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Wheat Production  
Prediction

Property of Many Special  
Types

Relation Polynomials in  
Number

Leucaena Leucocephala Leaf  
Meal



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IMAGE: OBSERVATORY WITH STAR  
TRAILS ON MOUNTAINS FOR  
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LONDON JOURNAL OF  
RESEARCH IN SCIENCE: NATURAL AND FORMAL

Volume 22 | Issue 15 | Compilation 1.0

Print ISSN: 2631-8490  
Online ISSN: 2631-8504  
DOI: 10.17472/LJRS





London Journal of Research in Science: Natural and Formal

Volume 22 | Issue 15 | Compilation 1.0

## PUBLISHER

London Journals Press  
1210th, Waterside Dr, Opposite Arlington Building, Theale, Reading  
Phone: +444 0118 965 4033 Pin: RG7-4TY United Kingdom

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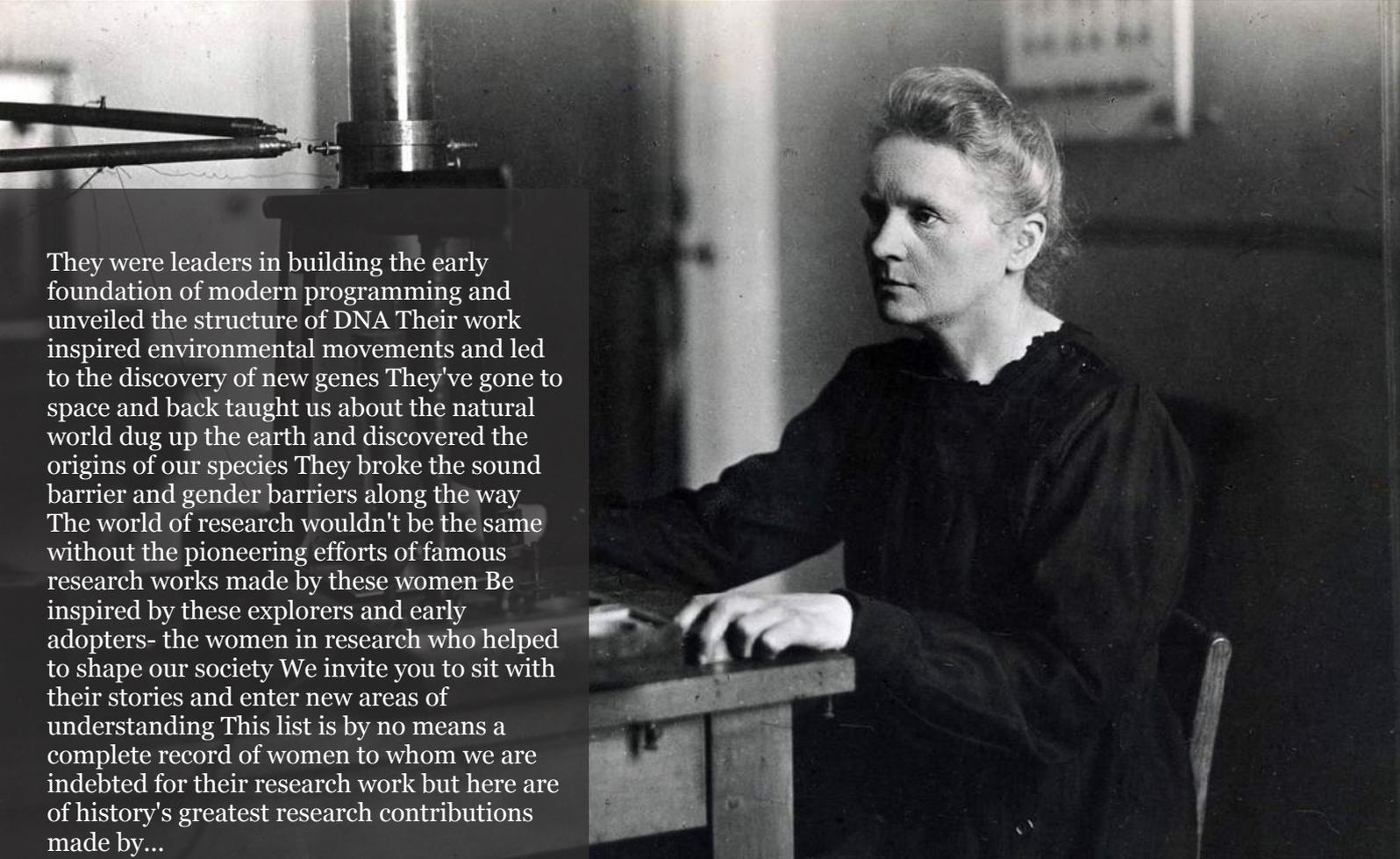
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# Some Physiological Responses of the Catfish, *Clarias Gariepinus* (Burchell 1822) Fed Cassava (*Manihot Esculenta*) Peel and *Leucaena Leucocephala* Leaf Meal

*Adewumi, A. A, Idowu, E.O., Obe, B.W, Abesin, O. & Odeyemi, O.M*

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## ABSTRACT

This study examined the physiological responses (hematology and enzyme characteristics) of *Clarias gariepinus* juveniles (mean weight  $29.69 \pm 0.91\text{g}$ ) fed diets with varying levels of fermented cassava (*Manihot esculenta*) peel and *Leucaena leucocephala* leaf meal (CPLM), for a period of 10 weeks in the laboratory. Seven iso-caloric and iso-nitrogenous diets were formulated containing 0%, 20%, 30%, 40%, 50%, 60% and 70% CPLM maize replacements tagged diets D<sub>0</sub>; D<sub>20</sub>; D<sub>30</sub>; D<sub>40</sub>; D<sub>50</sub>; D<sub>60</sub> and D<sub>70</sub> respectively.

The results showed that the final weight gain, DWG, PWG, FCR and PER of the fish fed diets diet D<sub>0</sub>, D<sub>20</sub>, D<sub>30</sub>, D<sub>40</sub>, D<sub>50</sub> were not significantly different ( $P > 0.05$ ) from one another, but were significantly higher ( $P < 0.05$ ) than those of the fish fed diets D<sub>60</sub>. The SGR and survival were not significantly different ( $P > 0.05$ ) from one another. The white blood cell (WBC) and lymphocytes ranged between  $7.35\text{-}8.14 \times 10^3 \text{ mm}^{-3}$  and 63.00-72.00% respectively.

**Keywords:** proximate, disorders, glucanase, amylase, anti-nutrients, haematology.

**Classification:** FOR Code: 069999

**Language:** English



London  
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LJP Copyright ID: 925651

Print ISSN: 2631-8490

Online ISSN: 2631-8504

London Journal of Research in Science: Natural and Formal

Volume 22 | Issue 15 | Compilation 1.0





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This study examined the physiological responses (hematology and enzyme characteristics) of *Clarias gariepinus* juveniles (mean weight 29.69 ± 0.91g) fed diets with varying levels of fermented cassava (*Manihot esculenta*) peel and *Leucaena leucocephala* leaf meal (CPLM), for a period of 10 weeks in the laboratory. Seven iso-caloric and iso-nitrogenous diets were formulated containing 0%, 20%, 30%, 40%, 50%, 60% and 70% CPLM maize replacements tagged diets D<sub>0</sub>; D<sub>20</sub>; D<sub>30</sub>; D<sub>40</sub>; D<sub>50</sub>; D<sub>60</sub> and D<sub>70</sub> respectively.

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farmers especially in resource poor regions of the world can take advantage of this ingredient as a replacement for more expensive maize when formulating feed for fish in aquaculture.

**Keywords:** proximate, disorders, glucanase, amylase, anti-nutrients, haematology.

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## I. INTRODUCTION

Feed is one of the major inputs in aquaculture production and fish feed technology is one of aquaculture least developed sectors of aquaculture, particularly in Africa and other developing countries of the world. FAO (1993) have emphasized the utmost importance of using local feed resources as the key driving force to increase the productivity of fish. High cost of fish feed was observed as one of the problems hampering aquaculture development in Nigeria (Adewumi, 2015). Expensive feeds will marginalize or even nullify the profitability of fish farming thereby incapacitating the expansion of farms to increase production resulting in the scarcity of the commodity (fish) and eventually high cost of the few available ones to the disadvantage of the consumers (Adikwu, 1992).

Cassava peel as a cheap carbohydrate source is capable of supplying adequate calories to *Clarias gariepinus* fingerlings/juvenile with improved

protein value, through fermentation with biomass from organic sources (Ijaiya, 2001). The starch in cassava is highly digestible when compared to that of maize due to the high content of amylopectin (Talthawan et al., 2002). However, cassava peel, as an energy component of the test diets contains some hydrogen cyanide (HCN) derivatives that have been shown to be toxic to livestock (McDonald et al., 1995) and therefore limits the use of cassava peels in the raw state, as feedstuff.

Phytate, found in cassava products binds with phosphorus in diets and render it non bio-available to any animal that is non-ruminant. Besides, phytate has also been reported to form complexes with proteins at both low and high pH values. These complex formations alter the protein structure, which may result in decreased protein solubility, enzymatic activity, and proteolytic digestibility. In order to prevent goitrogenetic and other neuropathological effect on animals, it will be necessary to process the peels before consumption.

Studies on the use of cassava meal in fish feed indicate that cassava can replace the conventional energy feed ingredients such as maize, broken rice and sorghum, which are commonly used in animal diet in most parts of Africa (Akinfala and Tewe, 2001). Cassava has been successfully used to replace maize in *Clarias gariepinus* fingerlings (Abu et al., 2010; Olukunle, 2006). Inclusion of whole cassava root meal in the diet of fish enhanced growth and survival. Though cassava is high in carbohydrate content, it is however low in protein content (Tewe and Egbunike, 2007) and has a very high crude fibre. Therefore, the need to fortify the peel with protein is necessary. *Glyricidia sepium* leaves have been chosen in this study to boost its protein value.

Hematological parameters are good indicators of physiological status of animals and have been found useful for disease prognosis and for therapeutic and feed stress monitoring (Togun et al., 2009; Aro and Akinmoegun (2012).

Hematological constituents reflect the physiological responsiveness of the animal to its internal and external environments which include

feeding. Hematological values could serve as baseline information for comparisons of nutrient deficiency. Haematological components of blood are also valuable in monitoring feed toxicity especially with feed constituents that affect the formation of blood in culture fisheries (Oyawoye and Ogunkunle, 1998). Animals with good blood composition are likely to show good performance (Isaac et al., 2013).

In view of the above, the present research was set up with the objective of determining the optimum replacement level of maize meal by fermented cassava peel meal (CPLM) and the effects on growth, haematology and digestive enzymes of *Clarias gariepinus*.

