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IMAGE: OBSERVATORY WITH STAR TRAILS ON MOUNTAINS FOR CLEAR SKY

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Two Generalizations of Brouwer Fixed Point Theorem

Bhamini M. P. Nayar

Morgan State University

ABSTRACT

The following fixed point theorems are given: (1) If X is a Hausdorff and compact space and $g : X \to X$ is a one-one continuous function, then g has a fixed point. (2) If X is a compact, Hausdorff and second countable space and $f : X \to X$ is a contraction mapping, then f has a fixed point. Two proofs of Theorem 1 are given, one using sequences and the other using ultrafilters. These theorems generalize the Brouwer Fixed Point Theorem.

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Bhamini M. P. Nayar

In loving memory of Prof. James E. Joseph

ABSTRACT

The following fixed point theorems are given: (1) If X is a Hausdorff and compact space and $g : X \to X$ is a one-one continuous function, then g has a fixed point. (2) If X is a compact, Hausdorff and second countable space and $f : X \to X$ is a contraction mapping, then f has a fixed point.Two proofs of Theorem 1 are given, one using sequences and the other using ultrafilters. These theorems generalize the Brouwer Fixed Point Theorem.

Keywords and phrases: Fixed point theorem; Brouwer.

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I. INTRODUCTION

Brouwer Fixed Point theorem states that *a continuous map of D, the unit disc $\{(x, y) \in R^2 | x^2 + y^2 \leq 1\}$ to itself has a fixed point* ([1], [2], [5], [6]). In this article Brower Fixed Point Theorem is extended to compact Hausdorff spaces.

The proof of Brower Fixed Point Theorem is generally given using the concept of homotopy and Retraction Theorem ([1], [2], [5], [6]). A subset A of X is called a *retract* of X, provided that there is a continuous function $f : X \to A$ such that $f(x) = x$ for all $x \in A$ and the function f is called a *retraction* [2]. The argument used in proving the Brower Fixed Point Theorem is that the boundary of the unit disc D, the unit circle, is not a retract of D. It is known that the unit circle, as well as the unit disc, is compact and Hausdorf where as the circle with the point $(0, 1)$ removed is not compact [4]. Since it is known that the unit circle $S = \{(x, y) \in \mathcal{R}^2 | x^2 + y^2 = 1\}$, with continuous function taking each point to its diametrically opposite point does not give a fixed point, and observing that such a function is not one-one, we arrive at the following result. It is well known that a continuous function from a compact space to a Hausdorff space is closed.

Theorem 1. Let X be Hausdorff, compact and $g : X \to X$ be continuous and one-one. Then g has a fixed point.

Proof. **Proof 1.** If g is constant, then g has a fixed point. In that case g is not on-one. Let g be non-constant. Since X is compact and Hausdorff, $g : X \to X$ is a closed function. Therefore, $g(X)$ is closed. So, $g(X)$ is open in $g(X)$ and is a closed subset of the space X. Choose an $x \in g(X) \subseteq X$ and form the sequence $\{g^n(x)\}$. Note that $g(X)$ being a closed subset of X, for all n , $\{g^n(x)\} \subseteq g(X)$. Moreover, it is compact, and hence is countably compact. So, there is a subsequence $\{g^{n_k}(x)\}$ of $\{g^n(x)\}$ in $g(X)$ such that $\{g^{n_k}(x)\} \to p$, $p \in g(X)$, $g(X)$ being closed. Also since g is one-one and $X - g(X)$ being open, and $\{g^n(x)\} \subseteq g(X)$ no subsequence of $\{g^n(x)\}$ in $X - g(X)$ will converge to $p \in g(X)$. So, $\{g^{n_k+1}(x)\} \to g(p)$, g being continuous. Thus $g(p) = p$, since $\{g^{n_k+1}(x)\}$ is a subsequence of $\{g^{n_k}(x)\}$. Hence there is a fixed point for g in $g(X) \subseteq X$.

Below, another proof for the Theorem 1 using ultrafilters is given.

Proof 2. Consider a space X which is compact and Hausdorff and let $f : X \to X$ be a continuous injection. Suppose $\mathfrak{F}$ is an ultrafilter on X. Since X is compact, $\mathfrak{F} \to x$ for some $x \in X$. Suppose that $\mathfrak{F}$ does not have a fixed point in X. Then for each $x \in X, x \neq f(x)$. The space X is Hausdorff and hence there exist open sets U and V in X such that $x \in U$ and $f(x) \in V, U \cap V = \emptyset$. Note that $\mathfrak{F} \to x$ and hence, there is an $F_1 \in \mathfrak{F}$ such that $F_1 \subseteq U$. Also, since f is continuous, $f(\mathfrak{F})$ is an ultrafilter and $f(\mathfrak{F}) \to f(x)$. Hence there is an $F_2 \in \mathfrak{F}$ such that $f(F_2) \subseteq V$. Let $F = F_1 \cap F_2$. Note that $F \in \mathfrak{F}$ and $F \subseteq U \subseteq X - V$. Also, $f(U) \subseteq V$, since f is continuous. Moreover, $f(U) \subseteq f(X - V) = f(X) - f(V)$. Therefore, $V \cap (f(X) - f(V)) \neq \emptyset$. That is, there exists a $t \in V$ such that $f(t) \notin f(V)$. That is, there is an $a \in V$ such that $a \neq f(t)$ for any $t \in V$ and $f(a) \in f(V)$, for all $a \in V$. Here we arrive at a contradiction since f is one-one and $V \cap (X - V) = \emptyset \Rightarrow f(V) \cap f(X - V) = f(V) \cap (f(X) - f(V)) = \emptyset$. Hence our assumption that for each $x \in X, x \neq f(x)$ is not true. So, there is an $x \in X$ such that $x = f(x)$ and hence f has a fixed point and the proof is complete.

The concepts defined in the rest of the article can be found in any of the referenced books and are not given individual references. A topological space is *metrizable* if there is a metric on X which gives the same collection of open sets as the topological space. In that case we say that the topology and the metric on X are compatible. While every metric space is a topological space, a topological space need not have a metric which provides the same collection of open sets. Investigating conditions on a space which guarantees the existence of a compatible metric is a significant investigative area, called *metrization*.

A space is *complete* if every Cauchy sequence in the space converges. It is well known that a compact metric space is complete. A space is *separable* if it has a countable dense subset and is *second countable* if it has a countable base. It is well known that a second countable space is separable, but a separable space need not be second countable. However, a separable metric space is second countable. Also, a compact Hausdorff space is regular. A function $f : (X, d) \to (Y, \rho)$ is a *contraction mapping*, if there is an $\alpha \in (0, 1)$ such that $\rho(f(x), f(y)) \leq \alpha d(x, y)$ for every $x, y \in X$. The *Urysohn metrization theorem* states the following:

Theorem 2 [2] Let X be a T_1-space. Then the following are equivalent:

(1) X is separable and metrizable;
(2) X is regular and second countable.

In view of the above result, It is clear that if a second countable topological space X is compact and Hausdorff, then it is metrizable. This guarantees the existence of a metric d on X such that the topology generated by the metric d is the topology on X. With this observation, the following definition is provided.

Definition Let X be a metrizable topological space. A function $f : X \to X$ is *a contraction mapping on* X, if it is a contraction mapping with respect to the metric on X which is compatible with the topology on X.

In view of the above, we have the following Theorem.

Theorem 3 Let X be compact, Hausdorff and second countable and let $f : X \to X$ be a contraction maping. Then f has a fixed point.

Proof. Given that X is compact and Hausdorff. Therefore X is regular and it is also second countable. So, by the Urysohn metrization theorem, the space is metrizable. That is, there is a metric d which is compatible with the topology. Choose $x \in X$ and consider the sequence $\{f^n(x)\}$. Since f is a contraction mapping, $\{f^n(x)\}$ is a Cauchy sequence. The space X being compact and regular, it is complete and hence the sequence $\{f^n(x)\}$ converges, say, to $p \in X$. So, $\{f^{n+1}(x)\} \to f(p)$. Hence $f(p) = p$ and f has a fixed point. The proof is complete.

Note that the unit disc D satisfies the conditions of the space X in Theorems 1 and 3. Hence these theorems genralize the Brouwer Fixed Point Theorem.

II. CONCLUSION

A word aboout the development of this article: When the work on the current article was started, our attempt was to use one of the characterizations of closed function provided in [3]. It states that: A function $g : X \to Y$ is a closed function if and only if $g(V) - g(X - V)$ is open in $g(X)$ whenever V is an open subset of X, where X and Y are topological spaces. When the space X is compact and Hausdorff, any continuous function $f : X \to X$ will be closed. Originally, we wanted to use this fact, along with the assumption that the space was connected and the function was onto. However, with the assumption that f is an injection, that will make f to be a homeomorphism.

It is well-known that corresponding to each real number t, we can find a point on the unit circle and each point on the unit circle associates with a real number. A function $f(x) = x + k$ is a continuous function on the set of reals, which does not have a fixed point. While the set of reals $\mathfrak{R}$ with usual topology is not compact, the line segment $[0, 2\pi]$, as a subset of the set of reals is compact and every continuous function on $[0, 2\pi]$ has a fixed point. However, the unit circle as a subset of the plane R^2 with Euclidean topology is compact and Hausdorff. A function which takes each point on the circle to its diametrically opposite point does not have a fixed point and it is continuous. These observations highlighted the periodic nature of the function which associates each real number to a point on the unit circle. Thus the assumption that the function on a compact Hausdorff space to be continuous and one-one is made.

The Brower Fixed Point Theorem is one among the significant fixed point theorems in Topology with its possibilities of applications in real world situations. Adams and Franzosa gave applications of Brower Fixed Point Theorem to identify equilibrium price distribution in Economics [1]. Its generalization to set-valued functions, Kakutani's Fixed Point Theorem, and its applications to Game Theory also are detailed in [1]. Considering such prominance, to have a straight forward proof of Borwer Fixed Point Theorem, even in a generalized form, is a significant addition to the literature.

ACKNOWLEDGMENTS

This article was started originally as a joint work by myself and Prof. James E. Joseph before his passing on December 8, 2022. In fact the first proof of Theorem 1 was outlined before he passed and also with the assumption that X as a connected space, using the above mentioned charecterization of a closed function. When I revised it recently, I introduced the function to be one-one and dropped the assumption that the space to be connected. The day before his passing, while discussing that theorem, he indicated to me that we

should give a proof using ultrafilters, but we did not discuss any details. I provide here a proof using ultrafilters and also added Theorem 3. He passed away peacefully in his sleep, the next day. Like several signifcant results in Topology, Brower Fixed Point Theorem was in his mind and as always he tried to give simpler proofs for such classical results.

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Tensor Triangulated Category to Quantum Version of Motivic Cohomology on etale Sheaves

Prof. Dr. Francisco Bulnes

ABSTRACT

The tensor structure of triangulated categories will be considered in derived categories of étale sheaves with transfers performed the tensor product of categories $X \otimes Y = X \times Y$, in the finite correspondences category Cor_kconsidering the product underlying of schemes on a field k.A total tensor product on the category $\mathrm{PSL}(k)$,is required to obtain the generalizations on derived categories using pre-sheaves, contravariant and covariant functors on additive categories of the type $\mathbb{Z}(A)$, or $A^{\oplus}$, to determine the exactness of infinite sequences of cochain complexes and resolution of spectral sequences. Then by a motives algebra, which inherits the generalized tensor product of $\mathrm{PSL}(k)$, is defined a triangulated category whose motivic cohomology is a hypercohomology from the category Sm_k, which has implications in the geometrical motives applied to bundle of geometrical stacks in field theory. Then are considered the motives in the hypercohomoloy to the category DQFT.A fundamental result in a past research was the creation of lemma that incorporates a 2-simplicial decomposition of $\Delta^3 \times A^1$, in four triangular diagrams of derived categories from the categorySm_k, this was with the goal to evidence the tensor structure of DQFT.

