IMAGE: A MAP OF THE STARS OF THE ORION CONSTELLATION

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Journal Content

In this Issue

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Rena Eldar kizi Kerbalayeva

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I. INTRODUCTION

Let $G \subset R^n$ and $1 \le s \le n$; s, n be naturals, where $n_1 + \cdots + n_s = n$. We consider the sufficient smooth function f(x), where the point $x = (x_1, \dots, x_s) \in R^n$ has coordinates

$$x_k = (x_{k,1}; \dots; x_{k,n_k}) \in R^{n_k} \ (k \in e_s = \{1,\dots,s\}).$$

More precisely,

$$R^n = R^{n_1} \times R^{n_2} \times \cdots \times R^{n_s}.$$

Thus we consider the fixed, non–negative, integral vector l=$(l_1, \dots, l_s)$ such that, $l_k = (l_{k.1}; \dots; l_{k,n_k})$, (k$\subset e_s$) that is, $l_{k,j} > 0$, $(j = 1, \dots, n_k)$ for all $k \in e_s$. Here we consider by Q the set of vectors i= $(i_1, \dots, i_s)$ where i_k= 1,2,…, n_k for every $k \in e_s$. The number of set Q is equal to:

$$|Q| = \prod_{k=1}^{s} (1 + n_k).$$

Therefore, to the vector i$=(i_1,\dots,i_s)\in Q$, we shall correspond the vector $l^i=\left(l_1^{i_1};\dots;l_s^{i_s}\right)$ of the set of non–negative, integral vectors l$=(l_1,\dots,l_s)$, where

$$l^0=(0,0,\dots,0),l_k^1=\left(l_{k,1},0,\dots,0\right),\dots,l_k^{i_k}=\left(0,0,\dots,l_{k,n_k}\right)$$

for all $k\in e_s$. Then to the vector e^i, we let correspond the vector $\bar{l}^i=\left(\bar{l}_1^{i_1},\bar{l}_1^{i_2},\dots,\bar{l}_1^{i_s}\right)$, where $\bar{l}_k^{i_k}=\left(\bar{l}_{k,1}^{i_1},\bar{l}_{k,2}^{i_2},\dots,\ \bar{l}_{k,n_k}^{i_k}\right)$ $(k\in e_s)$. Here the largest number $\bar{l}_{k,j}^{i_k}$ is less than $l_{k,j}^{i_k}$ for all $l_{k,j}^{i_k}>0$, when $l_{k,j}^{i_k}=0$ then we assume that $\bar{l}_{k,j}^{i_k}=0$ for all $k\in e_s$.

Theremore, we consider

$$D^{\bar{l}^i}f=D_1^{\bar{l}_1^{i_1}}\cdots D_s^{\bar{l}_s^{i_s}}f,\quad D_k^{\bar{l}_k^{i_k}}f=D_{k,1}^{\bar{l}_k^{i_k}}\cdots D_{k,n_k}^{\bar{l}_k^{i_k}}f,G_{t^{\varkappa}}=G\cap I_{t^{\varkappa}}(x),$$

$$I_{t^{\varkappa}}(x)=I_{t_1^{\varkappa_1}}(x_1)\times I_{t_2^{\varkappa_2}}(x_2)\times\cdots\times I_{t_s^{\varkappa_s}}(x_s),$$

$$I_{t_k^{\varkappa_k}}(x_k)=\left\{y_k:|y_k-x_k|<\frac{1}{2}t_k^{|\varkappa_k|},\ k\in e_s\right\}$$

and

$$|\beta_k|=\sum_{j=1}^{n_k}\beta_{k,j}^{i_k};\ |\beta_k^{i_k}|=\sum_{j=1}^{n_k}\beta_{k,j}^{i_k}\ \frac{dt_k}{t_k}=\prod_{j\in e_k^i}\frac{dt_{k,j}}{t_{k,j}},$$

we take $0<\beta_{k,j}^{i_k}=l_{k,j}^{i_k}-\bar{l}_{k,j}^{i_k}\le 1$, when $l_k^{i_k}>0$, but when $l_{k,j}^{i_k}=0$, then $\beta_{k,j}^{i_k}=0$; $t=(t_1,\dots,t_s)$, $t_k=\left(t_{k,1},\dots,t_{k,n_k}\right),\omega=(\omega_1,\dots,\omega_s),\omega_k=\left(\omega_{k,1},\dots,\omega_{k,n_k}\right)$ and we take

