semantic Web tooling can be used to query and integrate it—for instance via SPARQL endpoints and linked-data publication practices [20, 43].

7.2.1 Multi-Perspective Interpretation. The same knowledge graph can support multiple topoi, allowing scholars to explore how artworks are interpreted differently in different theoretical or cultural frameworks. For example, a feminist topos might highlight aspects of artworks related to gender and power; a postcolonial topos might reveal how artworks reflect or resist colonial ideologies; a formalist topos might focus on purely aesthetic properties. By constructing different topoi over the same knowledge graph, we can compare and contrast these interpretive perspectives, revealing what each makes visible and what each obscures [5, 6, 8, 33, 34].

7.2.2 Identifying Interpretive Tensions. Within a specified interpretive model, the framework can identify configurations in which existing sheaves fail to reconcile newly encountered local descriptions. Such cases can be treated as candidate sites of interpretive tension by monitoring additions to the knowledge graph, examining compatibility within the currently specified sheaves, and determining where their gluing conditions cease to be satisfied. These formal signals do not computationally establish that an artistic event has occurred in Badiou's philosophical sense. Rather, they identify structural tensions that may, together with art-historical interpretation, be investigated as possible moments of innovation or rupture [5, 6, 8, 33, 34].

7.2.3 Truth Procedure Tracing. We can model how the consequences of an event propagate through the art world, revealing which artworks become newly significant in light of the event. Among other effects, such propagation entails a construction of the new topos that emerges after an event; a re-evaluation of existing artworks according to new constitutive principles; an identification of which artworks are reinterpreted or revalued. By doing so, a formal method for studying the dynamics of art historical change is made available [8, 34].

7.2.4 Aesthetic Reasoning. The internal structure of the topos provides a formal system for reasoning about aesthetic judgments and their validity within specific interpretive contexts. Here are some examples of how it can be used to: to formalize critical arguments and evaluate their logical coherence; to explore the consequences of different aesthetic principles; to identify contradictions or tensions within interpretive frameworks. Thus, new possibilities for computational art criticism and digital aesthetics may open up [8, 34].

7.3 Challenges and Limitations

While the AOKG framework offers significant advantages, it also faces several challenges, which are addressed in the sequel.

7.3.1 Complexity. The mathematical machinery of topos theory is sophisticated and may be difficult for practitioners without a strong background in category theory.

Developing user-friendly tools and interfaces will be essential for making the framework accessible to art historians and digital humanists. The challenge is not insurmountable-category theory has been successfully applied in computer science and programming language theory, where similar pedagogical challenges have been addressed through careful exposition and the development of intuitive visual representations. We envision the creation of interactive tools that allow users to explore topoi and sheaves without needing to understand the underlying mathematical formalism in full detail [8, 34].

7.3.2 Data Requirements. Constructing a comprehensive knowledge graph requires significant amounts of high-quality data. While many art historical databases exist, they are often incomplete, inconsistent, or locked behind proprietary systems. Open data initiatives and collaborative curation will be necessary to realize the full potential of the framework. The Linked Open Data movement in cultural heritage has made significant progress in that direction, with projects such as Europeana and the Getty vocabularies providing freely accessible structured data. However, much work remains to be done in terms of data quality, coverage, and interoperability. Furthermore, the construction of sheaves requires not just factual data but also interpretive understanding, which is often implicit in scholarly discourse and difficult to formalize. Developing methods for extracting and representing such an interpretive understanding from art historical texts is an important area for future research [5, 6, 8, 14, 15, 33, 34].

7.3.3 Subjectivity. The choice of topology and the definition of sheaves involve subjective decisions. Different researchers may construct different topoi over the same knowledge graph, leading to different conclusions. This is not necessarily a problem—it reflects the genuine plurality of interpretive perspectives but it does mean that the framework cannot provide a single, definitive interpretation. Indeed, one of the strengths of the framework is that it makes these subjective choices explicit and allows for systematic comparison of different interpretive stances. By formalizing the structure of interpretation, we can better understand the assumptions and commitments that underlie different critical approaches. A meta-level perspective of such a kind is valuable in its own right, even if it does not resolve interpretive disputes

7.3.4 Computational Tractability. Computing with sheaves and structural logics can be computationally expensive, especially for large knowledge graphs. Developing efficient algorithms and approximations will be an important area of future research. One promising direction is to leverage recent advances in computational topology and topological data analysis, which have developed efficient methods for computing homology and persistent features of large datasets. Another approach is to use sampling and approximation techniques, computing sheaves on a representative subset of the knowledge graph rather than the entire graph. Finally, distributed and parallel computing architectures may be necessary for scaling the framework to very large cultural heritage datasets [5, 6, 8, 33, 34].