Keywords: DQFT, étale sheaves cohomology, hypercohomology, motivic cohomology, tensor triangulated category, quantum version of hypercohomology, simplicial.

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ABSTRACT

The tensor structure of triangulated categories will be considered in derived categories of étale sheaves with transfers performed the tensor product of categories $X\otimes Y = X \times Y$, in the finite correspondences category Cor_k considering the product underlying of schemes on a field k.A total tensor product on the category $PSL(k)$,is required to obtain the generalizations on derived categories using pre-sheaves, contravariant and covariant functors on additive categories of the type $\mathbb{Z}(A)$, or $A^{\oplus}$, to determine the exactness of infinite sequences of cochain complexes and resolution of spectral sequences. Then by a motives algebra, which inherits the generalized tensor product of $PSL(k)$, is defined a triangulated category whose motivic cohomology is a hypercohomology from the category Sm_k, which has implications in the geometrical motives applied to bundle of geometrical stacks in field theory. Then are considered the motives in the hypercohomoloy to the category $DQFT$. A fundamental result in a past research was the creation of lemma that incorporates a 2-simplicial decomposition of $\Delta^3 \times A^1$, in four triangular diagrams of derived categories from the category Sm_k, this was with the goal to evidence the tensor structure of $DQFT$. Now in this research we consider a theorem that relates the hypercohomology groups obtained with the spectrum through the its singular homology taking components $\mathbb{Z}_{tr}(k)$and the A^1 −homotopy in the action of the symmetric group on the derived category $DM_{Nis}^{eff,-}(k)$.Finally will give a crystallographic space-time model of simplicial type from the microscopic aspects that define it, and will be established under the dualities in field theory and the hypercohomology Nisnevich groups that the vertices in decomposition of the space $\Delta^3 \times A^1$,are equivalent to the field waves, for example gravitational waves.

Keywords: DQFT, étale sheaves cohomology, hypercohomology, motivic cohomology, tensor triangulated category, quantum version of hypercohomology, simplicial.

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I. INTRODUCTION

The category $\mathrm{PSL}(k)$, is Abelian [1] and therefore has enough injectives and projectives that can be used to create the conditions for the invariant presheaves of homotopy required to realization of the commutative diagrams in A^1 − homotopy of morphisms in the categorySm_k, of finite schemes X, and Y. For example, we have the correspondence between simplicials and the corresponding diagrams of A^1 −morphisms in the category $C_*\mathbb{Z}_{tr}(X \times \mathrm{A}^1)$.

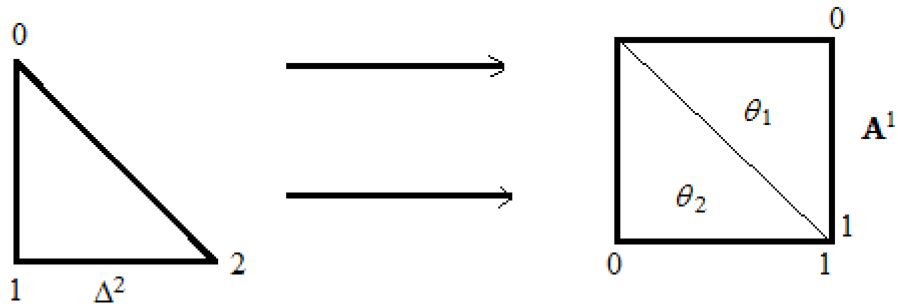


Figure 1. 2: Simplicial decomposition of $\Delta^2 \times \mathrm{A}^1$.

Or considering Δ^3, we have the correspondence:

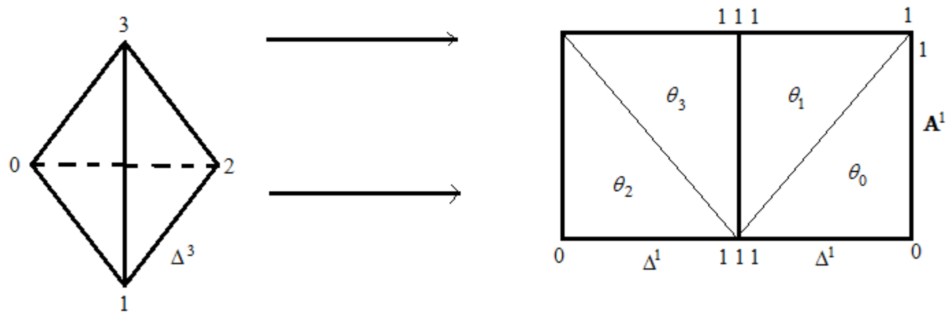


Figure 2: Simplicial decomposition of $\Delta^3 \times \mathrm{A}^1$.

This case will be used to obtain a general diagram that can be induced to the category DQFT, from a scheme of associated motives to a scheme X, (which is the class $m(X)$ of $C_*\mathbb{Z}_{tr}(X)$, which is clearly modulus A^1 –homotopy in an approximate triangulated category $\mathrm{DM}_{Nis}^{eff,-}(k, R)$,[1] constructed from the derived category of $\mathrm{PSL}(k)$.

We consider the following corollary to homotopy invariant presheaves [2, 3] and deduced from the fact that for every smooth scheme X, exists a natural homomorphism (is to say a homotopy) which explained a diagram that belongs to the correspondence planted in the figure 1, or as factor of the correspondence planted in the figure 2. Likewise we have:

Corollary 1. 1.$C_*\mathbb{Z}_{tr}(X \times \mathrm{A}^1) \to C_*\mathbb{Z}_{tr}(X)$, is a chain homotopy equivalence.

Then in the motives context and after of demonstrate the equivalences (in A^1 –homotopy) of the correspondences morphisms of injectives and projections, we can to have the motives scheme equivalence $m(X) \cong m(X \times \mathrm{A}^1)$, for all X, which helps to establish in a general way that any A^1 –homotopy equivalence $X \to Y$, induces an isomorphisms $m(X) \cong m(Y)$, considering inverses.

II. INDUCING A TOTAL TENSOR PRODUCT FROM DERIVED TENSOR PRODUCTS FOR THE REQUIRED DERIVED CATEGORIES

We consider $\mathcal{A}$, a small additive category and we define $\mathbb{Z}(\mathcal{A})$, to be the category of all additive presheaves on $\mathcal{A}$, on all conformed additive functors:

$$F: \mathcal{A} \to \mathrm{Ab}, \tag{1}$$

[1] This category has the total tensor product inherited from the total tensor product of $\mathrm{PSL}(k)$.

which is an Abelian category. We suppose that $\mathcal{A}$, has an additive symmetric monoidal structure [4, 5] (tensored) or given by $\otimes$, such that

$$\mathcal{A} = \mathrm{Cor}_k, \tag{2}$$

which means that $\otimes$, commutes with all direct sums $\mathcal{A}^{\oplus}$, corresponding to the presheaf

$$h_X = \otimes_i h_{X_i} \tag{3}$$

in $\mathbb{Z}(\mathcal{A})$. If R, is a ring we can define $R(\mathcal{A})$, to be the Abelian category of all additive functors

$$F: \mathcal{A} \to R - \mathrm{mod}, \tag{4}$$

Then we write h_X, for the functor with correspondence rule

$$\mathcal{A} \to R \otimes_{\mathbb{Z}} \mathrm{Hom}_{\mathcal{A}}(\mathcal{A}, X), \tag{5}$$

which is called representable. Then by [3] a representable preshea fh_X, is a projective object of $R(\mathcal{A})$, where every projective object of $R(\mathcal{A})$, is a direct summand of a direct sum of a representable functor, and every F, in $R(\mathcal{A})$, has a projective resolution.

The idea is consider the homotopy category established by all h_X, such that the tensor product $\otimes$, can be established to a total tensor product on the category $\mathrm{Ch}^- R(\mathcal{A})$, of bounded above cochain complexes

$$\cdots \to F \to 0 \to \cdots, \tag{6}$$

Our considerations of $\otimes$, is realized by the requirement due as follows: if X, and Y, are in $\mathcal{A}$; then the tensor product $h_X \otimes h_Y$, of their representable presheaves should be representable by the tensor product of schemes $X \otimes Y$.

Here is when we can appreciate the possibilities of extend $\otimes$, to a tensor product:

$$\otimes : \mathcal{A}^{\oplus} \times \mathcal{A}^{\oplus} \to \mathcal{A}^{\oplus}, \tag{7}$$

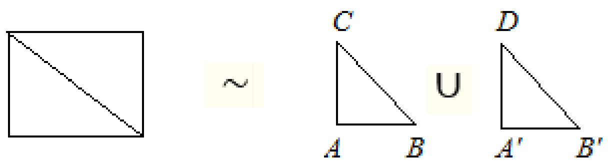
commuting with $\otimes$. Then if $L_1, L_2 \in \mathrm{Ch}^-(\mathcal{A}^{\oplus})$, of bounded above cochains complexes as (6) are the chain complexes as (6) are the chain complex $L_1 \otimes L_2$, can be defined as the total complex of the double complex $L_1^* \otimes L_2^*$. As has been mentioned and considering $L_1^* \otimes L_2^*$, can be extended the tensor $\otimes^{\mathbb{L}}$, to a total tensor product on the category $\mathrm{Ch}^- R(\mathcal{A})$. This could be the usual derived functor if $\otimes$, is well balanced and our construction is parallel. This last considering the corresponding homotopy.

Likewise, if $C \in \mathrm{Ch}^- R(\mathcal{A})$, then is quasi-isomorphism the application:

$$P \xrightarrow{\cong} C, \tag{8}$$

with$P$, a complex of projective objects. Any such complex P, is called a projective resolution of C, and therefore any other projective resolution of C, is homotopic chain to P.

Example. 2. 1. We can consider the simplicial Δ^3, and the factor diagram given by:



(9)

Then from $A \cong A'$, and $B \cong B'$, we can to have the equivalence between morphisms

$$A \to B \cong A' \to B', \tag{10}$$

Of fact, (10) is a A^1 –homotopy equivalence. Then we have the diagram figure 2.

Likewise, if $D \in \mathrm{Ch}^{-}R(\mathcal{A})$, and

$$Q \xrightarrow{\cong} D, \tag{11}$$

is a projective resolution too, we have with structure $\otimes^{\mathbb{L}}$, that:

$$C \otimes^{\mathbb{L}} D = P \otimes Q, \tag{12}$$

Then result several properties of extensions of mappings $C \otimes^{\mathbb{L}} D \to C \otimes D$, to the mappings

$$F \otimes^{\mathbb{L}} G = F \otimes G, \tag{13}$$

Other properties are obtained in the extension of projective resolutions.

The important result is given by the following proposition.

Proposition 2. 1. The derived category $D^{-}R(\mathcal{A})$, equipped with $\otimes^{\mathbb{L}}$, is a tensor triangulated category.