## II. MATERIALS AND METHODS

### 2.1 Procurement of Materials

The study was carried out at the Animal House of the Department of Zoology and Environmental Biology, Ekiti State University (EKSU), Ado Ekiti, Ekiti State, Nigeria. Fresh cassava peels (2kg) were obtained from Aba Ebira, Iworoko Ekiti, Ekiti State, and fresh *Leucaena leucocephala* leaves (1kg) were harvested from the Department of Plant Science, EKSU, Ado-Ekiti. Fresh poultry dropping (1kg) was collected from a poultry farm in Iworoko Ekiti. A total of healthy, one hundred and eighty (180) samples of juvenile *C. gariepinus* (av. weight  $29.69 \pm 0.91g$ ) were obtained from Mr. Olatunji's farm, Ibadan, Oyo State and transported, in an aerated container, to the Animal House of the Dept. of Zoology and Environmental Biology, EKSU, Ado-Ekiti.

### 2.2 Ingredients and Diet Preparation

Two kg of fresh cassava peel wastes collected were cut into small pieces of about 2cm<sup>2</sup> sizes. These were exposed to the sun for about 5hours, to wilt. The leucaena leaves were added to poultry droppings and mixed together with the cassava peels. These were packed into black polyvinyl bag, tied and left to ferment for 21 days. After 21 days, the mixture was spread out and sundried for four days after which it was milled, with a hammer mill, into powdery form, tagged fermented cassava peel meal (CPLM). The other feed

ingredients; maize, soya beans, 78% Danish fishmeal, ground nut oil, salt, fish premix, lysine and rice bran, were obtained from Metrovet Venture, Ado Ekiti. They were all in milled form, ready for use in feed formulation.

A 2g sample of the CPLM was taken to the lab for proximate analysis (moisture content, crude protein, ash, fibre, fat and carbohydrate) at the Central Science Laboratory of Federal University of Technology, Akure, Ondo state. Based on the crude protein content of the CPLM, seven diets containing different levels of the CPLM replacement for maize (0%, 20%, 30%, 40%, 50%, 60% and 70%) tagged D0, D20, D30, D40, D50, D60 and D70 respectively, were prepared, using Pearson's method (Table 1).

### 2.3 Experimental Set Up

The fish were acclimatized for 2 weeks in plastic aquarium tanks, supplied with clean water and fed with conventional commercial fish feed twice daily (8.00am - 9.00am) and (6.00-7.00pm), in order to adapt to the environmental condition before the commencement of the study. After 2 weeks of acclimatization, the fish were fed with the test diets for eight (8) weeks in each plastic bowls. Eighteen bowls were randomly allocated in triplicates, to six treatment diets, and the fish were randomly distributed into the bowls, at a stocking density of 15 juveniles per bowl. Feeding was carried out twice daily. The left-over feeds and faeces were siphoned off promptly and dead fish were promptly removed to prevent contamination.

**Table 1:** Composition of Experimental Diets (100-1dry Matter Basis) With Varying Inclusion Levels of Fermented Cassava Peel Meals (CPLM).

Ingredient	Diet						
	D0	D20	D30	D40	D50	D60	D70
Fish Meal	28	28	28	28	28	28	28
CPLM	0	3	6	9	12	15	18
Maize	30	27	24	21	18	15	12
Soybeans	19	19	19	19	19	19	19
Rice bran	20	20	20	20	20	20	20
Oil	1	1	1	1	1	1	1
Salt	1	1	1	1	1	1	1
Vit. premix	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Lysine	0.3	0.3	0.3	0.3	0.3	0.3	0.3
Methionine	0.2	0.2	0.2	0.2	0.2	0.2	0.2

### 2.3 Collection of Blood Samples

Blood samples of a set of three fish were collected at the beginning of the feeding trial (week 0) and at the end of trial (week 8) from each set in each bowl. This blood sample was withdrawn from the caudal peduncle, following the procedure described by Stockopf (1993), Joshi et al. (2000a) and Dienne and Olumuji (2014). Two ml of the blood sample from each fish was collected with 2ml syringe and needle and put in ethylenediamine tetra-acetic acid (EDTA) bottle. The samples were taken to the laboratory for hematological analysis which involves

measurement of erythrocyte values: Haemoglobin (Hb), estimated by cyanomethemoglobin method, red blood cells (RBC) and white blood cell (WBC) counted by Neubauer's improved haemocytometer, using Hyem's and Turks solution as a diluting fluid respectively. The absolute erythrocyte indices [mean corpuscular haemoglobin concentration (MCHC); mean corpuscular haemoglobin (MCH) and mean cell volume (MCV)] were calculated respectively using standard formula described by Dacie and Lewis (2001) as follows;

- $MCHC (\%) = \frac{Hb \times 10}{PCV}$
- $MCH (pg) = \frac{Hb \times 10}{RBC}$
- $MCV (fl) = \frac{PCV \times 10}{RBC}$

#### 2.4 Crude Enzyme Preparation

Ten experimental fish from each sample collection were slaughtered and the gut regions of the fish were pooled, homogenized in an ice cold 20mM phosphate buffer pH 7.0 and the homogenates were centrifuged at 1200 rpm for 30 minutes at 40C. The supernatants were used as crude enzyme extracts without further purification. Benedict's qualitative reagents were used for the qualitative assay of glycosidases (carbohydrases) following the methods used by Fagbenro et al., (2005) and Olatunde et al. (1988). Glycosidases (maltase, cellulase, gluconase) were assayed in a reaction mixture containing 2.0 ml of phosphate buffer (pH 7.0), 0.4 ml of 1 % of substrate and 0.2 ml of the enzyme extract. The test and control samples were incubated for one hour in a water bath at 370C. Hydrolysis of polysaccharides and non-reducing disaccharides were determined in terms of the appearance of reducing properties using Benedict's reagents. An aliquot of 5.0 ml of the alkaline copper reagent of Benedict was added to 1.0 ml of the reaction mixture and heated for 30 minutes in a water bath at 1000C. The appearance of brick red to cream yellow precipitate was taken as an index of positive

reaction. Quantitative assays were conducted using the dinitrosalicylate (DNS) methods described by Plummer (1978). Each reaction mixture comprised 0.4 ml of 1% substrate, 0.2 ml phosphate buffer (pH 7.0), 1.6 ml of alkaline 3, 5-dinitrosalicylic acid reagent (DNSA) and 0.2 ml of the enzyme extract. The reaction mixtures for test and control samples were heated for 30 minutes in a water bath at 100C. Each of the mixtures was made to 4.0 ml by diluting with 1.6 ml distilled water. The amount of reducing sugars produced on enzymatic reaction was estimated colorimetrically and the absorbance read at 550 nm on a spectrophotometer.

#### 2.5 Statistical Analysis

All data collected were subjected to analysis of variance (ANOVA). Comparisons among diets means were carried out by Duncan Multiple Range Test (Duncan 1955) at a significant level of 0.05. All computation was performed using statistical package SPSS 15.0 (SPSS Inc., Chicago, IL, U.S.A).

### III. RESULTS

Table 1 shows the percentage composition of the fermented and unfermented cassava peel meals respectively, while Table 2 shows the proximate compositions of the seven diets formulated for the feeding trial. The crude protein content of the diet ranged between 39.32 and 43.06%, crude lipid, 4.40-5.44% and crude fibre, 4.96-8.44%.

**Table 1:** The Proximate Composition (G/100g DM\*) of Cassava Peels Used in Formulating the Experimental Diets

Parameter	Composition	
	Unfermented	Fermented
Crude protein	5.3	11.4
Lipid	1.2	3.5
Ash	5.9	6.3
Crude Fibre	20.9	7.1
Moisture	5.2	5.7
Carbohydrate	61.5	66.0

DM = dry matter

**Table 2:** The Proximate Composition of the Experimental Diets Fed to *C. Gariepinus*, During the Period of Study

INGREDIENTS	DIETS						
	Do	D20	D30	D40	D50	D60	D70
Crude protein	40.24	43.06	41.56	40.79	39.77	39.78	39.32
Moisture	9.96	7.31	8.13	7.76	7.08	7.13	8.25
Ether extract	5.17	5.08	5.41	5.44	4.55	4.57	4.40
Crude fiber	4.96	5.85	6.82	6.91	7.02	7.65	8.44
Ash	5.78	5.67	5.32	5.34	5.72	5.89	6.91
Nitrogen Free extract	33.89	33.03	32.76	33.76	35.86	34.98	32.68

*NFE, Nitrogen free extract*

Table 3 shows the hematological composition of fish fed with varying fermented cassava peel meal-based diet during the experiment. The Packed cell volume (PCV) results showed that the fish fed the control and D20 had increase in PCV values (32.00% and 30.00% respectively) when compared with the initial value. These values were not statistically significant ( $p > 0.05$ ) from one another. The fish fed diets D30 to D60 showed a decrease in the PCV. White blood cells (WBC) result showed that fishes fed D30 to D60 had higher values than fishes fed control and D20 diets. The highest value of  $8.02 \times 10^3 \text{ mm}^3$  was recorded in fish fed diet D70.

The red blood cell (RBC) showed a decrease as fermented cassava peel meal increased in the diet. The fish fed control diet and D20 recorded values of  $3.60 \times 10^3 \text{ mm}^{-3}$  and  $3.20 \times 10^3 \text{ mm}^{-3}$  respectively and were not significantly different ( $P > 0.05$ ) from one another, but were significantly different from fish fed diet D30 to D60.

Hemoglobin (Hb) decreased in fishes fed diet containing D20 to D60. The fish fed the control diet and D20 recorded values of 9.20 g/100 ml and 9.00 g/100 ml respectively. These values showed a significant ( $P < 0.05$ ) difference from fishes fed diet containing D30 to D60.

Lymphocyte (LYMPH) count showed an increase as the level of CPLM increased in the diet. The highest value of 70.00% was recorded in fish fed diet D60 and D70 and the least value of 63.00% was recorded in fish fed the control diet. The highest value (30.77%) for MCHC was recorded in fish fed diet D30 and the lowest value (28.75%) was obtained in fish fed the control diet. The results obtained for MCH and MCV showed that the fishes fed diet D50 had the highest values of 47.00 pg and 160.00 fl for MCH and MCV respectively and the least values of 25.56 pg and 88.89 fl were recorded for MCH and MCV in fish fed D20 diet.