7.4 A Worked Example: Cubism as an Event

To further illustrate the framework, we present a detailed worked example, namely the emergence of Cubism as an artistic event [29, 32].

7.4.1 The knowledge Graph. Consider a knowledge graph of early 20th-century European art, containing entities such as the following. Artists: Pablo Picasso, Georges Braque, Paul Cézanne, Henri Matisse. Artworks: Les Demoiselles d'Avignon (1907), Houses at L'Estaque (1908), The Large Bathers (1906). Movements: Post-Impressionism, Fauvism, Cubism. Techniques: Oil painting, collage, multiple perspectives. Concepts: Representation, fragmentation, simultaneity, the fourth dimension [5, 6, 29, 32, 33].

The knowledge graph includes relationships such as "Picasso created Les Demoiselles d'Avignon," "Les Demoiselles d'Avignon belongs to Cubism," Picasso was influenced by Cézanne, and "Cubism employs multiple perspectives" [29, 32].

7.4.2 The Old Topos: Perspectival Representation. Before Cubism, the dominant topos for interpreting painting was the topos of perspectival representation. In it, key sheaves include the sheaf of "spatial coherence", which assigns to each artwork a single, consistent spatial viewpoint, the sheaf of "temporal unity", which assigns to each artwork a single moment in time, and the sheaf of "naturalism", which assigns to each artwork a degree of fidelity to visual appearance [8, 29, 32, 34].

Now, the topos's formal structure includes principles such as: an artwork depicts a scene from a single viewpoint; objects in the artwork maintain their spatial relationships as they would in reality; The artwork represents a single moment in time [8, 34].

7.4.3 The Event: Les Demoiselles d'Avignon. Picasso's Les Demoiselles d'Avignon (1907) produces substantial interpretive friction within a narrowly specified topos of perspectival representation. The painting depicts five female figures, but their bodies are fragmented and shown from multiple viewpoints simultaneously. Its spatial relations challenge the conventions of single-point perspective: some figures appear frontal, others in profile, and the faces on the right are rendered in a markedly different style, influenced by African masks. No single viewpoint straightforwardly organizes the scene [29, 32].

Within this deliberately delimited perspectival schema, the painting places productive pressure on inherited expectations of spatial coherence. The corresponding sheaf, as provisionally specified, cannot reconcile all of the painting's local descriptions under its existing constraints. This does not pronounce the work itself incoherent, nor does it reduce its complex reception to a binary truth failure. Rather, it models the interpretive tension characteristic of a transitional phase, in which established conditions of appearance are being tested and rearticulated [8, 34].

7.4.4 The New Topos: Cubist Simultaneity. The emergence of Cubism necessitates the construction of a new topos, in which a number of new sheaves are defined. The sheaf of "multiple perspectives" assigns to each artwork a set of simultaneous viewpoints. The sheaf of "temporal layering" associates to each artwork a synthesis of multiple moments. The sheaf of "conceptual structure" ascribes to each artwork an underlying geometric or conceptual form that transcends visual appearance [8, 29, 32, 34].

The internal logic of this topos encompasses new principles. For instance, an artwork can depict a scene from multiple viewpoints simultaneously. At the same time, spatial relationships can be rearranged to reveal underlying structure. Thus, the artwork represents not a single moment but a synthesis of temporal experience [8, 34, 35].

Within such a new topos, Les Demoiselles d'Avignon becomes interpretable. The fragmentation and multiple perspectives are not incoherence but a new form of coherence. The painting reveals the underlying structure of the figures and the space they occupy, freed from the constraints of a single viewpoint [8, 34].