Proof. [3]. ∎

The definition of the category $D^{-}R(\mathcal{A})$, is the space of projective elements:

$$D^{-}R(\mathcal{A}) = \left\{C, D \in \mathrm{Ch}^{-}R(\mathcal{A}) | C \otimes^{\mathbb{L}} D = P \otimes Q, \text{where} \otimes \text{ implies } P \xrightarrow{\cong} G,\ Q \xrightarrow{\cong} D\right\}, \tag{14}$$

Likewise, the category $\wp$, of projective objects in $R(\mathcal{A})$, is additive, symmetric, monoidal and $D^{-}R(\mathcal{A})$, is equivalent to the chain homotopy category $K^{-}(\wp)$ [4, 5]. Likewise, this category is a tensor triangulated category under $\otimes$.

Then the conclusion of the proposition 2. 1, is followed from the natural isomorphism

$$\otimes \cong \otimes^{\mathbb{L}}, \tag{15}$$

in $\wp$, of $C \otimes^{\mathbb{L}} D \to C' \otimes^{\mathbb{L}} D'$.. This is seemed through the fact of that

$$A \otimes B \to A' \otimes B', \tag{16}$$

always that

$$A \otimes^{\mathbb{L}} B \to A' \otimes^{\mathbb{L}} B', \tag{17}$$

in the example.

Now in the étale sheaves context, also is obtained that the derived category of bounded above étale sheaves of R –modules with transfers is a tensor triangulated category.

III. DERIVED TRIANGULATED CATEGORIES WITH STRUCTURE BY PRE-SHEAVES $\otimes^{L}$, AND $\otimes^{tr}_{L,et}$.

The tensor product of the derived category of bounded above complexes of étale sheaves of R –modules $\otimes^{tr}_{L,ét}$, preserves quasi-isomorphisms. Also the category of bounded above complexes of étale sheaves of R –modules with transfers is a tensor triangulated category [6, 7].

In particular, and by a motives algebra in the derived category of étale sheaves of $\mathbb{Z}/m$ – module with transfers, the operation

$$m \to m(1) = m \otimes^{tr}_{L,ét} \mathbb{Z}/M(1), \tag{18}$$

is inversible. Then $\forall\ E, F$, are bounded above complexes of locally constant étale sheaves of R −module $E\otimes_{L,\acute{e}t}^{tr}\ F$, is quasi-isomorphic to $E\otimes_{R}^{\mathbb{L}}F$, which is their total tensor product of complexes of étale sheaves of R −modules. Indeed, we consider the morphism $f: E \rightarrow E'$, of bounded above complexes of presheaves of R −modules with transfers. Then in particular for étale sheaves we have $E_{\acute{e}t} \rightarrow E_{\acute{e}t}$, then we have

$$E\otimes_{L,\acute{e}t}^{tr}F \rightarrow E' \quad \otimes_{L,\acute{e}t}^{tr}F,$$

It is a quasi-isomorphism for F. Now if F, is a locally complete étale sheaf of R −modules then $E'\otimes_{L,\acute{e}t}^{tr}F, \rightarrow E\otimes_{L,\acute{e}t}^{tr}F$, is a quasi-isomorphism for every étale sheaf with transfers E. But $\otimes \cong \otimes^{\mathbb{L}}$, in $\wp$, and using the a natural mapping of presheaves given by $\lambda : h_X\otimes_{\acute{e}t}^{tr}h_Y \rightarrow h_{X\otimes_{\acute{e}t}^{tr}Y}$, where every $h_{X_i} = R(X_i)$, having the right exactness of $\otimes_{R,}$, and $\otimes_{\acute{e}t}^{tr}$, and being E, F, are bounded above complexes of locally constant étale sheaves of R −module then $E\otimes_{L,\acute{e}t}^{tr}F \rightarrow E\otimes_{R}^{\mathbb{L}}F$, is a quasi-isomorphism.

Similarly as with the étale sheaves, a presheaf with functors F, is a Nisnevich sheaf with transfers if its underlying presheaf is a Nisnevich sheaf on Sm/k. Clearly every étale sheaf with transfers is a Nisnevich sheaf with transfers. In motives with $\mathbb{Q}$ −coefficients with transfers we have result:

Lemma 3. 1. Let F, be a Zariski sheaf of $\mathbb{Q}$ −modules with transfers. Then F, is also an étale sheaf with transfers.

Proof. [3].

Then is deduced from theorem that characterizes the Nisnevich sheaves [2, 3,6] whose category $Sh_{Nis}(Cor_k)$, and the before lemma 3. 1, the following corollary.

Corollary 3. 1. If F, is a presheaf of $\mathbb{Q}$ −modules with transfers then $F_{Nis} = F_{\acute{e}t}$.

For other side, the construction of a derived category as such $\mathrm{DM}_{Nis}^{eff.-}(k, R)$, is parallel to the construction of $\mathrm{DM}_{\acute{e}t}^{eff.-}(k, R)$. If k, admits regularizations of singularities then $\mathrm{DM}_{\acute{e}t}^{eff.-}(k, R)$, allows us to extend motivic cohomology to all schemes of finite type as a cdh, hypercohomology group.

If $\mathbb{Q} \subseteq R$,we we showed that $\mathrm{DM}_{Nis}^{eff.-}(k, R)$, and $\mathrm{DM}_{\acute{e}t}^{eff.-}(k, R)$, are equivalent [3]. Likewise, $\mathrm{D}^- = \mathrm{D}^-\left(Sh_{\acute{e}t}(Cor_k, R)\right)$, is a derived category which is a tensor triangulated category. The same is applicable in the Nisnevich topology for derived category $\mathrm{D}^-\left(Sh_{Nis}(Cor_k, R)\right)$.

Likewise, $\forall\ C, D \in \wp$, and therefor in $\mathrm{Ch}^- R(\mathcal{A})$, we have:

$$C\otimes_{L,Nis}^{tr}D \cong \left(C\otimes_{L}^{tr}D\right)_{Nis}, \tag{19}$$

In particular the derived category D^-, of bounded above complexes of Nisnevich sheaves with transfers is a tensor triangulated category under $\otimes_{L,Nis}^{tr}$. Then by the proposition that says that $h_X = R_{tr}(X)$, is projective if

$$R_{tr}(X)\otimes^{tr}R_{tr}(Y) = R_{tr}(X \times Y), \tag{20}$$

Then we have in the motives context

$$m(X)\otimes_{L,Nis}^{tr}m(Y) = m(X \times Y), \tag{21}$$

Likewise, we can to define the category$\mathrm{DM}_{gm}^{eff}(k,R)$, to be the thick subcategory of $\mathrm{DM}_{Nis}^{eff.-}(k,R)$, generated by the motives $m(X)$, where X, is smooth over k. Objects in $\mathrm{DM}_{gm}^{eff}(k,R)$, are the effective geometric motives, which will be the objects that we require in our motivic cohomology, that we obtain for resolution of the decomposing of $X \times \mathrm{A}^1$ in in $\mathrm{A}^1 -$ homotopy of morphisms in the category Sm_k.

IV. DEVELOPMENT OF THE MOTIVIC COHOMOLOGY REQUIRED

From the lemma 3. 1, and corollary 3. 1, (considering the theorems of characterization of Nisnevich sheaves of $\mathbb{Q}-$ modules with transfers), and the following definition of Motivic cohomology on $\mathbb{A}(q)$ –coefficients:

Def. 4. 1. For any abelian group A, we define the étale (or Lichtenbaum) motivic cohomology of X, as the hypercohomology of $A(q)$,

$$H_L^{p,q}(X,A) = \mathbb{H}_{\acute{e}t}^{p}(X, A(q)|_{X_{\acute{e}t}}), \tag{22}$$

If $q < 0$, then $H_L^{p,q}(X,A) = 0$, due to that $A(q) = 0$. If $q = 0$, then

$$H_L^{p,0}(X,A) \cong H_{\acute{e}t}^{p}(X,A),$$

when$A(0) = A$. Also considerations on prime integers to the characteristic of k, are considered [2, 3].

Then the étale (or Lichtenbaum) motivic cohomology $H_L^{p,q}(X,\mathbb{Q})$, is defined to be the etale hypercohomology of the complex $\mathbb{Q}(q)$:

Theorem 4. 1. Let k, be a perfect field. If K, is a bounded above complex of presheaves of $\mathbb{Q}$ –modules with transfers, then $K_{Nis} = K_{\acute{e}t}$, and

$$\mathbb{H}_{\acute{e}t}^{*}(X, K_{\acute{e}t}) = \mathbb{H}_{Nis}^{*}(X, K_{Nis}), \tag{23}$$

for every X, in Sm/k. In particular we have $H_L^{p,q}(X,\mathbb{Q}) = H^{p,q}(X,\mathbb{Q})$.

Proof. [2, 3].■

Now we consider the vanishing of components of the right functors.

Under considerations of the before section the tensor product as presheaf of étale sheaves can have a homology space of zero dimension that vanishes in certain component to the right exact functor.

$$\Phi(F) = R_{tr}(Y) \otimes_{\acute{e}t}^{tr} F, \tag{24}$$

from the category PST (k,R), of pre-sheaves of R –modules with transfers to the category of étale sheaves of R –modules with transfers. Therefore, each derived functor given $L_n\Phi$, vanishes on homology space $H^0(\tilde{C})$, for certain étale complex [8, 9].

Therefore, all functors $R_{tr}(Y) \otimes_{\acute{e}t}^{tr} F$, are acyclic. By this way, is demonstrated the functor exactness and resolution in modules inducing the tensor product $\otimes_{L,\acute{e}t}^{tr}$, (tensor triangulated structure) to a derived category more general than $\mathrm{D}^{-}R(\mathcal{A})$.

We consider the following lemma concern to the vanishing of a presheaf F, of R –modules with transfers.

Lemma 4. 1. Fix Y, and we have (24). If F, is a presheaf of of R −modules with transfers such that $F_{ét} = 0$, then $L_n\Phi(F) = 0$, $\forall n$.

The geometrical motives required in our research are a result of embeds the derived $\mathrm{DM}_{gm}^{-}(k, R)$, (geometrical motives category) in the derived category $\mathrm{DM}_{\text{ét}}^{eff,-}(k, \mathbb{Z}/m)$, considering the category of smooth schemes on the field k.

Also as discussed and exposed in [8] all functor $L_{\mathbb{A}^1} \in \mathrm{DM}_{-}^{eff}(k)$, induces a tensor operation on the category $\mathrm{D}_{\mathbb{A}^1}^{-}(\mathrm{Sh}^{Nis}(Cor(k)))$, making that itself is a tensor triangulated category. Likewise, explicitly in $\mathrm{DM}_{-}^{eff}(k)$, this give us the functor:

$$m: Sm_k \to \mathrm{DM}_{-}^{eff}(k), \tag{25}$$

V. RESULT

Under several considerations and studies realized in the book chapter [8] and the motivic cohomology treatment given in [2, 3, 6, and 10] as the embedding theorem in $\mathrm{DM}_{\text{ét}}^{eff}(k)$, we can consider the following triangulated diagram:

$$\begin{array}{ccc} Sm_k & \to & \mathrm{DM}_{\text{ét}}^{eff}(k) \\ m \searrow & & \downarrow \mathrm{Id}, \\ & \mathrm{DM}_{\text{ét}}^{eff}(k) & \end{array} \tag{26}$$

which has implications in the geometrical motives applied to bundle of geometrical stacks in mathematical physics, as has been studieded and showed in [8, 11,12].