**Table 4:** Hematological Composition of *Clarias Gariepinus* Juveniles Fed With Varying Levels of Fermented Cassava Peel Meal-Based Diet

Parameter	Do	D20	D30	D40	D50	D60	D70
PCV %	27.80 <sup>b</sup>	32.00 <sup>a</sup>	30.00 <sup>a</sup>	27.00 <sup>b</sup>	27.50 <sup>ab</sup>	24.00 <sup>c</sup>	24.00 <sup>c</sup>
WBC ( $10^3 \text{ mm}^{-3}$ )	7.20 <sup>c</sup>	7.45 <sup>bc</sup>	7.50 <sup>bc</sup>	7.60 <sup>b</sup>	7.60 <sup>b</sup>	8.00 <sup>a</sup>	8.02 <sup>a</sup>
RBC ( $10^6 \text{ mm}^{-3}$ )	2.80 <sup>b</sup>	3.60 <sup>a</sup>	3.20 <sup>a</sup>	2.00 <sup>b</sup>	1.90 <sup>b</sup>	1.70 <sup>b</sup>	1.50 <sup>bc</sup>
Hb (g/100ml)	8.00 <sup>b</sup>	9.20 <sup>a</sup>	9.00 <sup>a</sup>	8.10 <sup>b</sup>	8.00 <sup>b</sup>	7.00 <sup>c</sup>	7.05 <sup>b</sup>
LYMPH (%)	60.00 <sup>c</sup>	63.00 <sup>b</sup>	63.02 <sup>a</sup>	64.00 <sup>a</sup>	64.05 <sup>b</sup>	70.00 <sup>a</sup>	70.00 <sup>a</sup>
MCHC (%)	28.78 <sup>ab</sup>	28.75 <sup>ab</sup>	29.00 <sup>a</sup>	29.02 <sup>a</sup>	29.00 <sup>a</sup>	30.17 <sup>a</sup>	30.38 <sup>a</sup>
MCH (pg)	28.57 <sup>cd</sup>	25.56 <sup>d</sup>	28.13 <sup>cd</sup>	40.50 <sup>b</sup>	42.11 <sup>b</sup>	36.84 <sup>c</sup>	47.00 <sup>a</sup>
MCV (fl)	100.00 <sup>c</sup>	88.89 <sup>d</sup>	93.75 <sup>d</sup>	93.00 <sup>b</sup>	92.84 <sup>b</sup>	133.33 <sup>b</sup>	160.00 <sup>a</sup>

Figures on the same row having the same superscript are not significantly different ( $p > 0.05$ ).

PVC = Packed cell volume; WBC = white blood cell; RBC = red blood cell; Hb = hemoglobin; LYMP = lymphocyte; MCHC=mean corpuscular hemoglobin concentration; MCH = mean

corpuseular hemoglobin; MCV=mean corpuseular volume.

Digestive enzyme assays in the gut of *Clarias gariepinus* (Table 3) indicated significant difference ( $P < 0.05$ ) between the gut amylase activity of the fish fed the control diet and the other dietary treatments, with D60 showing the least activity. Sucrase activity increased with increase in CPLM level, while maltase activity was

not significantly different ( $P = 0.05$ ) from one another. The glucanase activity of the fish fed the various diets was not significantly different ( $P > 0.05$ ) from one another, but lower than the initial value. Cellulase activity decreased at lower CPLM level while there was no significant difference ( $P > 0.05$ ) at higher level of CPLM inclusion.

**Table 5:** Sugar Degrading Enzyme Activity From the Gut (G) of Catfish (*Clarias Gariepinus*)

Diet	Amylase	Sucrase	Maltase	Glucanase	Cellulase
Initial	36.64 <sup>b</sup>	4.51 <sup>b</sup>	4.23 <sup>b</sup>	4.23 <sup>b</sup>	1.83 <sup>b</sup>
Do	70.47 <sup>a</sup>	7.26 <sup>a</sup>	4.82 <sup>b</sup>	4.64 <sup>a</sup>	2.92 <sup>a</sup>
D20	66.95 <sup>a</sup>	5.17 <sup>c</sup>	5.07 <sup>a</sup>	4.65 <sup>a</sup>	2.58 <sup>a</sup>
D30	42.58 <sup>a</sup>	12.47 <sup>a</sup>	4.59 <sup>a</sup>	4.19 <sup>a</sup>	1.76 <sup>ab</sup>
D40	41.58 <sup>a</sup>	10.86 <sup>a</sup>	4.52 <sup>a</sup>	4.19 <sup>a</sup>	2.04 <sup>ab</sup>
D50	37.53 <sup>a</sup>	14.29 <sup>a</sup>	4.86 <sup>ab</sup>	3.34 <sup>a</sup>	1.56 <sup>ab</sup>
D60	36.57 <sup>b</sup>	14.22 <sup>a</sup>	4.77 <sup>ac</sup>	1.01 <sup>a</sup>	1.00 <sup>bc</sup>
D70	33.54 <sup>b</sup>	15.12 <sup>a</sup>	5.01 <sup>ac</sup>	1.32 <sup>a</sup>	1.57 <sup>bc</sup>

Note: Samples with the same letter in a column are not significantly different at 5% level

#### IV. DISCUSSION

The potential of a feedstuff such as cassava peel and leaf meal in fish diets can be evaluated on the basis of its proximate chemical composition, which comprises the moisture content, crude protein, crude fibre, crude fat, total ash, carbohydrate and nitrogen free extract. The proximate composition of fermented cassava peel meal in the present investigation revealed that the crude protein content was 28.03%, crude fibre 18.87%, crude fat 2.25%, carbohydrate 48.00 and total ash 6.81%. These values as observed in this study fell within the range obtained by Sotolu, (2010). This confirmed the potential of fermented cassava peel meal as adequate animal feedstuff from nutritional point of view. A trial conducted by Oboh and Akindaunsi (2003) on the fermentation of cassava peels with a consortium of microorganism indicated a significant increase in protein content and digestibility of the microbially treated peels, as against the untreated control. The authors concluded that such fermented cassava by-product could be a good supplement in compounding animal feed.

Antai and Mbongo (1994) observed that fermentation of cassava peels by pure culture of

*Saccharomyces cerevisiae* could increase its protein content from 2.4% in non-fermented cassava to 14.1% in fermented products. They reported that fermented cassava flour, with *S. cerevisiae*, enhanced the protein level (from 4.4% to 10.9%) and decreased the amount of cyanide content (Oboh and Akindahunsi, 2005).

Noomhorm et al. (1992) reported that the conversion of a part of the starch in cassava root meal to protein by microbes, during the process of solid-state fermentation, has great potential as a means of improving the feed value of cassava root meal. As observed in this work, Adeyemi and Sipe (2004) also reported an improvement in crude protein concentration of cassava root when fermented with rumen filtrate with or without ammonium sulphate as the source of nitrogen.

Adeyemi et al. (2004) obtained a value of 237.8 % increase in the crude protein value of whole cassava root meal fermented with rumen filtrate using caged layer waste as source of nitrogen. Ubalua and Ezeronye (2008) have identified fermentation as one of the less expensive means of increasing the protein quality of cassava and cassava wastes. Dried products from roots, which have been fermented or ensiled to detoxify the

HCN or to increase their protein content, are other ways of root processing (Khajareran and Khajareran, 2007).

All the haematological parameters measured in this study were within the recommended physiological ranges reported for *C. gariepinus*. The change in the blood characteristics of *C. gariepinus* caused by stress due to exposure to environmental pollutants, diseases or by pathogens have been studied by a number of workers especially in capture fisheries (Onusiriku and Ufodiye, 2000; Ezeri, 2001; Gabriel et al., 2001).

Blaxhall and Daisley (1973) reported the essence of using haematocrit to detect anaemic condition in fishes. The packed cell volume (PCV) range 24.00 to 32.00% observed in this study is within the range of 20 to 50% reported by Pietse et al. (1981), though, a decrease was observed in the level of PCV as the level of CPLM increased in the diet. Reduction in the concentration of the PCV in the blood usually suggests the presence of toxic factor. The decreasing trend observed in the PCV of this study may be attributed to the presence of remnants of some anti-nutrients, such as some levels of hydrogen cyanide, tannin and mimosine in the fermented cassava peel meal as reported by Oboh and Akindahunsi (2003).

White blood cells (WBC) and lymphocytes results recorded in this study showed an increase as the level of CPLM increased in the diet. White blood cells (WBC) and lymphocytes are the defence cells of the body. Douglas and Jane (2010) demonstrated that the amount has implication in immune responses and the ability of the animal to fight infection. High WBC count is usually associated with microbial infection in the circulatory system (Oyawoye and Ogunkunle, 1998). The value range of  $7.20 \times 10^3$  to  $8.02 \times 10^3$  mm<sup>-3</sup> recorded in this study for WBC was lower compared to  $16.13 \times 10^3$  to  $16.39 \times 10^3$  mm<sup>-3</sup> reported by Sotolu and Faturoti (2009).

Reduction in the red blood cells was observed as the level of fermented cassava peel meal increased in the diet. The range of RBC ( $1.50 \times 10^6$  to  $3.60 \times 10^6$  mm<sup>-3</sup>) recorded in this study is fairly

comparable with ( $1.70 \times 10^6$  to  $4.00 \times 10^6$  mm<sup>-3</sup>) reported by Bhasker and Rao (1990) and lesser than ( $2.24 \times 10^6$  to  $2.49 \times 10^6$  mm<sup>-3</sup>) reported by Sotolu and Faturoti (2009). The decrease in RBC could probably be due to the high concentration of anti-metabolites in the diet containing more fermented cassava peel meal.

The haemoglobin result showed a decrease as the CPLM increased in the diet. The haemoglobin range (7.00 – 9.20g/100ml) recorded were high and fell within the range (5.6 to 15.8 g/100 ml) reported for *Esox lucius* (Mulcahy, 1970). It also compared well with (8.70 g/100 ml) recorded for *C. gariepinus* (Sowunmi, 2003). These values were also higher than 4.46 g/100 ml reported for *Heterotis niloticus* (Fagbenro et al., 2000). The range of haemoglobin concentration recorded in this study is quite high and can be related to large anaerobic metabolism capacity of *C. gariepinus*. The decrease in the level of haemoglobin as CPLM increased in the diet could imply that diets having higher fermented cassava peel meal had negative effect on the blood.

The mean corpuscular volume (MCV) range (88.89 to 160.00 fl) recorded in this experiment was higher than (79.20 to 105.32fl) reported for *Heteroclaris* by Anyanwu et al. (2011), meanwhile the mean corpuscular haemoglobin concentration (MCHC) range (28.75 to 30.77%) recorded in this study compared fairly well with 30.70% reported for *C. gariepinus* from Asejire dam (Adedeji and Adegbile, 2011).

The MCH results showed that the fish fed diet D50 recorded the highest values. The MCH range (25.56 to 47.00 pg) obtained in this study was higher than the range (20.82 to 26.60 pg) reported for *Heteroclaris* fed *Carica papaya* leaf meal incorporated feed (Anyanwu et al., 2011). In recent years, good management practices have been advocated as effective ways of reducing stress in fish culture (Gabriel et al., 2007).

The enzymes present in the gut of the fish fed experimental diets were lower compared to the initial. The decreasing amylase activities in the fish, as the level of CPLM increases in the diets, can be explained by lower dietary lipid levels. As

reported here, Fountoulaki et al. (2005) also reported that in gilthead sea bream, amylase is affected by dietary fat level. Apata and Ojo (2000) suggested that the decrease in the effect of enzymes in the gut may be due to the change arising from the breaking down of high dietary fibre. *Clarias gariepinus* is physiologically equipped to cope with frequently and irregular meals as its digestive enzymes respond faster to feeding than those of eel (*Anguilla anguilla*) or carp (*Cyprinus carpio*) (Yalcin et al., 2001; Adeyemi et al., 2004).

From the above observations, it is evident that maize substitution with fermented cassava peel meal at a rate of up to 50% in catfish (*C. gariepinus*) fish feed have no adverse toxicological effect on the fish as revealed through the haematological indices and digestive enzyme assay. Essers et al., (1995) documented that 50% replacement of maize with cassava meal in broiler diet showed no depression in growth or unfavourable feed conversion ratio. This was also supported by Olurin et al. (2006) who reported a replacement level of 50% cassava meal for maize without a depressing growth in *Clarias gariepinus*.