7.4.5 The Truth Procedure. Following the event, a truth procedure unfolds. Consider Picasso and Braque, who produce a series of Cubist works that explore the possibilities of the new topos, thereby triggering a reinterpretation of earlier works. Hence, Cézanne's late paintings are seen as proto-Cubist, revealing aspects that were invisible in the old topos. The new topos is articulated in critical and theoretical discourse: critics develop the vocabulary of "multiple perspectives,"simultaneity," and "the fourth dimension." The new topos influences other domains: literature (e.g., the simultaneity of Joyce's Ulysses), music (e.g., the polyrhythms of Stravinsky), and philosophy (e.g., Bergson's theories of duration and memory). Accordingly, the truth procedure is a process of fidelity to the event, tracing its consequences and building a new body of engagement taking shape around it [8, 34].

7.5 Comparison with Other Frameworks

To situate our framework within the broader landscape of digital humanities and computational aesthetics, we compare it with several alternative approaches.

7.5.1 Traditional Ontologies. Traditional ontologies for art (e.g., CIDOC-CRM, ICON) provide structured vocabularies and taxonomies for describing artworks and their properties. While valuable, these ontologies are limited in their ability to represent context-dependent meaning and interpretive plurality. They typically assume a single, fixed interpretation of each concept, whereas our topos-theoretic framework allows for multiple, context-dependent interpretations [8, 12, 13, 34].

7.5.2 Semantic Web and Linked data. The Semantic Web and Linked Data approaches use RDF triples and ontologies to create interconnected knowledge graphs [19, 20, 21]. These approaches excel at representing factual relationships and enabling data integration across sources. However, they lack the formal machinery for modeling the logic of interpretation and the dynamics of meaning change. Our framework can be seen as an extension of Semantic Web approaches, adding a topos-theoretic layer that captures interpretive logic.

7.5.3 Machine learning and Embeddings. Machine learning approaches, particularly neural network embeddings, have been used to represent artworks and their relationships in high-dimensional vector spaces. These approaches can capture subtle similarities and patterns that are difficult to express in symbolic form. However, they are typically "black boxes" that do not provide interpretable explanations for their predictions. Our framework, by contrast, provides an explicit, interpretable model of meaning. A promising direction for future work is to combine the two approaches: using embeddings to learn sheaves from data, while using the topos-theoretic framework to provide interpretable structure [8, 24, 30, 33, 34].

7.5.4 Computational Hermeneutics. Computational hermeneutics is an emerging field that seeks to formalize interpretive processes using computational methods. Our framework can be seen as a contribution to computational hermeneutics, providing a rigorous mathematical foundation for modeling interpretation. Compared to other approaches in computational hermeneutics, which often focus on text analysis and natural language processing, our framework is specifically designed for the visual and aesthetic domain, taking into account the unique challenges of interpreting artworks [27, 39].

From the perspective of the arts, this line of work aligns with broader methodological shifts in the digital humanities—including "distant reading" and large-scale cultural analytics—which treat corpora of texts and images as analyzable datasets while retaining interpretive, contextual commitments [37, 40]

8 Related Work

8.1 Knowledge Graphs in Digital Humanities

The use of knowledge graphs in digital humanities has grown significantly in recent years [6]. Projects such as Linked Open Data for cultural heritage have created vast networks of interconnected information about artworks, artists, and cultural artifacts. Europeana, for example, provides access to millions of digitized cultural heritage objects from European museums, libraries, and archives, all connected through a knowledge graph structure. The Getty vocabularies (Art & Architecture Thesaurus, Union List of Artist Names, Getty Thesaurus of Geographic Names) provide controlled vocabularies that are widely used in the art world and serve as foundational elements for many knowledge graphs.

However, most of these projects focus on factual data—who created what, when, where, using what materials—and do not address the interpretive and aesthetic dimensions that our framework seeks to capture. They excel at answering questions like "Which artworks were created by Rembrandt in Amsterdam?" but struggle with questions like "How did the meaning of light symbolism change from the Renaissance to the Baroque?" Our framework aims to bridge the existing gap by adding a layer of interpretive logic on top of the factual knowledge graph [5, 6, 33].

8.2 Ontologies for Art

Several ontologies have been developed specifically for the art domain. CIDOC-CRM (Conceptual Reference Model) is a comprehensive ontology for cultural heritage that provides a formal structure for describing museum objects, their provenance, and their relationships. It has been widely adopted by museums and cultural institutions worldwide. The ICON ontology [12] extends CIDOC-CRM with specific structures for representing iconography and iconology, following Panofsky's three-level interpretive method. ICON distinguishes between pre-iconographic description (what is literally depicted), iconographic analysis (what symbols mean), and iconological interpretation (what deeper cultural meanings are expressed).