Theorem 5. 1 (F. Bulnes). Suppose that $\mathbb{M}$, is a complex Riemannian manifold with singularities. Let X, and Y, be smooth projective varieties in $\mathbb{M}$[2]. We know that solutions of the field equations $dda = 0$, [8, 11, and 12] are given in a category $\mathrm{Spec}(Sm_k)$, (see [11]). Solution context of the quantum field equations for $dda = 0$, is defined in hypercohomology on $\mathbb{Q}$ −coefficients from the category Sm_k, defined on a numerical field k, considering the derived tensor product $\otimes_{\text{ét}}^{tr}$, of presheaves. Then the following tensor triangulated diagram is true and commutative:

$$\begin{array}{ccc} & \mathrm{DQFT} & \\ i \swarrow & & \searrow F, \\ \mathrm{DM}_{gm}(\mathbb{Q}) & \to & \mathrm{DM}(\mathfrak{O}_Y) \end{array} \tag{27}$$

Proof. [8].■

The category $\mathrm{DM}_{gm}^{eff}(k, R)$, has a tensor triangulated structure and the tensor product of its motives is $m(X) \otimes m(Y) = m(X \times Y)$. Remember that the triangulated category of geometrical motives $\mathrm{DM}_{gm}(k, R)$, is defined formally inverting the functor of the Tate objects, which are objects of a motivic category called Tannakian category [12].

We enunciate the following result important in the technical detail of the topologies required to DQFT.

Theorem 5. 2. If $\mathbb{Q} \subseteq R$, then

[2] Singular projective varieties useful in quantization process of the complex Riemannian manifold. The quantization condition compact quantizable Käehler manifolds can be embedded into projective space.

$$\omega: \mathrm{DM}^{eff,-}_{Nis}(k,R) \to \mathrm{DM}^{eff,-}_{\acute{e}t}(k,R) \tag{28}$$

is an equivalence of tensor triangulated categories.

Proof. [12]. ■

We want to apply the considerations of before sections to give a tensor triangulated category to a quantum version of motivic cohomology on étale Sheaves, from $\Delta^3 -$ simplicial that shows the $\mathbb{A}^1$ –homotopy in an approximate triangulated category $\mathrm{DM}^{eff,-}_{Nis}(k,R)$, which for every Nisnevich sheaf with transfers that is an every étale sheaf with transfers, is a category $\mathrm{DM}^{eff,-}_{\acute{e}t}(k,R)$. The Nisnevich detail in the derived category is due to the importance in motivic homotopy theory of that the objects of interests are "spaces", which are simplicial sheaves of sets on the big Nisnevich site that is the category Sm/k.

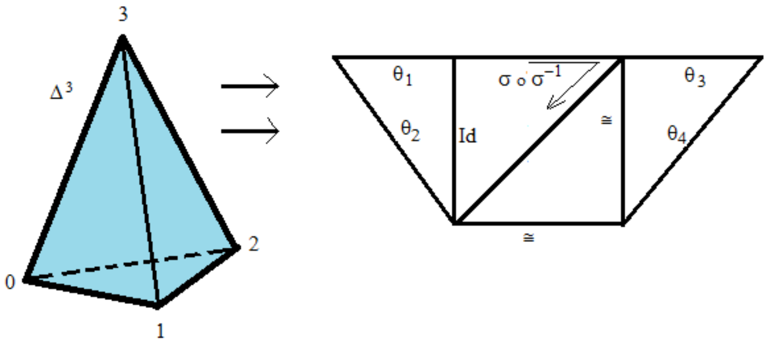


Figure 4: 2-Simplicial decomposition of $\Delta^3 \times \mathbb{A}^1$, for DQFT.

In reality we consider two topologies for aspects of localization and covering.

We have the following commutative diagram in the geometrical motives context that are useful to link the derived category DQFT.

Lemma 5. 1. The following diagram is commutative

$$\begin{array}{c} Sm_k \xrightarrow{i'} \mathrm{DM}^{eff}_{gm}(k) \xrightarrow{\sigma} \mathrm{DM}_{gm}(k) \xleftarrow{i} \mathrm{DQFT} \\ m \searrow \quad \updownarrow \mathrm{Id} \quad \sigma \swarrow \updownarrow \cong \quad \swarrow F, \\ \mathrm{DM}^{eff}_{gm}(k) \xrightarrow{\cong} \mathrm{DM}(\mathfrak{D}_Y) \end{array} \tag{29}$$

Proof. [13]. ■

The following aspects were considered before of its demonstration.

We say that a diagram in Cor_k, is homotopy commutative if every pair of composites $f, g: X \to Y$, ith the same source and target are $\mathbb{A}^1$ –homotopic. Any homotopy invariant presheaf with transfers identifies $\mathbb{A}^1$ – homotopic maps, and converts a homotopy commutative diagram into a commutative diagram.

We consider $\mathrm{QFT} \xrightarrow{i} \mathrm{DM}_{gm}(k) \xrightarrow{\sigma} DM^{eff}_{gm}(k)$, which is zero (see lemma 21.9 [3, 13]). Very helpful was the fact of the singular homology [14] to start $Cor_k/\mathbb{A}^1 -$ homotopy.

An corollary of the diagram (29) can be the re-interpretation from the étale sheaves and Simplicial decomposition of $\Delta^3 \times \mathbb{A}^1$, for DQFT, considering their spectrum of its singular homology. We consider from category of motives the following proposition worked in [15]:

Proposition 5. 1. If X, is any scheme of finite type over k, then

$$H_n^{sing}(X,R) \cong H_{n,0}(X,R), \tag{30}$$

Proof. [15].■

In the demonstration of (30) is considered the hyper-cohomology groups $\mathbb{H}^*_{Nis}(\mathrm{Spec}k, K) = H(K(\mathrm{Spec}k))$, which represent the spectrum of the corresponding singular homology. This spectrum can be a projective vector bundle used to work singularities. Oscillations and singularities can be the same in motivic cohomology? The answer is yes, although in duality.

Also is very helpful the following theorem.

Theorem 5. 3 (Projective Bundle Theorem). Let $p: \mathbb{P}(\mathcal{E}) \to X$, be a projective bundle associated to the vector bundle $\mathcal{E}$,of rank $n+1$.Then the canonical mapping

$$\oplus_{i=0}^{n} \mathbb{Z}_{tr}(X)(i)[2i] \to \mathbb{Z}_{tr}\big(\mathbb{P}(\mathcal{E})\big), \tag{31}$$

is an isomorphism in the category $\mathrm{DM}_{gm}^{eff}(k)$, and p,is the projection onto the factor $\mathbb{Z}_{tr}(X)$.

Proof. [15].■

Likewise we have the orthogonalizing composition[3]:

$$\mathbb{Z}_{tr}(\mathbb{P}_k^n) = \oplus_{i=0}^{n} \mathbb{Z}(i), \tag{32}$$

Oscillations and singularities can be the same in motivic cohomology?

We consider the following theorem proved in [12].

Theorem (F. Bulnes) 5. 3.$H^*(GL(n,k))$ has the decomposing in components $H^i(X)$, that are hyper-cohomology groups corresponding to solutions as $\mathbf{H}$ –states in $\mathrm{Vec}_{\mathbb{C}}$,for field equations $\mathrm{d}da = 0$.

Proof. [12].

In the before theorem was proved that the oscillations of **H**-states are the solutions of a big field equations class where these solutions are hyper-cohomology groups to superposition of **H**-states, considering a Hitchin base [12, 16, 17]. By duality in field theory, particle and wave are equivalent. Then oscillations are singularities in the space-timetoo. In our category of motives are undistinguishable. This can be demonstrated in terms of singular cohomology considering the proposition 5. 1, where is clear that:

$$H_n^{sing}(X,R) = \mathbb{H}_{Nis}^{-n}(\mathrm{Spec}k, C_* R_{tr}(X)), \tag{33}$$

Then considering the proposition 5. 1, the theorem 5. 3, and the A^1 – homotopy, between σ, and Id, onto its diagram, we can give a version of the theorem 4. 2, in the context of the group $SL_n(k)$,on $C_*\mathbb{Z}_{tr}(\mathbb{A}^n - 0)$,which is chain homotopic to the trivial action.

Corollary 4. 1. The action of the symmetric group Σ_n, on $\mathbb{Z}(n)$, is $\mathbb{A}^1$ –homotopic to the trivial action. Hence it is trivial in the category $\mathrm{DM}_{Nis}^{eff,-}(k)$, and on the motivic cohomology (hyercohomology) $\mathbb{H}^r(X, \mathbb{Z}(n))$.

[3] $\mathbb{Z}(n)$, is the motivic complex of singularities whose dual in hypersurfaces in amanifold (that our case we want with complex Riemannian with singularities) is the projective space $\mathbb{P}^n$.

Proof. [15]. ■

Theorem 4. 4. We consider $H^*(SL(n,k))$. This has a decomposing in components $\mathbb{Z}(i)[2i]$, [4]that are hypercohomology groups to solutions as **H**-states in $\mathrm{Vec}_{\mathbb{P},}$, to field equations $\mathrm{d}da = 0$.on singularities.

Proof. We consider the last triangle directly from diagram (29):

$$\begin{array}{ccc} \mathrm{DM}_{gm}(k) & \overset{i}{\leftarrow} & \mathrm{DQFT} \\ \updownarrow \cong & \swarrow F, & \\ \mathrm{DM}(\mathfrak{D}_Y) & & \end{array} \tag{34}$$

and we express this in the context of the singular homology components $\mathbb{Z}(i)[2i]$, and using its Spec relation given by (33) we have the triangle:

$$\begin{array}{c} H_n^{sing}(\mathrm{Spec}k, \mathbb{Z}) \sim \mathbb{Z}(n)[2n] \\ \gamma \ \swarrow \qquad \nwarrow \tilde{\gamma}, \\ C_*\mathbb{Z}_{tr}(\mathbb{A}^n)/\mathbb{Z}_{tr}(\mathbb{A}^n - 0) \overset{\cong}{\rightarrow} C_*\mathbb{Z}_{tr}(\mathbb{P}^n)/\mathbb{Z}_{tr}(\mathbb{P}^{n-1}) \cong \\ C_*\mathbb{Z}_{tr}(\mathbb{G}_m^{\wedge n})[n] \end{array} \tag{35}$$

But

$$M(\mathbb{P}^n) = \oplus_{i=0}^{n} \mathbb{Z}(i)[2i], \tag{36}$$

which are in the space $C_*\mathbb{Z}_{tr}(\mathbb{P}^n)$. Then by the corollary 4. 1, the action of Σ_n, on $\mathbb{A}^n$, extends to an action on $\mathbb{P}^n$, fixing $\mathbb{P}^{n-1}$. Then all states are in $\mathrm{Vec}_{\mathbb{P}}$. Finally we can consider $\mathrm{DQFT} \overset{i}{\rightarrow} \mathrm{DM}_{gm}(k) \overset{\sigma}{\rightarrow} DM_{gm}^{eff}(k)$, in the triangle context (34). Then can to define solutions in $\Omega^1[\mathbf{H}]$, due to that, we need solutions for $\mathrm{d}da = 0$. as cotangent vectors [12]. But this is obtained in the derived category $\mathrm{DM}(\mathfrak{D}_Y)$. ■

V. APPLICATIONS.

Example 6. 1. Rotations around of some vertex (sources) produce oscillations of **H**-states which in presence of electromagnetic fields or only one magnetic field produce a field torsion accompanied of gravitational waves. To quantum gravity, we want obtain a spectrum in the dual $\widehat{T}Bun_G$,[5] considering the triangle given in (26), whose geometrical motives will be stacks of holomorphic bundles.