## V. CONCLUSION

These investigations have revealed that up to 50% substitution rate of fermented cassava peel meal for fishmeal in catfish (*C. gariepinus*) fish feed produces no adverse toxicological effect on the fish as revealed through the haematological and digestive enzyme indices. By fermentation method, fish feed can therefore be produced at relatively cheaper cost by the use of commonly available cassava peels thus increasing the profits to fish farmers.

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# Wheat Production Prediction in India using ARIMA, Neural Network and Fuzzy Time Series

*S. Selvakumar & V. Kasthuri*

## ABSTRACT

A time series is a predetermination of data points that happen in repeated order of time. Forecasting productions play a necessary part in several fields such as, meteorological data, weather data, stock market data, rainfall data, agriculture data and so on. In recent years, fuzzy time series is used for forecasting. Song and Chissom (1993) proposed fuzzy time series for forecasting enrollments of data. In this paper, Autoregressive Integrated Moving Average Model (ARIMA), Neural networks for Radial Basis Function (RBF) and Multilayer Perceptron (MLP) and fuzzy time series for predicting wheat production of India were compared. Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) were compared. The results were displayed numerically and graphically.

*Keywords:* ARIMA, neural network, fuzzy time series, MAE, MAPE, RMSE, residual analysis and prediction.

*Classification:* FOR Code: 280212

*Language:* English



London  
Journals Press

LJP Copyright ID: 925652

Print ISSN: 2631-8490

Online ISSN: 2631-8504

London Journal of Research in Science: Natural and Formal

Volume 22 | Issue 15 | Compilation 1.0



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# Wheat Production Prediction in India using ARIMA, Neural Network and Fuzzy Time Series

S. Selvakumar<sup>α</sup> & V. Kasthuri<sup>σ</sup>

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## ABSTRACT

*A time series is a predetermination of data points that happen in repeated order of time. Forecasting productions play a necessary part in several fields such as, meteorological data, weather data, stock market data, rainfall data, agriculture data and so on. In recent years, fuzzy time series is used for forecasting. Song and Chissom (1993) proposed fuzzy time series for forecasting enrollments of data. In this paper, Autoregressive Integrated Moving Average Model (ARIMA), Neural networks for Radial Basis Function (RBF) and Multilayer Perceptron (MLP) and fuzzy time series for predicting wheat production of India were compared. Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) were compared. The results were displayed numerically and graphically.*

**Keywords:** ARIMA, neural network, fuzzy time series, MAE, MAPE, RMSE, residual analysis and prediction.

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## I. INTRODUCTION

The time series is a category of variables ordered in a specific order of time. Forecasting believes the future values of the time series. For the past decades, fuzzy time series has been widely used for predicting the historic data. Fuzzy time series is used to planning with forecasting difficulties in which the historical data are linguistic values. Song and Chissom (1993) proposed the fuzzy time series for the enrollments of university of Alabama. Wheat is the chief cereal crop in India. Wheat crop has well-known flexibility. Wheat is developed in a variety of soils of India. Yanpeng Zhang et.al(2020) proposed a novel fuzzy time series forecasting model by multiple linear regression and time series clustering for forecasting market prices. Singh, P. (2018) offered a new model to deal with four major issues of fuzzy time series (FTS) forecasting, viz., Wangren Qiu et.al(2015) proposed model was implemented in forecasting enrollment data at the University of Alabama. Ozge Cagcag Yolcu et.al(2016) proposed a novel high-order fuzzy time series approach that considers the membership values, where artificial neural networks are employed to identify the fuzzy relations. Adesh Kumar Pandey(2008) proposed fuzzy time series and neural network. The proposed method has been implemented in the historical data. Paarth Thadani(2021) presented non-linear forecasting models, including artificial neural networks, are popularly adopted in financial forecasting. Yousif Alyousifi et.al (2021) proposed Fuzzy Time Series Markov Chain – Transition Probability Matrix model is tested using two types of time series data, namely, air pollution index (API) data, and yearly enrollments for the University of Alabama. Wang et.al (2017) applied autoregressive integrated moving average model and artificial neural network for air pollution data. Alyousi et. al (2019) applied artificial neural network and Markov chain are applied for air pollution forecasting. In this work, ARIMA, Neural networks for Multilayer Perceptron and Radial Basics Function and fuzzy time series algorithms are used for wheat production prediction in India. Residual analysis for Mean

Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) were compared.

## II. METHODOLOGY

### 2.1 Autoregressive Integrated Moving Average Model

Autoregressive Integrated Moving Average Model (p, d, q), where p is Autoregressive and q is the Moving Average Model and d is the differencing. If d = 0, the data exhibits stationary and the order is denoted as (p, q), which is called ARMA process. If the data does not exhibit stationary, the first order differencing is carried out in converting it into a stationary, hence the model is denoted as (p, d, q).

### 2.2 Radial Basis Function (RBF)

RBF networks, a class of feed forward networks, called radial basis function that compute activations at the hidden neurons in a way that is different from what we have seen in the case of feed forward neural networks. Rather than employing an inner product between the input vector and the weight vector The RBF output layer results in a linear fashion. The output y is computed by

$$y_i(x) = \sum_{k=1}^{J_2} W_{ki} \phi(\|X - C_k\|)$$

For  $i = 1, \dots, J_3$  where  $y_i(x)$  is the  $i^{\text{th}}$  output of the RBF.  $W_{ki}$  is the connection weight from the  $k^{\text{th}}$  hidden to the  $i^{\text{th}}$  output unit  $C_k$  is the prototype or center of the  $k^{\text{th}}$  hidden unit, and  $\|\cdot\|$  denotes the Euclidean norms. The RBF  $\phi(\cdot)$  typically selected as the Gaussian function

$$\phi(x - c_i) = e^{-\|x - c_i\|^2 / 2\sigma^2}$$

Where  $c_i = (c_{i1}, c_{i2}, \dots, c_{in})$  is the center of the associated field, and  $\sigma$  is the width of the Gaussian function.

### 2.3 Multi Layer Perceptron

Artificial Neural Network(ANN) is an operational model which consists of a large number of interconnected nodes(neurons). Each node contains a specific output function which is called an activation function. The connection between every two nodes represents a weighted value that passes through the connection signal, which is called weight. Weight is equivalent to the memory of ANN. Multi Layer Perceptron(MLP) has many layers, the first layer is the input layer, the last layer is the output layer, the middle layers are called hidden layers, each layer includes several neurons. This calculation process is called feed forward process of MLP. If there is an MLP, which contains m hidden layers, its input and output dimensions are respectively equal to  $n_1$  and  $n_{m+2}$ . The number of nodes in each hidden layer is  $n_2, n_3, \dots, n_{m+1}$  respectively. In the feed forward process of this MLP, each node value is calculated using the following formula

$$X_{ij} = f(WX_{i-1} + b_{i-1})$$

f is the activation function.

Where  $X_{ij}$  represents the value of the  $j$  neuron  $i$  layer.  $W_i$  represents the weight vector of the  $j$  neuron in layer  $i-1$  to layer  $i$ .  $X_{i-1}$  represents the value vector of all neurons in layer  $i-1$ .  $b_{i-1}$  represents the bias of the  $i-1$  layer, and  $f$  is the activation function.

## 2.4 Fuzzy Time Series

It is the values of the observations of a special dynamic process are represented by linguistic values.

### Computational Algorithm for Fuzzy Time Series

The step by step process is as follows:

*Step1:* Calculate the first order variation of the historical data.

*Step2:* Define the universe of discourse,  $U$  based on the range of available historical data.

$$U=[D_{\min}-D_1, D_{\max}+D_2]$$

Where  $D_{\min}$  is the minimum value of the first order variation of the historical data,  $D_{\max}$  is the maximum value of the historical data and  $D_1, D_2$  are two positive integers.

*Step3:* Partition the universe of discourse  $U$  into equal length intervals:  $u_1, u_2, \dots, u_m$ .

*Step4:* The number of intervals will be in accordance with the number of linguistic variables (fuzzy sets)  $A_1, A_2, \dots, A_m$  to be considered.

*Step5:* Fuzzify the variations of the historical data and establish the fuzzy logical relationship is represented by  $A_i \rightarrow A_j$ .

*Step6:* Rules for forecasting:

$[A_j]$  is corresponding interval  $u_j$  for which membership in  $A_j$  is supremum (i.e., 1)

$L[A_j]$  is the highest value of the interval  $u_j$  having supremum value in  $A_j$ .

$M[A_j]$  is the mid value of the interval  $u_j$  having supremum value in  $A_j$ .

For a fuzzy logical relationship  $A_i \rightarrow A_j$ .

$A_i$  is the fuzzified wheat production of the current year  $n$ ;

$A_j$  is the fuzzified wheat production of the next year  $n+1$ ;

$D_i$  is the actual wheat production of the current year  $n$ ;

$D_{i-1}$  is the actual wheat production of the previous year  $n-1$ ;

$E_i$  is the variation wheat production of the current year  $n$ ;

$E_{i-1}$  is the variation wheat production of the previous year  $n-1$ ;

$F_j$  is the forecasted wheat production of the next year  $n+1$ ;

*Step7:* Forecasting wheat production for the year  $n+1$  is obtained from modified computational algorithm as follows;

Obtain the fuzzy logical relationship  $A_i \rightarrow A_j$ .

$$\begin{aligned} \text{If } E_i < M[A_j], \text{ then } F_j &= D_{i-1} + (M[A_j] - 1/6L[A_j]) \\ \text{else if } E_i > M[A_j], \text{ then } F_j &= D_{i-1} + (M[A_j] + 1/6L[A_j]) \\ \text{else } F_j &= D_{i-1} + M[A_j] \end{aligned}$$

*Step8:* Obtain the mean absolute error using actual values and forecasted values

$$MAE = \frac{1}{n} \sum_{t=1}^n |u_t|$$

Where n is the number of years and  $|u_t| = Y_t - \hat{Y}_t$ .  $Y_t$  is actual values at time t.  $\hat{Y}_t$  is predicted values at time t.

*Step9:* Obtain the mean absolute percentage error using actual values and forecasted values

$$MAPE = \frac{1}{n} \sum_{t=1}^n \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right|$$

*Step 10:* Obtain the root mean square error using actual values and forecasted values

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (Y_t - \hat{Y}_t)^2}{n}}$$

### III. RESULTS AND DISCUSSIONS

*Step1:* Compute the first order variation of the historical data.

*Step2:* The universe of discourse U is defined as+

$$U = [D_{\min} - D_1, D_{\max} + D_2]$$

$$U = [-9323 - 77, 11510 + 90] = [-9400, 11600]$$

Where  $D_{\min} = -9400$  is the minimum value of the first order variation of the historical data.

$D_{\max} = 11600$  is the maximum value of the first order variation of the historical data,

$D_1 = 77$  and  $D_2 = 90$  are two positive integers.  $D_1, D_2$  are choosing arbitrarily for the rounded off U value.