These ontologies provide valuable vocabularies and structures, and our framework is compatible with them—indeed, they can serve as the basis for constructing the knowledge graph layer. However, they do not incorporate the topos-theoretic machinery that allows us to model context-dependent meaning and artistic events. They provide a single, fixed interpretation of each concept, whereas our framework allows for multiple, context-dependent interpretations that can change over time [5, 6, 8, 33, 34].

8.3 Category theory in Computer Science

Category theory has been widely used in computer science, particularly in programming language semantics and type theory. The Curry-Howard correspondence, which relates logical propositions to types and proofs to programs, is fundamentally categorical in nature. Functional programming languages like Haskell make extensive use of categorical concepts such as functors, monads, and natural transformations.

The use of topos theory for knowledge representation has been explored in a few works [11]. Reformat and D'Aniello propose using topos categories to represent knowledge graphs, arguing that the categorical structure provides a more flexible and expressive framework than traditional graph databases. However, their work focuses on technical aspects of knowledge representation and query processing, and has not been applied specifically to the domain of art or aesthetics. Our framework builds on these technical foundations but extends them with philosophical and aesthetic concepts drawn from Badiou's work.

8.4 Computational approaches to Art History

Computational art history is an emerging field that applies computational methods to art historical questions. Recent work has used computer vision to analyze stylistic features of paintings, network analysis to study the structure of the art market, and natural language processing to extract information from art historical texts. These approaches have yielded valuable insights, but they typically treat artworks as data points to be analyzed statistically, without engaging with the interpretive and hermeneutic dimensions of art history [27, 42].

Our framework offers a different approach. Rather than replacing human interpretation with computation, it seeks to formalize the structure of interpretation itself, making it possible to reason about interpretive frameworks and their transformations. The approach aligns with the goals of digital humanities more broadly, which seeks to use computational methods to enhance rather than replace humanistic inquiry.

8.5 Badiou's Philosophy in Digital Humanities

While Badiou's philosophy has been influential in literary theory and cultural studies, its application to computational methods is still nascent. Some work has explored the use of Badiou's concepts in analyzing digital culture and new media art, examining how digital technologies create new possibilities for artistic events and truth procedures. However, to our knowledge, this is the first attempt to formalize Badiou's aesthetics using topos theory together with knowledge graphs in a systematic way [5, 6, 8, 33, 34].

The challenge in applying Badiou's philosophy computationally is that his concepts—event, truth procedure, fidelity—are intentionally abstract and resist straightforward formalization. Our approach is not to reduce Badiou's philosophy to a computational algorithm, but rather to use the mathematical structures he employs (set theory, topos theory) as a bridge between philosophical concepts and computational implementations. By formalizing the logic of appearing as a topos, we remain faithful to Badiou's mathematical approach while making his ideas amenable to computational exploration [8, 34].

8.6 Philosophical Aesthetics and Formal methods

The relationship between aesthetics and formal methods has a long history. Leibniz dreamed of a characteristica universalis that could resolve all disputes through calculation [18]. In the 20th century, logical positivists sought to apply formal logic to philosophical problems, though they largely avoided aesthetics as too subjective. More recently, formal methods have been applied to ethics (deontic logic) and epistemology (epistemic logic), but aesthetics has remained relatively untouched.

Our framework can be seen as a contribution to formal aesthetics, using the tools of category theory and topos theory to provide a rigorous mathematical foundation for aesthetic concepts. However, unlike earlier attempts at formal aesthetics (e.g., Birkhoff's aesthetic measure), we do not seek to reduce aesthetic value to a formula. Rather, we formalize the structure of interpretive frameworks and the dynamics of their transformation, leaving room for the irreducible plurality and historicity of aesthetic judgment [8, 34]

9 Conclusion

This paper has introduced a novel framework for Artistic-Oriented Knowledge Graphs that synthesizes knowledge graph technology, category theory, and the philosophical aesthetics of Alain Badiou. By distinguishing between the knowledge graph layer, understood as a typed operational presentation of an art-historical situation, and the topos layer, which formalizes a corresponding "logic of appearing," we have developed a model capable of capturing both structured art-historical information and the context-dependent, transformative character of artistic meaning [5, 6, 8, 33, 34].