Example 6. 2. In much topological models of the space-time, are proposed some types of algebraic tools based on schemes in which the discrimination of singularities within objects is based on the space-time-spin group $SL(n,k)$ [17, 18]. Such topological objects possess an homotopy structure encoded in their fundamental group and the related $SL(n,k)$,multivariate polynomial character variety contains a plethora of singularities somehow analogous to the frequency spectrum in time structures [17].

Example 6. 3.In the QFT-applications, the singular homology groups of $\Delta^3 \times A^1$, for DQFT, are dual to the corresponding **H**-states in $\mathrm{Vec}_{\mathbb{C}}$, to the motivic co homology corresponding to the representation of the cosmic Galois group[6] as was demonstrated in the theorem 4. 2, [13].

[4] These are the Spec of the corresponding Chow groups. We consider the following *Corollary* [15]. There is quasi-isomorphism

$$M(\mathbb{P}^n) = C_*\mathbb{Z}_{tr}(\mathbb{P}^n) \rightarrow \mathbb{Z} \oplus \mathbb{Z}(1)[2] \oplus, \ldots, \oplus \mathbb{Z}(n)[2n].$$

[5] Dual image of the lines bundle which is divisor of holomorphic bundles. This is stack.

[6] $K_{2n-1}(\mathbb{K}) \otimes \mathbb{Q}, = \mathrm{H}_\bullet(GL(n,k))$, is the linear group of entries in k.

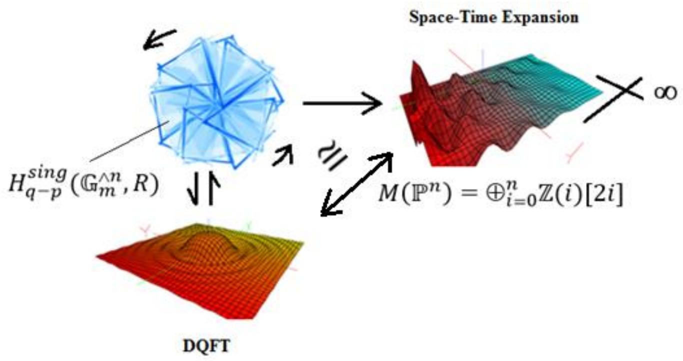


Figure 5: Triangle (35) and the Chow ring of the hypersurface modeled considering the space-time expansion, we will consider this sequence as a sequence of coherent sheaves in $\mathbb{P}^n$. The Cohomology of coherent sheaves is the same that the cohomology to étale sheaves.

VI. CONCLUSIONS

This help us to have a quantum field theory of simplicial geometry and construct model of the Universe on the simplicial frameworks and establish morphism of homotopy commutative relations which can induce to a hypercohomology to the solution of some field equations and aspects of gravitationat least in a microscopic level. For example, we consider ones of the field theories as the Schwinger-Dyson equation in three-dimensional simplicial quantum gravity, established by the paper novedous triangle relations and absence of Tachyons in Liouville string field theory [19], where could be contained in the derived category $\mathrm{DM}(\mathfrak{D}_Y)$, or the diagrams of the Polyakov string theory [20], with Polyakov integrals as intertwining operators between strings and particles (sources as vertices); can be used the simplicial geometry and its decomposition in triangulated diagrams of schemes belonging to the category Sm_k, and morphisms between schemes of the category Cor_k, all with the total tensor product on the category $\mathrm{PSL}(k)$,as example its component elements $\mathbb{Z}_{tr}(k)$, to obtain the generalizations on derived categories using sheaves (étale or Nisnevich) or pre-sheaves and contravariant and covariant functors on additive categories to define the exactness of infinite sequences and resolution their spectral sequences. The advantages from the tensor triangulated category to a quantum version considering a motivic cohomology on étale Sheaves is the respective factorization algebras in QFT, where is necessary consider the combined observation measures from many components with an commutative property for their diagrams between their derived categories. Likewise the theorem 4. 4, the $\mathbb{A}^1$ –homotopy in theaction of the symmetric group on the derived category $\mathrm{DM}_{Nis}^{eff,-}(k)$,is trivial and on corresponding motivic hypercohomology too.

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Human Well-Being in the Age of Artificial Lighting

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I. CIRCADIAN SYSTEM, MELATONIN, CHRONODISRUPTION, AND EXTERNAL INPUTS

The human visual system consists of delicate diverse cells that take external information from the surroundings to provide internal responses via a delicate input-response circle manner. It is widely known that rods and cones are the main input cells for the visual system. However, a third set of cells that show higher photosensitivity compared to rods and cones are discovered in the biome which plays a crucial role in providing input to the circadian system. These third sets of cells are widely known as intrinsically photosensitive ganglion cells (ipRGCs). Furthermore, these cells show peak sensitivity to short-wavelength light at 480nm. Due to their lower quantity compared to rods and cones and their significant role in the human circadian system, molecular protection of ipRGCs is crucially important to have a healthy circadian system.

The circadian system or rhythm (CS) is controlled by the suprachiasmatic nuclei (SCN) section of the brain and plays a crucial role in the homeostasis of the body by controlling the secretion of melatonin hormone. Disruption of the circadian system has been linked to a number of physiological consequences, such as sleep-wake disorders, cardiovascular disease, immunological disorders, metabolic disorders, obesity, and cancer progression (Zubidat & Haim, 2017, p. 295-313). Melatonin secretion reaches its peak levels at night and decreases to its lowest point during the day. Hence, one can think of melatonin as the hormone of the darkness.

The input of the light is transferred from the external sources to the SCN via the hypothalamic tract according to the input received by ipRGCs. This connection is important as any ill time input from ipRGCs significantly affects the secretion of melatonin. Recently artificial light at night (ALAN) has been the most prominent input for the disruption of the circadian rhythm therefore for the secretion of the melatonin. Furthermore, in 2013 American Medical Association (AMA) declared light at night as environmental pollution due to its consequences on melatonin secretion (Haim & Zubidat, 2015). This assessment from AMA shows that ALAN can be a dangerous input for human well-being by creating significant misalignment in the circadian system causing ill-timed behavioral and psychological responses and not properly responding the environmental changes. An interesting correlation has been shown between the recent pandemic COVID-19 and the damaged circadian system in both bats and humans. From the bat's perspective, the caged animals are being exposed to constant artificial lighting in animal markets which disrupts healthy circadian rhythm and prevents the full recovery and reduction of the oxidative stress. Eventually, this circle results in genetic mutation of the diseases that animals are carrying. At the other end of the spectrum from a human's perspective, a disrupted circadian rhythm results in a more vulnerable immune system and eventually creates a host for genetically mutated viruses such as COVID-19 (Khan et al., 2020).

Furthermore, it must be noted that the circadian feedback loop is present not only in the pineal gland but also in all cells in peripheral tissues such as the heart, spleen, lung, liver, endocrine glands, and rest of the organs (Zubidat & Haim, 2017, p. 295-313). Any disruption of circadian rhythm can cause bigger health problems if not taken seriously.

Commonly studies related to the ALAN are being performed on rodents, however, it is reported that SCN in humans has between two and a half to four times fold neurons compared to rodents which have between 8000 and 20000 neurons at the SCN legion (Fonken & Nelson, 2014, p. 648-670). This shows that the effects of the ALAN and circadian disruption might be more significant for humans compared to the data sets that are obtained from rodent-based studies. Furthermore, it must be noted that recent studies show that artificial light even dim indoor lighting, especially short-wavelength, can still penetrate the eyelid and disrupt the circadian entrainment by entraining the sensitive ipRGCs in healthy humans and even can show negative effects on totally blind humans (Haim & Zubidat, 2015). This shows significant negative effects of the ALAN even in indoor settings regardless of the higher illuminance levels that have been associated with the disruption of the healthy circadian rhythm. It must be noted that regardless of the timing of the exposure to higher illuminance levels as well as exposure to artificial lighting due to the night shift work or lack of darkness can create cumulative effects of a severely damaged circadian system while causing retinal damage or advancing diseases that are related to high oxidative stress (Contín et al., 2015, p. 255-263). This correlation can be explained by the antioxidant properties of melatonin and its importance in adjusting the redox status of mitochondria. Furthermore, it is reported that melatonin can scavenge up to ten reactive oxygen species (ROS) and reactive nitrogen species (RNS) compared to traditional antioxidant pills which are only able to scavenge a few ROS (Minich et al., 2022). This enhanced role of melatonin in the reduction of cellular oxidative stress and helping to keep the homeostatic level can be crucial for the treatment and/or prevention of diseases such as cancer, Alzheimer's and Parkinson's Diseases. Two decades ago Lissoni et al.'s research on cancer patients showed that 20mg of intramuscular injection of melatonin and 10mg daily oral dose as a follow-up was effective in controlling the tumor growth as well as improving the life quality of the patients (Minich et al., 2022). However, it must be noted that the amount of taken dose for melatonin must be decided by a health professional in order to provide the correct dosage use of the melatonin supplements. Unfortunately, it is widely believed that taking a higher dosage of melatonin supplements that are commercially available would provide more benefits in rapid response, however, it must be noted that this belief can create negative effects on the natural secretion of the melatonin hence damaging the natural circadian rhythm.

Aside from cancer or neurodegenerative diseases, it is also noted that the misaligned or disrupted circadian rhythm causes the development of Type II Diabetes Mellitus (T2DM) in laboratory conditions in fat sand rats (Bilu et al., 2022). Further epidemiological studies show that night shift work significantly increases T2DM compared to non-night shift work conditions by five percent per every five years of night shift work. Furthermore, rotating shifts are further increasing T2DM possibility due to the excessive misalignment of the circadian system (Bilu et al., 2022). Also, recent studies noted that being exposed to light in the spectrum between 415 to 465nm can over-activate the opsin3 protein which regulates melanin production and the persistent pigment darkening and hence hyperpigmentation, which used to be considered UVA exposure dependent (de Gálvez et al., 2022). Also, it is noted that light that has a peak spectrum below 453nm increases oxidative stress equivalent to one-fourth of the oxidative stress that is caused by UVA exposure (Nakashima, Ohta, & Wolf, 2017). Looking at Cardiovascular Diseases, sleep disorders, oxidative stress, and circadian misalignment, the common problem found to be an increase in oxidative stress at the cellular level in the biological system (Wei et al., 2022, p. 297-305). As one of the most important roles of melatonin in reducing oxidative stress and the proven correlation between oxidative stress and detrimental diseases, the healthy secretion regulation of melatonin by the circadian rhythm inputs becomes more important than it's ever thought to be.

II. CURRENT DOCUMENTATION OF ARTIFICIAL LIGHT AT NIGHT VALUES AND CORRELATION OF LED USAGE

Considering those noted crucial effects of melatonin on human well-being and the sensitive nature of the circadian system to light we must consider the level of light pollution that we are experiencing in the modern world. It is noted that one-fifth of the European population and almost every individual in the US is living under excessively night polluted sky and being exposed to ALAN (Falchi et al., 2016). This shows that almost every individual human is experiencing a chronodisruption to a certain degree due to even dim light being able to penetrate eyelids and alter the healthy circadian rhythm.