*Step3:* The universe of discourse U is partitioned into five equal length of intervals.

$$U_1 = [-9400, -5200] \quad U_2 = [-5200, -1000] \quad U_3 = [-1000, 3200] \quad U_4 = [3200, 7400] \quad U_5 = [7400, 11600]$$

*Step4:* Define five fuzzy sets  $A_1, A_2, \dots, A_5$  having some linguistic values on the universe of discourse U. The linguistic values are as follows:

$A_1 =$  very poor     $A_2 =$  poor     $A_3 =$  Moderate     $A_4 =$  Good     $A_5 =$  Very Good

*Table 1:* Fuzzified Wheat Production(tonnes) on Variations

Year	Wheat production(tonnes)	Variations	Fuzzified variations
2001	69681	-	-
2002	72766	3085	A3
2003	65761	-7005	A1
2004	72156	6395	A4
2005	68637	-3519	A2
2006	69355	718	A3
2007	75807	6452	A4
2008	78570	2763	A3
2009	80679	2109	A3

2010	80804	125	A3
2011	86874	6070	A4
2012	94882	8008	A5
2013	93506	-1376	A2
2014	95850	2344	A3
2015	86527	-9323	A1
2016	87000	473	A3
2017	98510	11510	A5
2018	99870	1360	A3
2019	103600	3730	A4
2020	107860	4260	A4
2021	109520	1660	A3

*Step5:* The membership of above mentioned linguistic variables is assigned through the Trapezoidal membership function by fixing the values arbitrarily. The memberships of Linguistic variables are as follows.

$$\begin{aligned}
 A_1 &= 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 \\
 A_2 &= 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 \\
 A_3 &= 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 \\
 A_4 &= 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 \\
 A_5 &= 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5
 \end{aligned}$$

*Step6:* The historical variations of the time series data are fuzzified in order to have the fuzzy logical relations obtained as follows: Variations in the fuzzy logic relationships

$$\begin{aligned}
 &A_3 \rightarrow A_1, A_1 \rightarrow A_4, A_4 \rightarrow A_2, A_2 \rightarrow A_3, A_3 \rightarrow A_4, A_4 \rightarrow A_3 \\
 &A_3 \rightarrow A_3, A_3 \rightarrow A_3, A_3 \rightarrow A_4, A_4 \rightarrow A_5, A_5 \rightarrow A_2, A_2 \rightarrow A_3 \\
 &A_3 \rightarrow A_1, A_1 \rightarrow A_3, A_3 \rightarrow A_5, A_5 \rightarrow A_3, A_3 \rightarrow A_4, A_4 \rightarrow A_4 \\
 &A_4 \rightarrow A_3
 \end{aligned}$$

*Step-7:* The forecasted values have been obtained by using the computational algorithm. Then the forecasted output while comparing with different models given as table 2.

*Table 2:* Forecasted Wheat Production (tonnes) by Different Models

Year	Actual Wheat Production	ARIMA	RBF	MLP	Fuzzy Time Series
2001	69681	70444	71136	68012	---
2002	72766	72919	70476	68511	71314
2003	65761	69333	68013	69209	66333
2004	72156	72296	67256	70171	72294
2005	68637	71247	67285	71465	69223
2006	69355	71458	69203	73150	69204
2007	75807	75788	76098	75258	74655

2008	78570	79249	78514	77761	77440
2009	80679	81977	79655	80561	80203
2010	80804	83173	82161	83488	81246
2011	86874	87615	86382	86339	87337
2012	94882	94466	90346	88932	94441
2013	93506	96192	93535	91147	91949
2014	95850	98445	91769	92941	95139
2015	86527	93432	85975	94331	87683
2016	87000	92056	87682	95372	87094
2017	98510	99045	98117	96132	98433
2018	99870	102568	100114	96676	100143
2019	103600	106355	104423	97061	103937
2020	107860	110576	108673	97331	107667
2021	109520	113292	108709	97518	109493

Table 2 shows that wheat production of actual values and forecasted values using different models, namely, ARIMA, RBF,MLP and fuzzy time series.

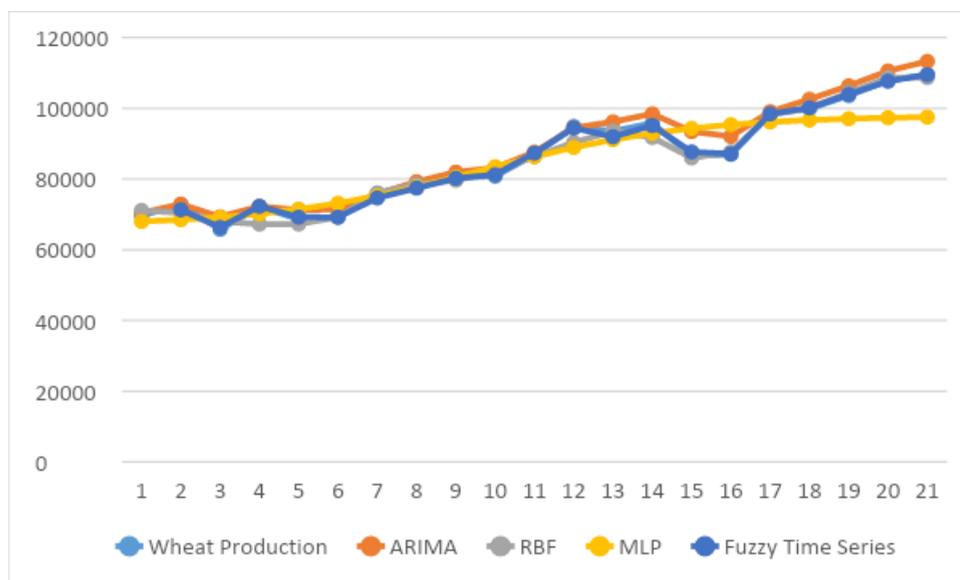


Figure 1 Shows that actual and forecasted values of wheat production of various models were compared by using line chart.

*Figure 1:* Actual and Forecasted Values of Wheat Production (tonnes)

Table 3: Residual Analysis by Different Models

Models	MAE	MAPE	RMSE
ARIMA	3453.9	4.188	4907.1
RBF	1361.19	1.668	1972.72
MLP	4033.857	4.5134	5171.001
Fuzzy Time Series	571.4	0.6976	733.569

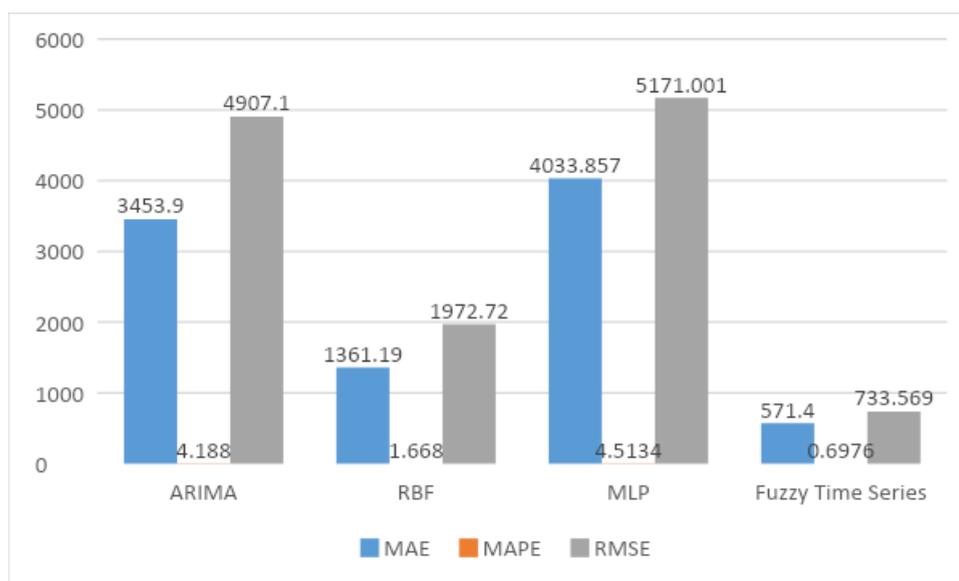


Figure 2: Residual Analysis by Using ARIMA, RBF, MLP and Fuzzy Time Series

Figure 2 shows that the mean absolute error, mean absolute percentage error and root mean square error obtained by using ARIMA and neural networks for Radial Basics Function and Multilayer Perceptron and fuzzy time series for wheat production prediction. Mean absolute error, mean absolute percentage error and root mean square error of fuzzy time series is less values when compared to ARIMA and neural networks. Mean absolute error, mean absolute percentage error and root mean square error show that the performance of fuzzy time series is better than that of ARIMA and neural networks.

### III. CONCLUSION

In this work, three models, namely ARIMA, neural networks and fuzzy time series were used for wheat production prediction in India. Residual analysis for mean absolute error, mean absolute percentage error and root mean square error were compared using bar charts. Mean absolute error, mean absolute percentage error and root mean square error were minimum for fuzzy time series when compared to ARIMA and neural networks. Fuzzy time series is performed better than that of ARIMA and neural networks.

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# Application and Property of Many Special Types Recurrence Relation Polynomials in Number Theory and their Special Representation

*Mannu Arya & Vipin Verma*  
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## ABSTRACT

It develops a formula that explicitly expresses the general term of a linear recurrent sequence, allowing us to generalize J. McLaughlin's original finding on powers of 2 matrices to the case of a square matrix of size  $\leq 2$  matrix. The identities of Fibonacci and Stirling numbers, as well as a variety of combinatorial relations, are deduced. It uses two-variable Hermit polynomials and their operational laws to derive integral representations of Chebyshev polynomials. Most of the Chebyshev polynomial properties can be obtained using the Hermit polynomials  $H_n(x, y)$  definitions and formalism. They also show how to use these results to introduce valid generalizations of these polynomial groups and derive new identities and integral representations for them. For Chebyshev polynomials of the first and second kinds, its present new generating functions. A recurrence relation is an important mathematical concept. Recurrence relations are used in a variety of fields, including mathematics, economics, physics, and other sciences. It presents a significant finding on the convergence of recurrence relation sequences as a function of the recurrence relation coefficient.

*Keywords:* chebyshev polynomial, first kind, second kind, properties and applications.

*Classification:* FOR Code: 230102

*Language:* English



London  
Journals Press

LJP Copyright ID: 925654  
Print ISSN: 2631-8490  
Online ISSN: 2631-8504

London Journal of Research in Science: Natural and Formal

Volume 22 | Issue 15 | Compilation 1.0



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Mannu Arya<sup>α</sup> & Vipin Verma<sup>ο</sup>

## ABSTRACT

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*The major goal of this study is to use Girard and Waring's numerical and numerical adoption to the problem of Chebyshev polynomials' potential values and to identify certain structures that may be lost. As a specific application of our concept, we discovered two concurrent outcomes involving Fibonacci and Lucas numbers. The major goal of this work is to employ a few Chebyshev polynomial features to focus on the challenges of mixing the  $\text{Sin}x$  And  $\text{Cos}x$  energies, as well as to identify other interesting applications.*

**Keywords:** chebyshev polynomial, first kind, second kind, properties and applications.

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## I. INTRODUCTION

It is in the area of mathematical analysis that the orthogonal Polynomial and Dynamic functions have been established continuously and orthogonal analysis is not an exception. Orthogonal polynomials are the object of extensive employment, in particular and classical orthogonal polynomials have. Several problems apply to proven math, theory physics, chemistry, approximate principles, etc, with the accidentally of numbers of problems and other disciplines as well.