The formalization of artistic events as changes of topos provides a rigorous way to understand how art produces new truths by forcing the reconstruction of the very logic through which we perceive the world. The proposed approach respects the autonomy of art as a truth procedure while providing the formal tools necessary for computational analysis and reasoning [8, 34].

The framework opens new possibilities for digital humanities, computational aesthetics, and the formal study of cultural exposure. It allows us to model multi-perspective interpretation, examine candidate sites of interpretive rupture, trace the modeled consequences of truth procedures, and reason formally about aesthetic judgments. While challenges remain— particularly in terms of complexity, data requirements, and computational tractability—the framework provides a solid foundation for future research.

Ultimately, the present work demonstrates that the rigorous formalisms of mathematics can illuminate the most subtle and elusive aspects of human culture. By bringing together knowledge graphs, category theory, and philosophical aesthetics, we have shown that it is possible to model not only what art is, but how it means and how it transforms our understanding of the world. A synthesis of the computational and the humanistic, the formal and the interpretive, points towards a future in which digital tools enhance rather than diminish our appreciation of art's profound capacity to generate truth [5, 6, 33].

10 Future Directions

The AOKG framework opens up numerous avenues for future research, spanning theoretical developments, computational implementations, and empirical applications.

10.1 Theoretical Extensions

10.1.1 Higher Category theory. Our framework uses ordinary category theory and topos theory, but recent developments in higher category theory (2-categories, ∞-categories) offer even richer structures for modeling artistic practice. In higher category theory, we can represent not just objects and morphisms, but also morphisms between morphisms (2-morphisms), and so on. An approach of such a kind could allow us to model not just interpretations, but also relationships between interpretations, and transformations of interpretive frameworks. For example, a 2-morphism could represent the relationship between two different critical approaches to the same artwork, capturing how one interpretation can be seen as a refinement or critique of another [8, 34].

10.1.2 Homotopy Type theory. Homotopy type theory (HoTT) is a recent development that unifies logic, type theory, and homotopy theory. In HoTT, equality is not a simple yes/no proposition but a space of paths between objects. HoTT could provide a natural framework for modeling the gradual evolution of artistic meanings and the continuity between different interpretive stances. Rather than discrete changes of topos, we could model smooth deformations of interpretive frameworks, capturing the gradual shifts in aesthetic sensibility that characterize art historical periods [8, 34].

10.1.3 Quantum Logic and Non-Classical Logics. The conceptual structure of a topos is intuitionistic, but other non-classical logics (e.g., quantum logic, paraconsistent logic) might be appropriate for modeling certain aspects of artistic interpretation. Quantum logic, with its non-distributive lattice structure, could capture the way that certain artistic meanings are fundamentally incompatible or complementary, analogous to the complementarity of position and momentum in quantum mechanics Paraconsistent logic, which tolerates contradictions, could model the way that artworks can simultaneously support contradictory interpretations without collapsing into incoherence [8, 34, 35].

10.2 Computational Developments

10.2.1 Proof Assistants and Formal Verification. The underlying logic of a topos can be implemented in proof assistants such as Coq, Agda, or Lean, something which would allow us to formalize aesthetic arguments and verify their logical correctness automatically. Although the approach may seem overly formal for the humanities, it could be valuable for clarifying complex theoretical debates and identifying hidden assumptions. Moreover, the process of formalization itself can be illuminating, forcing us to make explicit the logical structure of our arguments [8, 34].

10.2.2 Machine learning for Sheaf Construction. One challenge in implementing the AOKG framework is the specification of sites and sheaves. Machine learning techniques, particularly unsupervised methods such as topic modeling and clustering, may assist this process by identifying candidate thematic groupings, stylistic families, or local interpretive patterns in large corpora. Such statistically inferred structures should not themselves be identified with sheaves. Rather, they can provide data-assisted hypotheses from which domain experts may specify candidate contexts, coverage relations, and local sections. A genuine sheaf construction additionally requires explicit restriction maps and verification of the relevant compatibility and gluing conditions. Developing principled interfaces between statistical pattern discovery and mathematically specified sheaf structures therefore remains an open methodological problem [8, 23, 34].