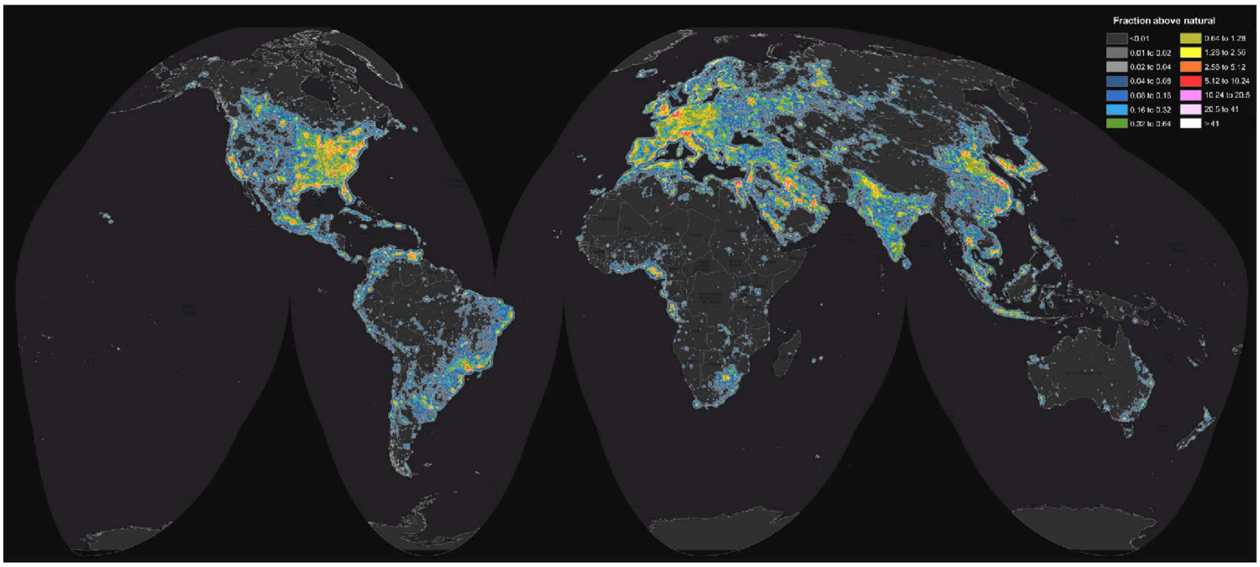


Figure 1: World map of artificial sky brightness (Falchi et al., 2016)

Generally, light pollution maps are being created with the information captured by the Suomi International Polar-orbiting Partnership (NPP) due to its reliability at the band of Visible Infrared Imaging Radiometer Suite (VIIRS). However, one-third of the radiant power of the white LED is not able to be captured by the day/night band of the VIIRS due to the short wavelength of the LED being out of the spectral curve of the day/night band (Cao & Bai, 2014, p. 11915-11935). Even though significant steps are being taken by governing bodies to prevent an increase in light pollution and ALAN, the rapidly increasing use of LED-based luminaires will increase light pollution to higher levels. One can argue that the current luminaires that are in use have been tested to prevent direct illumination to the sky to prevent further increase of light pollution, however, the current system at NPP is not able to capture the correct information as explained earlier.

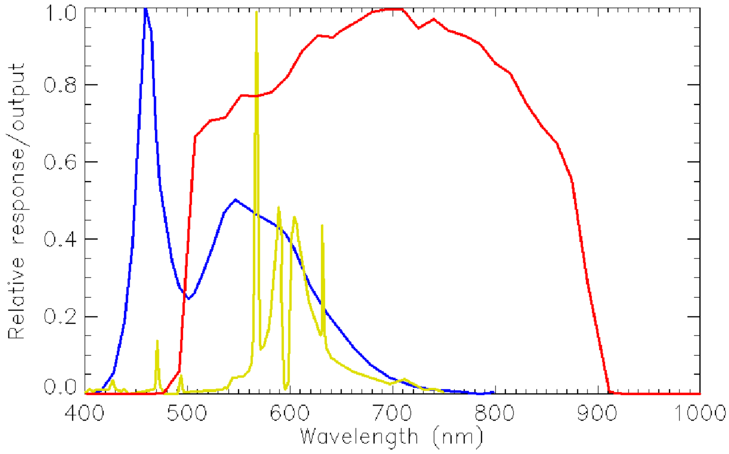
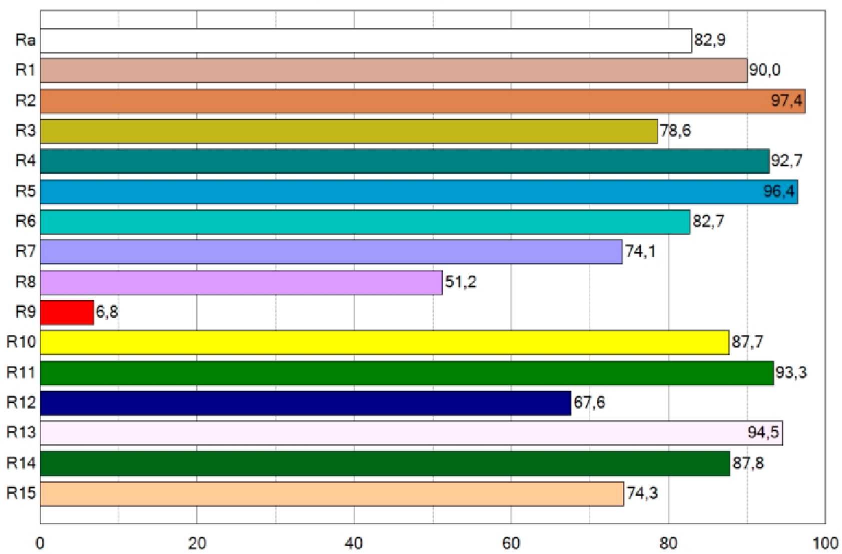


Figure 2: Spectral response curve of DNB (red), spectral power distribution curve of LED (blue), and spectral power distribution curve of High Pressure Sodium (yellow) (Cao & Bai, 2014, p. 11915-11935).

On the other hand, in one clinical research condition that included sixteen males and sixteen female juvenile monkeys to assess myopia development under 12-hour light and 12-hour dark conditions using an incandescent lamp (2700K CCT), LED fixtures 3000K, 4000K, and 5000K CCT as light sources which provided 50fc (500lux) illuminance at the center of the cages, researchers concluded that higher CCT light sources affected the elongation of the eye physiology hence creating a higher possibility for myopia development after only one year of exposure. The shortest elongation is noted for the test group which was located in the room that is illuminated by incandescent lamps (Hu et al., 2022, p. 229-233). This further proves that higher irradiance values in short-wavelength have significant effects on human well-being.

III. POSSIBLE FUTURE TECHNOLOGY FOR ARTIFICIAL LIGHTING

Current technology highly depends on LED technology due to its higher energy efficiency and compact size compared to legacy light sources. However high peak of irradiance at a shorter wavelength section can be harmful to human well-being to a degree and detrimental to health hence it is important to improve the spectral power distribution of the LED with combinations of newly developing technology such as quantum dots and organic dyes. According to a study that is conducted by Menéndez-Velázquez et al., by using a multilayer luminescent molecular system approach the team was able to create a white LED that had 2187 CCT and 82.9 CRI (Menéndez-Velázquez et al., 2022).



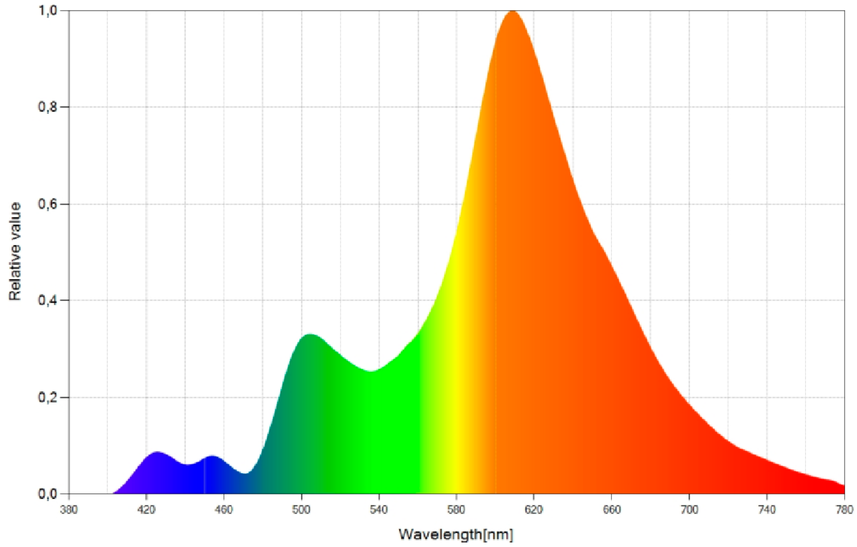


Figure 3 & 4: Spectral Power Distribution and CRI values of the Multilayer Luminescent Molecular System white LED (Menéndez-Velázquez et al., 2022).

It must be noted that the R9 value of the created LED is showing a significantly low value and that would affect the visual perception of the objects that are illuminated by this light source, this experiment shows a promising future for the reduction of the short wavelength irradiance values of the LED-based luminaire. Furthermore, the development of this technology might reduce the negative effects of the current LEDs on human well-being.

IV. CONCLUSION

We are currently living in a world where our bodies are under constant input from external resources. Artificial light is the most prominent input to the well-being of humans by affecting the circadian rhythm causing misalignments and increasing the risk of getting exposed to health concerning diseases like cancer and cardiovascular diseases. The current technology and rapid adaptation to LED-based luminaires are increasing these detrimental health effects for humans. Recent studies are showing that short-wavelength sections of the LEDs can be curbed by using additional layers of quantum dots and organic dyes to create less invasive artificial lighting for well-being firstly for humans and secondly for the rest of the fauna and flora. The development of this new style of white LEDs is dependent on further research.

Conflicts of Interest

The author declares no conflict of interest.

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Quasi-P-Normal and n-Power Class Q Composite Multiplication Operators on the Complex Hilbert Space

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ABSTRACT

In this paper, the condition under which composite multiplication operators on $L^2(\mu)$ -space become Quasi-P-Normal operators and n-Power class Q operator have been obtained in terms of radon-nikodym derivative f_0.

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ABSTRACT

In this paper, the condition under which composite multiplication operators on $L^2(\mu)$-space become Quasi-P-Normal operators and n-Power class Q operator have been obtained in terms of radon-nikodym derivative f_0 .

Keywords: composite multiplication operator, conditional expectation, Quasi-p-normal, multiplication operator, class Q operator.

I. INTRODUCTION

Let (X,Σ,μ) be a σ-finite measure space. Then a mapping T from X into X is said to be a measurable transformation if $T^{-1}(E)\in\Sigma$ for every $E\in\Sigma$.A measurable transformation T is said to be non-singular if $\mu(T^{-1}(E))=0$ whenever $\mu(E)=0$. If T is non-singular then the measure μT^{-1} defined as $\mu T^{-1}(E)=\mu(T^{-1}(E))$ for every E in Σ , is an absolutely continuous measure on Σ with respect to μ .Since μ is a σ-finite measure, then by the Radon-Nikodym theorem, there exists a non-negative function f_0 in $L^1(\mu)$ such that $\mu T^{-1}(E)=\int_E f_0\,d\mu$ for every $E\in\Sigma$. The function f_0 is called the Radon-Nikodym derivative of μT^{-1} with respect to μ.