A significant subject in mathematics is a recurrent relationship. It is necessary for a recurrence relationship to the subject of mathematics. In both mathematics and economy, recurrent relations are used Physics and others in subject areas. Number theories are the principal field of study which this paper explores. "Learning to the popular theorist Carl Friedrich Gauss: "Mathematics is the queen of all science, and Numbers theory is the queen of Mathematical Studies. The study of numerology is the examination of the characteristics of integer and rational numbers, which exceed the habitual tricks of math." This in which nutritional philosophy will go into his history and not like "because of set" and

Save in. Relationships between repeated subjects are implemented in both mathematics and economics. The key effects of the convergence of the series of recurrence depend on the recovery coefficient. We're talking about a few examples. We will use recurrence in network marketing in this document. Recurrence relationships are very useful subjects for mathematics that solve many real-life problems by repeat relations. In modern times, network marketing is a very well-known business practice for a lot of people. The use of a recurrence relationship method which results based on the recurrence coefficient of the network marketing enterprise. This approach is therefore very useful in evaluating the benefit of any network marketing provider. There are some major concepts developed for recurrence relations. These personalities are very valuable in seeking the terms in any series for the recurrent correlation. In this recurring method, to find out any terms all previous terms need to be found and that result is very necessary. A partnership between an individual has a significant property terms and root polynomial values of a recurrence relations for Second Order relationship, we gave the same recurrence relationship property of all Higher Order recurrence relations. We may eventually conclude that all recurrence relationship involve roots are different, this concept, being true, all order. We may therefore presume that the text of a joint relationship between a number of variables, the values, and roots of a polynomial relationship.

### *First Order Recurrence Relation*

In the first order recurrence relation only one initial term is given. For example

$$a_{n+1} = a_n + 5, n \geq 1, a_0 = 0$$

we can find the terms

$$a_1 = 5, a_2 = 10, a_3 = 15$$

### *Second Order Recurrence Relation*

In the second order recurrence relation new term depend on two previous terms and two initial terms are given.

For example

### *Third Order Recurrence Relation*

In the third order recurrence relation new term depends on three previous terms with three initial terms are given.

For example

$$a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3}, n \geq 3$$

With the initial terms  $a_0 = 0, a_1 = 1, a_2 = 2$

$$a_n = a_{n-1} + 2a_{n-2}, n \geq 2$$

With the initial terms  $a_0 = 0, a_1 = 1$

### *Third Order Recurrence Relation*

In the 3rd order recurrence relation new term is depend on previous three terms. For example

$$a_n = a_{n-3} + 2a_{n-2} + a_{n-1}, n \geq 3$$

With the initials terms  $a_0 = 0, a_1 = 1, a_2 = 2$ .

The relation between primary numbers and perfect numbers, including composite numbers, exists.

The first term will be  $2 \times 2 = 4$ , next  $2 \times 3 = 6$ ,  $3 \times 3 = 9$ , etc. in composite numbers by means of primaries; 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 etc. and by means of a composite count number, in line with the formula for  $m$  and  $n$  composites, where  $m$  and  $n$  shall be both primarily numbers. The processing is such that there is no number between them, such as  $3 \times 3$ , and multiply 2 by 5, to get 10. However, it is laborious to achieve composite numbers that are far-reaching, so that the  $n$ th composite number ( $C_n$ ) can be produced with the Wolfram Language Code as above. It is a code for the computer. A manual algorithm for extrapolating large composite numbers is exceedingly difficult to develop. But the generating function of Dirichlet is used for the characteristic function of the composite numbers. The relationship can also be dealt with as follows in ideal numbers;  $(\text{Power of Two}) \times (\text{Double that Power} - 1)$ . The prime number formula is given by  $(2n-1)$ . To get the Perfect Number, the formula becomes  $(2n-1) \times (2n-1)$ . Getting the  $n$ th sequence of a perfect number is dependent on the equivalent  $n$ th sequence of prime numbers. The research finally concludes with the call for researchers to team up to make more explicit, the recurrence relation in composite numbers. The study focuses its intention on recurrence relation in perfect and composite numbers. It is hinged at analyzing the mathematical relationships between them, as well as asking new questions about them and to prove that these relationships are true.

Recurrence Relation of Network Marketing is the sector came into being, marketing has become one of the main data mining applications. The determination whether a specific individual is to advertise is usually based solely on its characteristics or those of the population segments to which they belong (direct marketing) (mass marketing). This also results in optimum targeting choices since the impact of a business consumers on each other's buying decisions is not taken into consideration. Customers are heavily affected by the views of their partners in many markets. Viral marketing does this to sell a commodity cheaply, mostly through marketing people with the highest consumer power.

## II. RESEARCH METHODOLOGY

### 3.1 Introduction

Methodology is a "focused paradigm" for science, a consistent and rational approach centered on opinions, and principles that direct researchers and other users to choose. It contains theoretical analyses of the community of methods and concepts linked to a branch of expertise, which differs according to their historical creation in the different disciplines. These methods, described in the methodology, define the means or modes of data collection or, sometimes, how a specific result is to be calculated. Methodology does not define specific methods, even though much attention is given to the nature and kinds of processes to be followed in a particular procedure or to attain an objective.

A methodology is a system of methods and principles for doing something, for example for teaching or for carrying out research. choosing a wholly suitable and sound method that is right for research project will give the path to help succeed. A methodology will give guidelines to make the project manageable, smooth, and effective.

Research methodology defines the pattern of performing research. The research opinion preferred for this chapter is based on the type of methodology selected. With the requirements and prerequisites that are defined by an individual, a specific methodology can be selected for testing the research questions and find results accordingly. A specific time limit is defined on every segment of the study and the desired target is achieved.

1. The proposed methodology of work will be phase wise as described in the following sequence: I. Development of techniques / recent advances for application in different fields.
2. Collection of literature regarding with the apropos research work.
3. Derivation of new results related with study of generalization of Fibonacci sequence.

There are two main directions in which the Fibonacci polynomials can be generalized either the recurrence relation can be generalized and extended, or the recurrence relation is preserved but the first two terms are replaced by arbitrary terms.

Research methodology refers to the steps, procedures and strategies used for gathering data during the research investigation. This chapter is concerned with methodology used in achieving the research aim. It also contains focus the of study and instrument of data collection used.

Recurrence relation is especially useful topic of mathematics. It is an equation that defines a sequence based on a method that gives the next term as relation of the previous terms. Recurrence relation is especially useful in mathematics as well as economics. This can calculate growth in economics by recurrence techniques. In recurrence relation for finding any term of sequence it needs to find all previous terms of sequence but by using this theorem it can find direct any term of sequence. Recurrence relation is especially useful in real life problems. Should be most of the people in the network marketing are honest, trying to earn a living to provide a high-quality lifestyle for themselves and their families.

Number Theory is the study of positive numbers (1,2,3,4,5,6,7...) which scrutinize the properties of integers, the natural numbers which is common as -1,-2,0,1,2 and so forth. It is part theoretical and part experimental, as mathematics seeks to discover fascinating and even unexpected mathematical interactions. It is the study of subtle and far-reaching relationships of numbers. This far end reaching relationships have its application in computer algorithm as in Fibonacci numbers.

Since ancient times, people have separated the natural numbers into a variety of different types which includes but not restricted to odd numbers, cube numbers, prime numbers, composite numbers, 1(modulo 4) number, 3(modulo 4) numbers, triangular numbers, perfect numbers, Fibonacci numbers, etc.

### 3.2 Focus of the Study

1. The study focuses on the number and polynomial number recurrence relation. It is concerned with analyzing and asking new questions about the mathematical relationships between them and showing that these connections are valid.
2. Recurrences are an extremely useful tools (sometimes unique) to solve many counter problems, where the use of established combinatorial techniques makes it challenging (or impossible) to count items. Therefore, the RR and its solution complement the experience of combinatorics and therefore in the philosophy of probability and statistics. In the well-known books of Combinatorics, the subject of recurrences and their resolving takes place by chance.
3. The recurrence is causally linked to the recursion, which the names indicate. The RR is used less or more in recurrence for learners and is considered in several textbooks together. The recursive computing of  $n!$  the  $n$ th number of Fibonacci, etc. are classic examples of recurring functions in any programming textbook. These features are more naturally based on the respective recurrences (i.e., recursive definitions). They can then be used to explain recursion and the main steps it takes forward steps and backward steps in the simple case (below) of the recursion (optional in some recursions). For e.g., recursive calls are defined by the subsequent recurrence and the steps forward correspond to the extension of the recurrence terms as per the iteration process (but it happens automatically, by pushing stack frames in the program stack).

As a simple case of recursion, the original conditions are used. The number and polynomial measures are inductive for the computer – beginning from the initial words, when all terms are calculated up to the  $(n - 1)$  set, the  $n$ th term is calculated (the subject “recursion and iteration” is essential and

comprehensive, so further consideration is needed). The Fibonacci number illustration is an inefficient recursion classical example. The best way to show and clarify that this recursion is inefficient is using the appropriate recursion tree. By it can infer for the utility of related function: "Let the recurrence of form (1) of order  $k > 1$  be defined and its initial conditions. For each recurrence word, a recursive function that determines the  $n$ th term of this series with recursive calls is unsuccessful. Here are two established strategies to prevent the unsuccessful recursion: memorization (where a certain value is calculated, stored in an array that will be used any time a recurrence attempts to compute it) and repetition through iteration and usage of a stack if required (i.e., applying the bottom-up approach). The recurrences are used in the analysis of complexity of algorithms, mostly recursive. As it mentioned above, the recurrences and the recursion trees are appropriate, powerful, and unique tools for investigation the time-complexity of algorithms, based on a strategy "divide-and-conquer".

### III. RESULT ANALYSIS

*Analysis-* Generalized Fibonacci sequences: Now, let us consider for  $q \geq 1$ , the "multibonacci" sequence  $(\phi_n^{(q)})_{n \geq -q}$  defined:

by

$$\left\{ \begin{array}{l} \phi_{-q}^{(q)} = \dots = \phi_{-2}^{(q)} = \phi_{-1}^{(q)} = 0, \\ \phi_0^{(q)} = 1, \\ \phi_n^{(q)} = \phi_{n-1}^{(q)} + \phi_{n-2}^{(q)} + \dots + \phi_{n-q-1}^{(q)} \text{ for } n \geq 1. \end{array} \right.$$

In Belbachir and Bencherif showed that

$$\phi_n^{(q-1)} = \sum_{k_1+2k_2+\dots+qk_q=n} \binom{k_1+k_2+\dots+k_q}{k_1, k_2, \dots, k_q},$$

And, for  $q \geq 1$ ,

$$\phi_n^{(q-1)} = \sum_{k=0}^{\lfloor n/(q+1) \rfloor} (-1)^k \frac{n-k(q-1)}{n-kq} \binom{n-kq}{k} 2^{n-1-k(q+1)},$$

Belongs to

$$\sum_{k_1+2k_2+\dots+qk_q} \binom{k_1+k_2+\dots+k_q}{k_1, k_2, \dots, k_q} = \sum_{k=0}^{\lfloor n/(q+1) \rfloor} (-1)^k \frac{n-k(q-1)}{n-kq} \binom{n-kq}{k} 2^{n-1-k(q+1)}$$

Theorem 4.1. the identity is as following

$$\phi_n^{(q)} = \sum_{l=0}^{qm-r} \binom{n-l}{l}_q \quad (4.1)$$

Where  $m$  is given for division via the extended euclidean algorithm:  $n = m(q+1)-r, 0 \leq r \leq q$ .