10.2.3 Interactive Visualization Tools. To make the framework accessible to art historians and digital humanists, we need to develop interactive visualization tools that allow users to explore knowledge graphs, topoi, and sheaves without needing to understand the underlying mathematics. These tools could use graph visualization techniques to display the knowledge graph, with overlays showing the values of different sheaves in different regions. Users could interactively construct new topoi by selecting different sheaves and topologies, and immediately see how the interpretation of artworks changes. Such tools would make the framework not just a theoretical construct but a practical instrument for art historical research [5, 6, 8, 33, 34].

10.3 Empirical applications

10.3.1 Large-Scale Art Historical Studies. One of the most promising applications of the AOKG framework is large-scale art historical studies that would be infeasible with traditional methods. For example, we could analyze the evolution of iconographic meanings across centuries and cultures, tracing how symbols migrate and transform as they move through different interpretive contexts. We could study the dynamics of artistic influence at a global scale, identifying patterns of diffusion and innovation. We could compare different critical traditions (e.g., formalism, iconology, social art history) by constructing different topoi and seeing what each makes visible [8, 34].

10.3.2 Museum and Curatorial applications. The framework could be used to develop new tools for museum curation and exhibition design. By modeling the interpretive frameworks of different audiences (e.g., specialists vs. general public, different cultural backgrounds), curators could design exhibitions that are sensitive to these differences. The framework could also support personalized museum experiences, where visitors explore artworks through topoi that align with their interests and background involvement [8, 34].

10.3.3 Art Education. The framework could be used in art education to help students understand the plurality of interpretive perspectives and the historical contingency of aesthetic categories. By exploring different topoi, students could see how the same artwork can be interpreted in radically different ways, and how these interpretations are shaped by broader cultural and historical contexts. In doing so, the approach could foster a more sophisticated and reflective engagement with art [8, 34].

10.4 Interdisciplinary Connections

10.4.1 Philosophy of Science. The AOKG framework has interesting connections to philosophy of science, particularly debates about scientific realism, theory change, and incommensurability. Badiou's concept of the event and the change of topos is reminiscent of Kuhn's notion of paradigm shifts in science. By formalizing these concepts using topos theory, we might gain new insights into the structure of scientific revolutions and the nature of scientific progress. Conversely, insights from philosophy of science could inform our understanding of artistic innovation and change [8, 34].

10.4.2 Cognitive Science and Psychology. The framework could be connected to cognitive science and psychology of art, which study how people actually perceive and interpret artworks. The sheaves in our framework could be seen as formal models of interpretive schemas or mental models that people use to make sense of art. Empirical studies could test whether the topoi predicted by the framework align with the interpretive strategies that people actually employ. The proposed approach could lead to a more psychologically grounded theory of aesthetic interpretation [8, 34].

10.4.3 Literary theory and Cultural Studies. While our framework is developed for visual art, the underlying ideas are applicable to other cultural domains. Literary theory has long grappled with questions of interpretation, meaning, and the relationship between text and context. The AOKG framework could be adapted to model literary interpretation, with knowledge graphs representing intertextual relationships and topoi representing different critical approaches (e.g., New Criticism, deconstruction, New Historicism). Similarly, the framework could be applied to music, film, and other cultural forms, providing a unified mathematical language for the study of meaning across media [5, 6, 8, 33, 34].

Beyond its foundational role, the framework is designed to speak directly to everyday art-historical and curatorial workflows. Interpretive claims in catalogues raisonnés, exhibition narratives, conservation reports, and provenance dossiers are typically produced locally, under discipline-specific assumptions and evidentiary standards, and only later compared, negotiated, or revised. Encoding these local interpretive states as sections over contexts makes the act of scholarly "agreement" mathematically explicit namely when locally compatible interpretations glue, the framework represents a stable, jointly supported reading and when they fail to glue, the obstruction records a precise location of disagreement rather than collapsing it into noise. On the computational side, the framework suggests practical tools for transparent digital scholarship, including provenance-aware tracking of interpretive revisions, context-sensitive querying, and principled detection of inconsistency across interpretive communities.