Every non- singular measurable transformation T from X into itself induces a linear transformation C_T on $L^p(\mu)$ defined as $C_T f = f\circ T$ for every f in $L^p(\mu)$. In case C_T is continuous from $L^p(\mu)$ into itself, then it is called a composition operator on $L^p(\mu)$ induced by T. We restrict our study of the composition operators on $L^2(\mu)$ which has Hilbert space structure. If u is an essentially bounded complex-valued measurable function on X, then the mapping M_u on $L^2(\mu)$ defined by $M_u f = u\cdot f$, is a continuous operator with range in $L^2(\mu)$. The operator M_u is known as the multiplication operator induced by u.
A composite multiplication operator is linear transformation acting on a set of complex valued Σ measurable functions f of the form

$$M_{u,T}(f)=C_T M_u(f)=u\circ T\ \ f\circ T$$

Where u is a complex valued, Σ measurable function. In case $u=1$ almost everywhere, $M_{u,T}$ becomes a composition operator, denoted by C_T.

In the study considered is the using conditional expectation of composite multiplication operator on L^2 -spaces. For each $f\in L^p(X,\Sigma,\mu)$, $1\le p\le\infty$, there exists an unique $T^{-1}(\Sigma)$-measurable function $E(f)$ such that

$$\int_A g f\,d\mu=\int_A g E(f)\,d\mu$$

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for every $T^{-1}(\Sigma)$-measurable function g, for which the left integral exists. The function $E(f)$ is called the conditional expectation of f with respect to the subalgebra $T^{-1}(\Sigma)$. As an operator of $L^p(\mu)$, E is the projection onto the closure of range of T and E is the identity on $L^p(\mu)$, $p \geq 1$ if and only if $T^{-1}(\Sigma) = \Sigma$. Detailed discussion of E is found in [1-4].

1.1 Normal operator

Let H be a Complex Hilbert Space. An operator T on H is called normal operator if $T^*T = TT^*$

1.2 Quasi-normal operator

Let H be a Complex Hilbert Space. An operator T on H is called Quasi-normal operator if $T\,T^*T = T^*T\,T$ ie, T^*T commute with T

1.3 Quasi p-normal operator [13]

Let H be a Complex Hilbert Space. An operator T on H is called Quasi-normal operator if $T^*T\left(T+T^*\right)=\left(T+T^*\right)T^*T$

1.4 Power -normal operator

Let H be a Complex Hilbert Space. An operator T on H is called 2 power-normal operator if $T^2\,T^* = T^*T^2$

1..5 Class Q-operator [14]

Let H be a Complex Hilbert Space. An operator T on H is called Quasi-normal operator if $T^{*2}\,T^2 = \left(T^*\,T\right)^2$

II. RELATED WORK IN THE FIELD

The study of weighted composition operators on L^2 spaces was initiated by R.K. Singh and D.C. Kumar [5]. During the last thirty years, several authors have studied the properties of various classes of weighted composition operator. Boundedness of the composition operators in $L^p(\Sigma)$, $(1 \leq p < \infty)$ spaces, where the measure spaces are σ-finite, appeared already in [6]. Also boundedness of weighted operators on $C(X,E)$ has been studied in [7]. Recently S. Senthil, P. Thangaraju, Nithya M, Surya devi B and D.C. Kumar, have proved several theorems on n-normal, n-quasi-normal, k-paranormal, and (n,k) paranormal of composite multiplication operators on L^2 spaces [8-12]. In this paper we investigate composite multiplication operators on $L^2(\mu)$-space become Quasi-P-Normal operators and n-Power class Q operator have been obtained in terms of radon-nikodym derivative f_0.

III. CHARACTERIZATION ON COMPOSITE MULTIPLICATION OF QUASI P NORMAL OPERATORS ON L^2 SPACE

3.1 Proposition

Let the composite multiplication operator $M_{u,T} \in B(L^2(\mu))$. Then for $u \geq 0$

(i) $M^*_{u,T}\,M_{u,T}\,f = u^2 f_0\,f$

(ii) $M_{u,T}\,M^*_{u,T}\,f = u^2 \circ T \cdot f_0 \circ T \cdot E(f)$

(iii) $M^n{}_{u,T}(f) = (C_T M_u)^n(f) = u_n \ (f \circ T^n)$, $u_n = u \circ T . u \circ T^2 . u \circ T^3 .... u \circ T^n$

(iv) $M^*{}_{u,T} \ f = u f_0 \cdot E(f) \circ T^{-1}$

(v) $M^{*^n}{}_{u,T} \ f = u f_0 \cdot E(u f_0) \circ T^{-(n-1)} \cdot E(f) \circ T^{-n}$

where $E(u f_0) \circ T^{-(n-1)} = E(u f_0) \circ T^{-1} \cdot E(u f_0) \circ T^{-2} .... E(u f_0) \circ T^{-(n-1)}$

Theorem 3.1

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then the following statements are equivalent

(i) $M_{u,T}$ is Quasi p-normal operator

(ii) $u \circ T \ u^2 \circ T \ h \circ T \ f \circ T + h u E\left(h u^2 f\right) \circ T^{-1} = h u^2 \ u \circ T \ f \circ T + h^2 \ u^3 \ E(f)$

Proof:

For $f \in L^2(\mu)$, $M_{u,T}$ is Quasi P-normal operator if

$\left(M_{u,T} + M^*{}_{u,T}\right)\left(M^*{}_{u,T} M_{u,T}\right)f = \left(M^*{}_{u,T} M_{u,T}\right)\left(M_{u,T} + M^*{}_{u,T}\right)f$ and we have,

$$\left(M_{u,T} + M^*{}_{u,T}\right)\left(M^*{}_{u,T} M_{u,T}\right)f = M_{u,T}\left(M^*{}_{u,T} M_{u,T}\right)f + M^*{}_{u,T}\left(M^*{}_{u,T} M_{u,T}\right)f$$
$$= M_{u,T} \ M^*{}_{u,T} \left(u \circ T \ f \circ T\right) + M^*{}_{u,T} \ M^*{}_{u,T} \left(u \circ T \ f \circ T\right)$$
$$= M_{u,T}\left[h u E\left(uf \circ T\right) \circ T^{-1}\right] + M^*{}_{u,T}\left[h u E\left(uf \circ T\right) \circ T^{-1}\right]$$
$$= M_{u,T}\left[h u^2 f\right] + M^*{}_{u,T}\left[h u^2 f\right]$$
$$= u \circ T\left(h u^2 f\right) \circ T + h u E\left(h u^2 f\right) \circ T^{-1}$$
$$= u \circ T \ u^2 \circ T \ h \circ T \ f \circ T + h u E \left(h u^2 f\right) \circ T^{-1}$$

Consider

$$\left(M^*{}_{u,T} M_{u,T}\right)\left(M_{u,T} + M^*{}_{u,T}\right) f = \left(M^*{}_{u,T} M_{u,T}\right) M_{u,T} \ f + \left(M^*{}_{u,T} M_{u,T}\right) M^*{}_{u,T} \ f$$
$$= \left(M^*{}_{u,T} M_{u,T}\right)\left(u \circ T \ f \circ T\right) + \left(M^*{}_{u,T} M_{u,T}\right)\left(h u E(f) \circ T^{-1}\right)$$
$$= M^*{}_{u,T} \ u \circ T \ \left(u \circ T \ f \circ T\right) \circ T + M^*{}_{u,T} \ u \circ T \ \left(h u E(f) \circ T^{-1}\right) \circ T$$
$$= h u E\left[u \circ T \ u \circ T^2 \ f \circ T^2\right] \circ T^{-1} + h u E\left[u \circ T \ h \circ T \ u \circ T \ E(f)\right] \circ T^{-1}$$
$$= h u^2 \ u \circ T \ f \circ T + h^2 \ u^3 \ E(f)$$

Suppose, $M_{u,T}$ is Quasi P-normal operator. Then

$$\left(M_{u,T} + M^*{}_{u,T}\right)\left(M^*{}_{u,T} M_{u,T}\right)f = \left(M^*{}_{u,T} M_{u,T}\right)\left(M_{u,T} + M^*{}_{u,T}\right)f$$
$$\Leftrightarrow u \circ T \ u^2 \circ T \ h \circ T \ f \circ T + h u E \left(h u^2 f\right) \circ T^{-1} = h u^2 \ u \circ T \ f \circ T + h^2 \ u^3 \ E(f)$$

almost everywhere.

Corollary 3.2

The composition operator C_T on $B(L^2(\mu))$ is Quasi p-normal operator if and only if

$h\circ T\ f\circ T + h\,u E\left(h\ f\right)\circ T^{-1} = h f\circ T + h^2\ E(f)$ almost everywhere.

Proof:

The proof is obtained from Theorem 3.1 by putting $u = 1$.

Theorem 3.3

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then the following statements are equivalent

(i) $M^*_{u,T}$ is Quasi p-normal operator

(ii)

$$h^2 u^3 E(f)\circ T^{-1} + h\circ T^2\ u\circ T\ u^2\circ T^2\ E(f)\circ T$$
$$= h\circ T\ u^2\circ T\ E(h)\ E(u) E(f)\circ T^{-1} + h\circ T\ u\circ T\ u^2\circ T\ f\circ T$$

almost everywhere.

Proof:

For $f\in L^2(\mu)$, $M^*_{u,T}$ is Quasi P-normal operator if

$$\left(M^*_{u,T} + M_{u,T}\right)\left(M_{u,T}\ M^*_{u,T}\right) f = \left(M_{u,T}\ M^*_{u,T}\right)\left(M^*_{u,T} + M_{u,T}\right) f$$

and then we have

$$\left(M^*_{u,T} + M_{u,T}\right)\left(M_{u,T}\ M^*_{u,T}\right) f = M^*_{u,T}\left(M_{u,T}\ M^*_{u,T}\right) f + M_{u,T}\left(M_{u,T}\ M^*_{u,T}\right) f$$
$$= M^*_{u,T} M_{u,T}\left[h\,u\,E(f)\circ T^{-1}\right] + M_{u,T}\ M_{u,T}\left[h\,u\,E(f)\circ T^{-1}\right]$$
$$= M^*_{u,T}\ u\circ T\left[h\,u\,E(f)\circ T^{-1}\right]\circ T + M_{u,T}\ u\circ T\left[h\,u\,E(f)\circ T^{-1}\right]\circ T$$
$$= M^*_{u,T}\left[u\circ T\ h\circ T u\circ T\ E(f)\right] + M_{u,T}\left[u\circ T\ h\circ T u\circ T\ E(f)\right]$$
$$= h\,u\,E(u\circ T\ h\circ T u\circ T\ E(f))\circ T^{-1} + u\circ T\left[u\circ T\ h\circ T u\circ T\ E(f)\right]\circ T$$
$$= h^2\ u^3\ E(f)\circ T^{-1} + h\circ T^2\ u\circ T\ u^2\circ T^2\ E(f)\circ T$$

Consider

$$\left(M_{u,T}\ M^*_{u,T}\right)\left(M^*_{u,T} + M_{u,T}\right) f = \left(M_{u,T}\ M^*_{u,T}\right) M^*_{u,T}\ f + \left(M_{u,T}\ M^*_{u,T}\right) M_{u,T}\ f$$
$$= \left(M_{u,T}\ M^*_{u,T}\right) h\,u\,E(f)\circ T^{-1} + \left(M_{u,T}\ M^*_{u,T}\right)(u\circ T\ f\circ T)$$
$$= M_{u,T}\ h\,u\,E\left(h\,u\,E(f)\circ T^{-1}\right)\circ T^{-1} + M_{u,T}\ h\,u\,E\left(u\circ T\ f\circ T\right)\circ T^{-1}$$
$$= M_{u,T}\ h\,u\,E(h)\circ T^{-1} E(u)\circ T^{-1} E(f)\circ T^{-2} + M_{u,T}\ h\,u^2\ f$$
$$= u\circ T\left(h\,u\,E(h)\circ T^{-1}\ E(u)\circ T^{-1}\ E(f)\circ T^{-2}\right)\circ T + u\circ T(h\,u^2 f)\circ T$$
$$= h\circ T\ u^2\circ T\ E(h)\ E(u) E(f)\circ T^{-1} + h\circ T\ u\circ T\ u^2\circ T\ f\circ T$$

Suppose $M^*_{u,T}$ is Quasi p-normal operator. Then

$$\left(M^*_{u,T} + M_{u,T}\right)\left(M_{u,T}\ M^*_{u,T}\right) f = \left(M_{u,T}\ M^*_{u,T}\right)\left(M^*_{u,T} + M_{u,T}\right) f$$
$$\Leftrightarrow h^2\ u^3\ E(f)\circ T^{-1} + h\circ T^2\ u\circ T\ u^2\circ T^2\ E(f)\circ T$$
$$= h\circ T\ u^2\circ T\ E(h)\ E(u) E(f)\circ T^{-1} + h\circ T\ u\circ T\ u^2\circ T\ f\circ T$$

almost everywhere.