Proof. It says that

$$\begin{aligned} \phi_n^{(q)} &= \sum_{k_1+2k_2+\dots+qk_q=n} \binom{k_1+k_2+\dots+k_{q+1}}{k_1, k_2, \dots, k_{q+1}} \\ &= \sum_{L \geq 0} \sum_{k_1+2k_2+\dots+(q+1)k_{q+1}=n} \binom{L}{k_1, k_2, \dots, k_{q+1}} \\ &= \sum_{L \geq 0} \sum_{k_1+2k_2+\dots+(q+1)k_{q+1}=n-L} \binom{L}{L-k_2-\dots-k_{q+1}, k_2, \dots, k_{q+1}} \\ &= \sum_{L \geq 0} \binom{L}{n-L}_q \\ &= \sum_{L \geq \frac{n}{q+1}}^n \binom{L}{n-L}_q, \end{aligned}$$

the fact that is using as  $\binom{L}{a}_q = 0$  for  $a < 0$  or  $a > qL$

Consider the unique text of  $n$  given by the extended euclidean division algorithm

$$:n=m(q+1)-r, 0 \leq r < q+1 \text{ then } \frac{n}{q+1} = m - \frac{r}{q+1}, \text{ which gives}$$

In<sup>1</sup>Belbachir and Bencherif showed that

$$\phi_n^{(q)} = \sum_{l=0}^{qm-r} \binom{m+k}{qm-r-k}_q = \sum_{l=0}^{qm-r} \binom{m+k}{(q+1)k+r}_q = \sum_{l=0}^{qm-r} \binom{n-l}{l}_q.$$

This is obtain the following identities as an immediate consequence of Theorem 4.1.

$$\phi_{(q+1)m}^{(q)} = \sum_{l=0}^{qm} \binom{(q+1)m-l}{l}_q = \sum_{k=0}^{qm} \binom{m+k}{(q+1)k}_q,$$

$$\phi_{(q+1)m-1}^{(q)} = \sum_{l=0}^{qm-1} \binom{(q+1)m-l-1}{l}_q = \sum_{l=0}^{qm} \binom{m+k}{(q+1)k+1}_q,$$

$$\phi_{(q+1)m-r}^{(q)} = \sum_{l=0}^{qm-1} \binom{(q+1)m-l-r}{l}_q = \sum_{l=0}^{qm} \binom{m+k}{(q+1)k+1}_q,$$

The classic sequence Fibonacci is found for  $q = 1$ :

$$F_{-1} = 0, F_0 = 1, F_{n+1} = F_n + F_{n-1}, \text{ for } n \geq 0.$$

Thus, it achieve the common identity  $F_n = \sum_{l=0}^{\lfloor n/2 \rfloor} \binom{n-l}{l}$ .

Recently, Generalized Pascal triangle and sequences  $T_n$ . The following combinatorial interpretation occurs of the elements  $\binom{n}{k}_s$  of the generalized Pascal triangle. The word  $\binom{n}{k}_s$  assigns

the number of ways of distributing uniform objects  $k$  to  $n$  boxes, which may have a maximum number of objects in each box. Clearly,  $0 \leq k \leq sn$ . In other words,  $\binom{n}{k}_s = |\{f: \{0, \dots, n-1\} \rightarrow \{0, \dots, s\} \mid \sum_{i=0}^{n-1} f(i) = k\}|$ .

For example, if  $s = 2$  the triangle is achieved

$$\begin{array}{ccccccc}
 & & & & & & 1 \\
 & & & & & & & 1 & 1 & 1 \\
 & & & & & & & 1 & 2 & 3 & 2 & 1 \\
 & & & & & & & 1 & 3 & 6 & 7 & 6 & 3 & 1
 \end{array}$$

Where  $\binom{n}{k}_2 = \binom{n-1}{k-2}_2 + \binom{n-1}{k-1}_2 + \binom{n-1}{k}_2$ , if the value is zero if  $v < 0$  or  $2u < v$ , assumes that the  $u$  in  $\binom{u}{v}_2$  is not negative. It will skip subscript 1 and write only for the normal binomial coefficients  $\binom{n}{k}$  if  $s = 1$ . Now in further it is formulating a lemma for generalized binomial coefficients that is helpful in the theorem proof 4.2

*Lemma 4.2.* If  $s \geq 2$  then it will as

$$\binom{n}{k}_s = \sum_{k_1=\lfloor \frac{k}{s} \rfloor}^{\min\{k,n\}} \binom{n}{k_1} \binom{k_1}{k-k_1}_{s-1} \tag{4.2}$$

*Proof of Lemma 4.2.* If it chooses to distribute  $k$  elements, then it selects  $k_1$  boxes, with at most  $s - 1$  element per box, and then distribute the other  $k - k_1$  elements among the  $k_1$  boxes specified.  $\binom{u}{v}_s = 0$ .

Notice that the limit indication in sum (4.2) can be ignored by reminding the coefficient  $\binom{u}{v}_s = 0$  for an unremarkable if the integer  $v$  is outside the range  $0, \dots, su$ .

The generalized Pascal triangle  $\binom{n}{k}_s$ ,  $n \in \mathbb{N}$ ;  $0 \leq k \leq sn$  is linked to the linear recurrence  $\{T_n\}$  given by (1) and (2) via the diagonal sum,

$$\sum_{k_1=0}^{\lfloor \frac{sn}{s+1} \rfloor} \binom{n-1}{k_1}_s = T_{n+s} \tag{4.3}$$

The case  $s=1$  returns the nice identify,

$$\sum_{k_1=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-k_1}{k_1}_s = F_{n+1}$$

A000045 number for Fibonacci, while  $s=2$  is related to A000073 number for Tribonacci numbers. Generalized numbers associated with Fibonacci. More broadly, the unimodalities of all Pascal generalised rays are determined by showing that the sequence  $w_k = \binom{n+\alpha k}{m+\beta}_q$  is log concave, then unimodal.

The Fibonacci polynomials: Notes that if  $k$  is a real variable of  $x$ , then  $F_{k;n} = F_{x;n}$  is the polynomials defined by Fibonacci,

$$F_{n+1}(x) = \begin{cases} 1 & \text{if } n=0 \\ x & \text{if } n=1 \\ x(F_n(x) + F_{n-1}(x)) & \text{if } n \geq 2 \end{cases}$$

from where The first polynomials of Fibonacci are

$$F_1(x) = 1$$

$$F_2(x) = x$$

$$F_3(x) = x^2 + 1$$

$$F_4(x) = x^3 + 2x$$

$$F_5(x) = x^5 + 4x^3 + 3x$$

And more it can write  $k$ -Fibonacci numbers from these expressions:

$$F_{n+1}(x) = \sum_{i=0}^{\lfloor n/2 \rfloor} \binom{n-i}{i} x^{n-2i} \quad \text{for } n \geq 0$$

Notice is that  $F_{2n}(0) = 0$  and  $x = 0$  is the only real root, while  $F_{2n+1}(0) = 1$  to with no real roots.

Also for  $x = k \in \mathbb{N}$  The  $k$ -Fibonacci sequence elements are obtained.

*Analysis:* Chebyshev polynomials show integral representations of the hermit polynomials and the generation process will add the new representations of Chebyshev polynomials. Chebyshev polynomials after the second kind  $U_n(x)$

$$U_n(x) = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{(-1)^k (n-k)! (2x)^{n-2k}}{k!(n-2k)!}$$

*Proposition 1.* The two polynomials of Chebyshev satisfy the following integral characterization:

$$U_n(x) = \frac{1}{n!} \int_0^{+\infty} e^{-t} t^n H_n(2x, -\frac{1}{t}) dt$$

*Proof:* By taking note of this

$$n! = \int_0^{+\infty} e^{-t} t^n dt$$

It can write as

$$(n - k)! = \int_0^\infty e^{-t} t^{n-k} dt$$

The explicit form of  $U_n(x)$  the Chebyshev polynomials, and the standard two-variable Hermit polynomials:

We know that

$$H_n(x, y) = n! \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(y)^k (x)^{n-2k}}{k! (n - 2k)!} \quad (2.1)$$

$$U_n(x) = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^k (n - k)! (2x)^{n-2k}}{k! (n - 2k)!} \quad (2.2)$$

In (2.1)  $x$  replace by  $2x$  and  $y$  replace by  $-\frac{1}{t}$  we will get

$$H_n\left(2x, -\frac{1}{t}\right) = n! \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^k (2x)^{n-2k} t^{-k}}{k! (n - 2k)!} \quad (2.3)$$

Now (2.3) multiplying both side by  $e^{-t} t^n$  and integrating limit 0 to  $\infty$

We will get

$$\int_0^\infty e^{-t} t^n H_n\left(2x, -\frac{1}{t}\right) dt = n! \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^k (2x)^{n-2k}}{k! (n - 2k)!} \int_0^\infty e^{-t} t^{n-k} dt \quad (2.4)$$

using  $(n - k)! = \int_0^\infty e^{-t} t^{n-k} dt$  in (2.4) we will get

$$\int_0^\infty e^{-t} t^n H_n\left(2x, -\frac{1}{t}\right) dt = n! \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^k (2x)^{n-2k} (n-k)!}{k! (n-2k)!}$$

So we have

$$U_n(x) = \frac{1}{n!} \int_0^{+\infty} e^{-t} t^n H_n\left(2x, -\frac{1}{t}\right) dt$$

and then the study.

**Theorem 2:** The Chebyshev polynomials  $T_n(x)$  and  $U_n(x)$  satisfy the following recurrence relations:

$$\frac{d}{dx} U_n(x) = nW_{n-1}(x)$$

$$U_{n+1}(x) = xW_n(x) - \frac{n}{n+1} W_{n-1}(x)$$

Where,

$$W_n(x) = \frac{2}{(n+1)!} \int_0^{+\infty} e^{-t} t^{n+1} H_n\left(2x, -\frac{1}{t}\right) dt.$$

*Proof-* In the above section the recurring relations of the standard hermit polynomials  $H_n(x, y)$  can be costumed as follows.