The point is not simply that category theory can be applied to cultural data, but that, phrased in Badiou's way, the mathematics itself captures a distinctive feature of aesthetic meaning by operating at the level of the situation and reconfiguring the space of presentation in which meanings appear. Topoi provide a rigorous bridge between local artistic interpretation and global coherence, allowing mathematical notions such as locality, gluing, and logical internality to be read as structural correlates of art-historical reasoning. In such terms, the proposed AOKG formalism contributes to a mathematics of artistic meaning, where formal structure does not replace judgment but clarifies its conditions, its limits, and its modes of change. Accordingly, the framework should not be read as a reductive algorithm for producing artistic events or truths. Its role is to illuminate the formal conditions under which interpretations cohere or are reconfigured, while preserving art's autonomous capacity to generate singular truths.

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REFERENCES

[1] Badiou, A. (2005). Being and Event. Continuum. (Translated by Oliver Feltham)

[2] Badiou, A. (2005). Handbook of Inaesthetics. Stanford University Press. (Translated by Alberto Toscano)

[3] Badiou, A. (2009). Logics of Worlds: Being and Event II. Continuum. (Translated by Alberto Toscano)

[4] Angot-Pellissier, R. (2015). The Relation Between Logic, Set Theory and Topos Theory as it Is Used by Alain Badiou. In A. Koslow & A. Buchsbaum (Eds.), The Road to Universal Logic: Festschrift for the 50th Birthday of Jean-Yves Béziau, Volume II (Studies in Universal Logic, pp. 181–200). Birkhäuser, Cham. doi: 10.1007/978-3-319-15368-1_7.

[5] Ehrlinger, L., & Wöß, W. (2016). Towards a Definition of Knowledge Graphs. SEMANTiCS (Posters, Demos, SuCCESS), 48, 1–4.

[6] Hogan, A., Blomqvist, E., Cochez, M., et al. (2021). Knowledge Graphs. ACM Computing Surveys, 54(4), 1–37.

[7] Lan, J. (2014). Inaesthetics and Art as Procedure of Truth: An Introduction to Badiou's Art Thought. Theoretical Studies in Literature and Art, 34(2), 182-189.

[8] Mac Lane, S., & Moerdijk, I. (1992). Sheaves in Geometry and Logic: A First Introduction to Topos Theory. Springer-Verlag.

[9] Mac Lane, S. (1998). Categories for the Working Mathematician (2nd ed.). Springer-Verlag.

[10] Panofsky, E. (1955). Meaning in the Visual Arts. Doubleday Anchor Books.

[11] Reformat, M. Z., D'Aniello, G., & Gaeta, M. (2018). Knowledge Graphs, Category Theory and Signatures. In 2018 IEEE/WIC/ACM International Conference on Web Intelligence (WI) (pp. 480–487). IEEE Computer Society. doi: 10.1109/WI.2018.00-49.

[12] Sartini, B., Baroncini, S., van Erp, M., Tomasi, F., & Gangemi, A. (2023). ICON: An Ontology for Comprehensive Artistic Interpretations. ACM Journal on Computing and Cultural Heritage, 16(3), Article 59.

[13] CIDOC CRM Special Interest Group. (2025). Versions of the CIDOC-CRM (draft releases, including v7.3 and v7.3.1). Retrieved from https://cidoc-crm.org/versions-of-the-cidoc-crm

[14] Europeana. (2013). Europeana Data Model (EDM) Primer. Retrieved from https://pro.europeana.eu/page/edm-documentation.

[15] Getty Research Institute. (n.d.). Art & Architecture Thesaurus (AAT). Retrieved from https://www.getty.edu/research/tools/vocabularies/aat/.

[16] Kant, I. (1998). Critique of Pure Reason. (P. Guyer & A. W. Wood, Eds. & Trans.). Cambridge University Press.

[17] Kant, I. (2000). Critique of the Power of Judgment. (P. Guyer & E. Matthews, Trans.). Cambridge University Press.

[18] Leibniz, G. W. (1989). Philosophical Essays. (R. Ariew & D. Garber, Eds. & Trans.). Hackett Publishing.

[19] Berners-Lee, T., Hendler, J., & Lassila, O. (2001). The Semantic Web. Scientific American, 284(5), 34–43. doi: 10.1038/scientificamerican0501-34.

[20] Cyganiak, R., Wood, D., & Lanthaler, M. (2014). RDF 1.1 Concepts and Abstract Syntax. W3C Recommendation. Retrieved from https://www.w3.org/TR/rdf11-concepts/.