Corollary 3.4

The composition operator $C^{*}{}_{T}$ on $B(L^2(\mu))$ is Quasi P-normal operator if and only if

$$h^2\ E(f)\circ T^{-1}+h\circ T^2\ E(f)\circ T=h\circ T\ E(h)\ E(f)\circ T^{-1}+h\circ T\ \ f\circ T$$

almost everywhere.

Proof:

The proof is obtained from Theorem 3.3 by putting $u=1$.

III. CHARACTERIZATIONS ON N POWER CLASS Q COMPOSITE MULTIPLICATION OPERATOS ON L²-SPACE

Theorem 4.1

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then $M_{u,T}$ is n power class Q composite multiplication operator if and only if

$$h\,u\,E(h)\circ T^{-1}\ E(u)\circ T^{-1}\ E(u_{2n})\circ T^{-2}\ f\circ T^{2n-2}$$
$$=h\,u\,E(u_n)\circ T^{-1}\ h\circ T^{n-1}\ u\circ T^{n-1}\ E(u_n)\circ T^{n-2}\ f\circ T^{2n-2}$$

Proof:

Now Consider,

$$M^{*2}{}_{u,T}\,M^{2n}{}_{u,T}\ f=M^{*2}{}_{u,T}\left[u_{2n}f\circ T^{2n}\right]$$

where $u_{2n}=u\circ T^2\ u\circ T^4\ldots\ldots\ldots\ldots\ldots\ u\circ T^{2n}$

$$=M^{*}{}_{u,T}\left(\,h\,u\,E\left(u_{2n}\ f\circ T^{2n}\right)\circ T^{-1}\right)$$
$$=M^{*}{}_{u,T}\ h\,u\,E\,(u_{2n})\circ T^{-1}\ f\circ T^{2n-1}$$
$$=h\,u\,E\left(h\,u\,E\,(u_{2n})\circ T^{-1}\ f\circ T^{2n-1}\right)\circ T^{-1}$$
$$=h\,u\,E(h)\circ T^{-1}\ E(u)\circ T^{-1}\ E(u_{2n})\circ T^{-2}\ f\circ T^{2n-2}$$

Next we consider,

$$\left(M^{*}{}_{u,T}M^{n}{}_{u,T}\right)^2 f=\left(M^{*}{}_{u,T}M^{n}{}_{u,T}\right)\left(M^{*}{}_{u,T}M^{n}{}_{u,T}\right)f$$
$$=\left(M^{*}{}_{u,T}M^{n}{}_{u,T}\right)M^{*}{}_{u,T}\left(u_n\ f\circ T^{n}\right)$$

where $u_n=u\circ T\ u\circ T^2\ldots\ldots\ldots\ldots\ldots\ u\circ T^{n}$

$$=\left(M^{*}{}_{u,T}M^{n}{}_{u,T}\right)h\,u\,E\left(u_n\ f\circ T^{n}\right)\circ T^{-1}$$
$$=\left(M^{*}{}_{u,T}M^{n}{}_{u,T}\right)h\,u\,E(u_n)\circ T^{-1}\ f\circ T^{n-1}$$
$$=M^{*}{}_{u,T}\ \ u_n\left(\,h\,u\,E(u_n)\circ T^{-1}\ f\circ T^{n-1}\right)\circ T^{n}$$
$$=M^{*}{}_{u,T}\ \ u_n\ h\circ T^{n}\ u\circ T^{n}\ E(u_n)\circ T^{n-1}\ f\circ T^{2n-1}$$
$$=h\,u\,E\left(u_n\ h\circ T^{n}\ u\circ T^{n}\ E(u_n)\circ T^{n-1}\ f\circ T^{2n-1}\right)\circ T^{-1}$$
$$=h\,u\,E(u_n)\circ T^{-1}\ h\circ T^{n-1}\ u\circ T^{n-1}\ E(u_n)\circ T^{n-2}\ f\circ T^{2n-2}$$

Given $M_{u,T}$ is n power class Q composite multiplication operator

$$\Leftrightarrow M^{*2}{}_{u,T}\ M^{2n}{}_{u,T}\ f=\left(M^{*}{}_{u,T}M^{n}{}_{u,T}\right)^2 f$$

$$\Leftrightarrow h\,u\,E(h)\circ T^{-1}\,E(u)\circ T^{-1}\,E(u_{2n})\circ T^{-2}\,f\circ T^{2n-2} \\ = h\,u\,E(u_n)\circ T^{-1}\,h\circ T^{n-1}\,u\circ T^{n-1}\,E(u_n)\circ T^{n-2}\,f\circ T^{2n-2}$$ almost everywhere.

Corollary 4.2

The composition operator C_T on $B(L^2(\mu))$ is n power class Q if and only if

$h\ E(h)\circ T^{-1}\ f\circ T^{2n-2} = h\ h\circ T^{n-1}\ f\circ T^{2n-2}$

almost everywhere.

Proof:

The proof is obtained from Theorem 4.1 by putting $u = 1$.

Theorem 4.3

Let the $M_{u,T}$ be a composite multiplication operator on $L^2(\mu)$. Then $M^*{}_{u,T}$ is n power class Q composite multiplication operator if and only if

$$u\circ T\ u^2\circ T^2\ h\circ T^2\ E(u\,h)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)} \\ = u^2\circ T\ h\circ T\ E(u\,h)\circ T^{-(n-2)}\ h\circ T^{-(n-2)}\ E(u\,h)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$

Proof:

Now if we consider

$$M^2{}_{u,T}\,M^{*2n}{}_{u,T}\ f = M^2{}_{u,T}\left(\ h\,u\,E(h\,u)\circ T^{-(2n-1)}\ E(f)\circ T^{-2n}\right)$$
$$= M_{u,T}\left(u\circ T\left(\ h\,u\,E(h\,u)\circ T^{-(2n-1)}\ E(f)\circ T^{-2n}\right)\circ T\right)$$
$$= M_{u,T}\left(h\circ T\ \ u^2\circ T\ E(h\,u)\circ T^{-(2n-2)}\ E(f)\circ T^{-(2n-1)}\right)$$
$$= u\circ T\ \left(h\circ T\ \ u^2\circ T\ E(h\,u)\circ T^{-(2n-2)}\ E(f)\circ T^{-(2n-1)}\right)\circ T$$
$$= u\circ T\ u^2\circ T^2\ h\circ T^2\ E(u\,h)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$

and we consider

$$\left(M_{u,T}\,M^{*n}{}_{u,T}\right)^2 f = \left(M_{u,T}\,M^{*n}{}_{u,T}\right)\left(M_{u,T}\,M^{*n}{}_{u,T}\right) f$$
$$= \left(M_{u,T}\,M^{*n}{}_{u,T}\right) M_{u,T}\ \ u\,h\,E(u\,h)\circ T^{-(n-1)}\ E(f)\circ T^{-n}$$
$$= \left(M_{u,T}\,M^{*n}{}_{u,T}\right) u\circ T\left(u\,h\,E(u\,h)\circ T^{-(n-1)}\ E(f)\circ T^{-n}\right)\circ T$$
$$= M_{u,T}\,M^{*n}{}_{u,T}\left(u^2\circ T\ \ h\circ T\ \ E(u\,h)\circ T^{-(n-2)}\ \ E(f)\circ T^{-(n-1)}\right)$$
$$= M_{u,T}\ u\,h\,E(u\,h)\circ T^{-(n-1)}\ E\left(u^2\circ T\ \ h\circ T\ \ E(u\,h)\circ T^{-(n-2)}\ \ E(f)\circ T^{-(n-1)}\right)\circ T^{-n}$$
$$= M_{u,T}\ \left(u\,h\,E(u\,h)\circ T^{-(n-1)}\ u^2\circ T^{-(n-1)}\ h\circ T^{-(n-1)}\ E(u\,h)\circ T^{-(2n-2)}\ E(f)\circ T^{-(2n-1)}\right)$$
$$= u\circ T\left(\ u\,h\,E(u\,h)\circ T^{-(n-1)}\ u^2\circ T^{-(n-1)}\ h\circ T^{-(n-1)}\ E(u\,h)\circ T^{-(2n-2)}\ E(f)\circ T^{-(2n-1)}\right)\circ T$$
$$= u^2\circ T\ \ h\circ T\ \ E(u\,h)\circ T^{-(n-2)}\ h\circ T^{-(n-2)}\ E(u\,h)\circ T^{-(2n-3)}\ E(f)\circ T^{-(2n-2)}$$

Since $M_{u,T}$ is a Composite multiplication operator, by definition

$$\Leftrightarrow\ M^2{}_{u,T}\,M^{*2n}{}_{u,T}\ f = \left(M_{u,T}\,M^{*n}{}_{u,T}\right)^2 f$$

$$\Leftrightarrow \; u \circ T \, u^2 \circ T^2 \, h \circ T^2 \, E(u\,h) \circ T^{-(2n-3)} \, E(f) \circ T^{-(2n-2)}$$
$$= u^2 \circ T \; h \circ T \; E(u\,h) \circ T^{-(n-2)} \; h \circ T^{-(n-2)} \; E(u\,h) \circ T^{-(2n-3)} \; E(f) \circ T^{-(2n-2)}$$

almost everywhere.

Corollary 4.4

The composition operator $C^*{}_T$ on $B(L^2(\mu))$ is n power class Q if and only if

$$h \circ T^2 \, E(h) \circ T^{-(2n-3)} \, E(f) \circ T^{-(2n-2)}$$
$$= h \circ T \; E(h) \circ T^{-(n-2)} \; h \circ T^{-(n-2)} \; E(h) \circ T^{-(2n-3)} \; E(f) \circ T^{-(2n-2)}$$

almost everywhere.

Proof:

The proof is obtained from Theorem 4.3 by putting $u = 1$.

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