First we will prove identity (2.5) and (2.6)

$$\left[2x + \frac{1}{-t} \frac{\partial}{\partial x}\right] H_n\left(2x, -\frac{1}{t}\right) = H_{n+1}\left(2x, -\frac{1}{t}\right) \tag{2.5}$$

$$\frac{1}{2} \frac{\partial}{\partial x} H_n\left(2x, -\frac{1}{t}\right) = n H_{n-1}\left(2x, -\frac{1}{t}\right) \tag{2.6}$$

Consider

$$\frac{\partial H_n\left(2x, -\frac{1}{t}\right)}{\partial x} = n! \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{2(-1)^k (2x)^{n-2k-1} (n-2k) t^{-k}}{k! (n-2k)!}$$

$$\frac{\partial H_n\left(2x, -\frac{1}{t}\right)}{2\partial x} = n! \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{(-1)^k (2x)^{n-2k-1} t^{-k}}{k! (n-2k-1)!}$$

So we have

$$\frac{1}{2} \frac{\partial}{\partial x} H_n\left(2x, -\frac{1}{t}\right) = n H_{n-1}\left(2x, -\frac{1}{t}\right)$$

So (2.6) has proved some process we can prove (2.5)

By above theorem we have

$$U_n(x) = \frac{1}{n!} \int_0^{+\infty} e^{-t} t^n H_n(2x, -\frac{1}{t}) dt$$

Differentiation both side with respect to  $x$  we get

$$\frac{d}{dx} U_n(x) = \frac{1}{n!} \int_0^{+\infty} e^{-t} t^n \frac{\partial}{\partial x} H_n(2x, -\frac{1}{t}) dt$$

Now using identity (2.5) we will get

$$\frac{d}{dx} U_n(x) = \frac{2n}{n!} \int_0^{+\infty} e^{-t} t^n H_{n-1}(2x, -\frac{1}{t}) dt \tag{2.7}$$

The relation above provides a link between polynomials  $T_n(x)$  and  $U_n(x)$  however, as:

$$U_{n-1}(x) = \frac{1}{(n-1)!} \int_0^{+\infty} e^{-t} t^{n-1} H_{n-1}(2x, -\frac{1}{t}) dt \tag{2.8}$$

Using (2.8) in (2.7) immediately get:

$$\frac{d}{dx} T_n(x) = n U_{n-1}(x).$$

By using Second kind of Chebyshev polynomials in the first identity

$$U_{n+1}(x) = \frac{1}{(n+1)!} \int_0^{+\infty} e^{-t} t^{n+1} H_{n+1}(2x, -\frac{1}{t}) dt$$

Using  $\left[2x + \frac{1}{-t} \frac{\partial}{\partial x}\right] H_n(2x, -\frac{1}{t}) = H_{n+1}(2x, -\frac{1}{t})$  in (2.9)

$$U_{n+1}(x) = \frac{1}{(n+1)!} \int_0^{+\infty} e^{-t} t^{n+1} \left[2x + \frac{1}{-t} \frac{\partial}{\partial x}\right] H_n(2x, -\frac{1}{t}) dt$$

That is

$$U_{n+1}(x) = x \frac{2}{(n+1)!} \int_0^{+\infty} e^{-t} t^{n+1} H_n(2x, -\frac{1}{t}) dt - \frac{1}{(n+1)!} \int_0^{+\infty} e^{-t} t^n \frac{\partial}{\partial x} H_n(2x, -\frac{1}{t}) dt \tag{2.10}$$

Using  $\frac{1}{2} \frac{\partial}{\partial x} H_n(2x, -\frac{1}{t}) = n H_{n-1}(2x, -\frac{1}{t})$  in (2.10)

We have

$$U_{n+1}(x) = x \frac{2}{(n+1)!} \int_0^{+\infty} e^{-t} t^{n+1} H_n(2x, -\frac{1}{t}) dt - \frac{2n}{(n+1)!} \int_0^{+\infty} e^{-t} t^n H_{n-1}(2x, -\frac{1}{t}) dt \tag{2.11}$$

Using  $W_n(x) = \frac{2}{(n+1)!} \int_0^{+\infty} e^{-t} t^{n+1} H_n(2x, -\frac{1}{t}) dt$  in (2.11) we have

$$U_{n+1}(x) = xW_n(x) - \frac{n}{n+1}W_{n-1}(x)$$

Also we have

$$\frac{d}{dx}U_n(x) = \frac{2n}{n!} \int_0^{+\infty} e^{-t} t^n H_{n-1}(2x, -\frac{1}{t}) dt$$

And

$$W_n(x) = \frac{2}{(n+1)!} \int_0^{+\infty} e^{-t} t^{n+1} H_n(2x, -\frac{1}{t}) dt$$

Replace  $n + 1$  by  $n$

We get

$$W_{n-1}(x) = \frac{2}{(n)!} \int_0^{+\infty} e^{-t} t^n H_{n-1}(2x, -\frac{1}{t}) dt$$

So we have

$$\frac{d}{dx}U_n(x) = n W_{n-1}(x)$$

**Generation of functions:** The first and second type of Chebyshev polynomials can draw a slightly different links from these polynomials and their generative functions using the integrated representations in the previous section and related recurrence relations.

For the  $U_n(x)$  Polynomials of Chebyshev, we notice that both sides of the equation by  $\xi^n \mid \xi \mid < 1$  and it follow by sum marizing over  $n$

$$\sum_{n=0}^{+\infty} \xi^n U_n(x) = \int_0^{+\infty} e^{-t} \sum_{n=0}^{+\infty} \frac{(t\xi)^n}{n!} H_n(2x, -\frac{1}{t}) dt$$

By remembering the polynomials of the  $\sum_{n=0}^{+\infty} \frac{t^n}{n!} H_n(x, y) = e^{(xt+yt^2)}$  in the above relation generation and  $t$  integration, we end.

$$\sum_{n=0}^{+\infty} \xi^n U_n(x) = \frac{1}{1-2\xi x + \xi^2}$$

By using the results seen in the previous theorem, we can now state the respective generation function for the first Chebyshev polynomial  $T_n(x)$  and  $U_n(x)$

## IV. APPLICATIONS OF RECURRENCE RELATIONS

- *Biology*

Some of the most recognised variability calculations arise in attempts to shape population dynamics. The Fibonacci numbers, for examples, were once utilised as an example of growing the population of rabbit. Integrodifference equations are a significant type of recurrent relationship to spatial ecology. This and other variability computations are especially suitable for modelling in univoltine populations.

- *Computer Science*

Recurrence relation is also an important value in algorithm analysis. If an algorithm is intended to break a problem into more low (the divide and conquer) problems it run at times will be described by a relationship repetition. Simpler instances are that it takes the time and find an item with  $n$  elements in the order vector (in case no components can take such a density). One component at a time, from left to right, will be sought with a native algorithm. The worst case would be where the desired substance is the last element, so the range of contrasts is  $n$ . Binary search is the better algorithm. It takes a single, ordered vectors. That checks whether the element is in centre of the vector first. If not, then whether the centre component is greater than or less than the desired component is monitored.

- *Digital Signal Processing*

Digital signal processors) is the digital processing method for a wide variety of signal processing activities, as on the part of computer machines or more specialised digital signal processors. The digit this way is a set of numbers that is a series of digits forming samples of a continuous variable in a particular domain such as time, area, space, or frequency in this order. In digital signal processing, repetition links may impact feedback in a process where outputs, at the same time, become products. Thus, they occur in digital filters of infinite impulse response (IIR). Multi-use improvements can enhance the cryptographic safety of digital signals the encrypted and decrypted algorithms.

- *Economics*

In both theoretical and empirical economies, recurrent relationships, particularly straight recurrence interactions, are widely used. On a macroeconomic stage in particular, a model regions of big economy (the financial sector, the commodity area and the labour market etc.) might be developed based in that some actors' action depends on lagging trend modalities. You would be willing to solve the model to fix main variables' current value with regard to past and actual values (interest rate, true GDP, etc.) in other variables.<sup>23</sup>

- *Network Marketing*

Network business is a business method or activity in which people are compensated for not only work created by themselves, but for the work also produced by others. "The network business model is referred to as "down line model," since it is developed by distributors with multiple levels of compensation and "several" levels in.

All sorts of network enterprise platforms are there. Employees directly sell products to customers through link referrals and word of voice marketing in most network business platforms. Many grid marketing organizations aimed to build opportunities for individuals who would not otherwise have them, including those who. Less privacy in running their own company, Do not have limited cash, Compatible with their present labor level, Has been unwise with own companies failed to work. Recurrence is a very helpful subject in mathematics which resolves many issues of real lives many through repeated interactions.

A lot of individuals are involved in network marketing companies in modern time Network Marketing is very renowned company direction.

## V. CONCLUSION

Recurrence relation is very useful topic of mathematics many problems of real life many be solved by recurrence relations but in recurrence relation there is a major difficulty in the recurrence relation if we want find 100th term of sequence then we need to find all previous 99 terms of given sequence then we can get 100th term of sequence but above theorem is very useful if coefficients of recurrence relation of given sequence are satisfied the condition of the above theorem then we can apply above theorem and we can find direct any term of sequence without find all previous terms. There is important property of a relation between coefficients of recurrence relation terms and roots of a polynomial for second order relation but in this paper, we gave this same property of recurrence relation of all higher order recurrence relation. So finally, we can say that this theorem is valid all order of recurrence relation only condition that roots are distinct. So, we can say that this paper is generalization of property of a relation between coefficients of recurrence relation terms and roots of a polynomial.

The relation between primary numbers and perfect numbers, including composite numbers, exists.

The first term will be  $2 \times 2 = 4$ , next  $2 \times 3 = 6$ ,  $3 \times 3 = 9$ , etc. in composite numbers by means of primaries; 2, 3,5,7,11,13,17,19,23,29,31 etc. and by means of a composite count number, in line with the formula for m and n composites, where m and n shall be both primarily numbers. The processing is such that there is no number between them, such as  $3 \times 3$ , and multiply 2 by 5, to get 10. However, it is laborious to achieve composite numbers that are far-reaching, so that the nth composite number (Cn) can be produced with the Wolfram Language Code as above. It is a code for the computer. A manual algorithm for extrapolating large composite numbers is exceedingly difficult to develop. But the generating function of Dirichlet is used for the characteristic function of the composite numbers. The relationship can also be dealt with as follows in ideal numbers; (Power of Two)  $\times$  (Double that Power - 1). The prime number formula is given by  $(2n-1)$ . To get the Perfect Number, the formula becomes  $(2n-1) \times (2n-1)$ . Getting the nth sequence of a perfect number is dependent on the equivalent nth sequence of prime numbers. The research finally concludes with the call for researchers to team up to make more explicit, the recurrence relation in composite numbers. The study focuses its intention on recurrence relation in perfect and composite numbers. It is hinged at analyzing the mathematical relationships between them, as well as asking new questions about them and to prove that these relationships are true.

Recurrence Relation of Network Marketing is the sector came into being, marketing has become one of the main data mining applications. The determination whether a specific individual is to advertise is usually based solely on its characteristics or those of the population segments to which they belong (direct marketing) (mass marketing). This also results in optimum targeting choices since the impact of a business consumers on each other's buying decisions is not taken into consideration. Customers are heavily affected by the views of their partners in many markets. Viral marketing does this to sell a commodity cheaply, mostly through marketing people with the highest consumer power.

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