[21] W3C OWL Working Group. (2012). OWL 2 Web Ontology Language Document Overview (Second Edition). W3C Recommendation. Retrieved from https://www.w3.org/TR/owl2-overview/.

[22] Federico Battiston, Giulia Cencetti, Iacopo Iacopini, Vito Latora, Maxime Lucas, Alice Patania, Jean-Gabriel Young, and Giovanni Petri. Networks beyond pairwise interactions: Structure and dynamics. Physics Reports 874, 1–92, 2020. doi: 10.1016/j.physrep.2020.05.004.

[23] David M. Blei, Andrew Y. Ng, and Michael I. Jordan. Latent Dirichlet allocation. Journal of Machine Learning Research 3, 993–1022, 2003.

[24] Antoine Bordes, Nicolas Usunier, Alberto Garcia-Duran, Jason Weston, and Oksana Yakhnenko. Translating embeddings for modeling multi-relational data. In Advances in Neural Information Processing Systems (NeurIPS), 2013.

[25] Alain Bretto. Hypergraph Theory: An Introduction. Springer, 2013. doi: 10.1007/978-3-319-00080-0.

[26] Herbert Edelsbrunner and John Harer. Computational Topology: An Introduction. American Mathematical Society, 2010.

[27] Hans-Georg Gadamer. Truth and Method. 2nd revised edition. Edited by Joel Weinsheimer and Donald G. Marshall. Continuum, 2004.

[28] Chad Giusti, Robert Ghrist, and Danielle S. Bassett. Two's company, three (or more) is a simplex. Journal of Computational Neuroscience 41(1), 1–14, 2016. doi: 10.1007/s10827-016-0608-6.

[29] John Golding. Cubism: A History and an Analysis, 1907–1914. 3rd edition. Belknap Press of Harvard University Press, 1988.

[30] Nelson Goodman. Languages of Art: An Approach to a Theory of Symbols. 2nd edition. Hackett Publishing Company, 1976.

[31] Allen Hatcher. Algebraic Topology. Cambridge University Press, 2002.

[32] Linda Dalrymple Henderson. The Fourth Dimension and Non-Euclidean Geometry in Modern Art. Revised edition. The MIT Press, 2013.

[33] Shaoxiong Ji, Shirui Pan, Erik Cambria, Pekka Marttinen, and Philip S. Yu. A survey on knowledge graphs: Representation, acquisition, and applications. IEEE Transactions on Neural Networks and Learning Systems 33(2), 494–514, 2022. doi: 10.1109/TNNLS.2021.3070843.

[34] P. T. Johnstone. Topos Theory. Academic Press, 1977.

[35] J. Lambek and P. J. Scott. Introduction to Higher-Order Categorical Logic. Cambridge University Press, 1986.

[36] Kazimir Malevich. The Non-Objective World: The Manifesto of Suprematism. Dover Publications, 2003.

[37] Lev Manovich. Cultural Analytics. The MIT Press, 2020.

[38] Mike Mintz, Steven Bills, Rion Snow, and Dan Jurafsky. Distant supervision for relation extraction without labeled data. In Proceedings of the Joint Conference of the 47th Annual Meeting of the ACL and the 4th International Joint Conference on Natural Language Processing (ACL-IJCNLP), pages 1003– 1011, 2009.

[39] John W. Mohr, Robin Wagner-Pacifici, and Ronald L. Breiger. Toward a computational hermeneutics. Big Data & Society 2(2), 2015. doi: 10.1177/2053951715613809.

[40] Franco Moretti. Distant Reading. Verso, 2013.

[41] David Nadeau and Satoshi Sekine. A survey of named entity recognition and classification. Lingvisticae Investigationes 30(1), 3-26, 2007. doi: 10.1075/li.30.1.03nad.

[42] Babak Saleh and Ahmed Elgammal. Large-scale classification of fine-art paintings: Learning the right metric on the right feature. International Journal for Digital Art History (2), 2016. doi: 10.11588/dah.2016.2.23376.

[43] Steve Harris and Andy Seaborne (eds.). SPARQL 1.1 Query Language. W3C Recommendation, 21 March 2013. W3C. Available at https://www.w3.org/TR/sparq111-query/.