$$\omega_{k,j}=1,\text{ when k}\in e^i,$$

or we give

$$\omega_{k,j}=0,\text{ when k}\in e_s/e^i,$$

$$e^i=supp\bar{l}^i=suppl^i=supp\omega,1\le\theta\le\infty;\ 1\le p<\infty.$$

Here $t_0=\left(t_{0,1},\dots,t_{0,s}\right),t_{0,k}=\left(t_{0,k,1},\dots,t_{0,k,n_k}\right)$ –is fixed vector and $\varkappa\in(0,\infty)^n$, a$\in[0,1],\tau\in[1,\infty],[t_k]_1=\min\{1,t_k\},\ k\in e_s$. Here

$$\Delta^{\omega}(t)f=\Delta_1^{\omega_1}(t_1)\cdots\Delta_s^{\omega_s}(t_k)f,$$

when $2\omega = (2, 2, \ldots, 2)$, and

$$\Delta_k^{\omega_k}(t_k)f = \Delta_{k,1}^{\omega_{k,1}}(t_{k,1}) \cdots \Delta_{k,n_k}^{\omega_{k,n_k}}(t_{k,n_k})f, (k \in e_s),$$

following $\Delta_{k,j_k}^{\omega_{k,j_k}}(t_{k,j_k})f$ are finite difference function, which has direction with variables t_{k,j_k} and with order ω_{k,j_k}, by step t_{k,j_k} for $j = 1, \ldots, n_k$ and for all and $k \in e_s$, following

$$\Delta_{k,j_k}^1(t_{k,j_k})f(\cdots, x_{k,j_k}, \cdots) =$$

$$f(\cdots, x_{k,j_k} + t_{k,j_k}, \cdots) - f(\cdots, x_{k,j_k}, \cdots),$$

and

$$\Delta_{k,j_k}^{\omega_{k,j_k}}(t_{k,j_k})f(\cdots, x_{k,j_k}, \cdots) =$$

$$\Delta_{k,j_k}^1(t_{k,j_k})\left\{\Delta_{k,j_k}^{\omega_{k,j_k}-1}(t_{k,j_k})f(\cdots, x_{k,j_k}, \cdots)\right\},$$

but when $\omega_{k,j_k} = 0$, then

$$\Delta_{k,j_k}^0(t_{k,j_k})f(\cdots, x_{k,j_k}, \cdots) = f(\cdots, x_{k,j_k}, \cdots).$$

[10, 24, 25, 27]

Let us assume that we have a basis functions $\Phi = \{\varphi_n\}_{n=1}^{\infty}$. Given function f(x) can be rewritten with this basis: $f(x) = \sum_{n=1}^{\infty} c_n \varphi_n(x)$. Hence we obtain

$$\langle \varphi_k, f \rangle = \langle \varphi_k, \sum_{n=1}^{\infty} c_n \varphi_n(x) \rangle = \sum_{n=1}^{\infty} c_n \langle \varphi_k, f \rangle =$$

$$\sum_{n=1}^{\infty} c_n \langle \varphi_k, \varphi_n \rangle = \langle \varphi_k, \sum_{k=1}^{\infty} c_k \varphi_k(x) \rangle.$$

If introducing basis is an orthogonal basis, then we get $\langle \varphi_k, \varphi_n \rangle = n_k \delta_{kn}$, where δ_{kn} is the Kronecker delta:

$$\delta_{jn} = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases}.$$

Then we hold

$$\langle \varphi_k, f \rangle = \sum_{n=1}^{\infty} c_n \langle \varphi_k, \varphi_n \rangle = \sum_{n=1}^{\infty} c_n n_k \delta_{kn}.$$

That is,

$$c_k = \frac{\langle \varphi_k, f \rangle}{n_k} = \frac{\langle \varphi_k, f \rangle}{\langle \varphi_k, \varphi_k \rangle}, k = 1, 2, \dots$$

Let $\{\varphi_n\}_{n=0}^{\infty}$ be a linearly independent sequence of continuous functions defined for $x \in R^n$. The an orthogonal basis of functions can be found following

$$\varphi_0 = f_0, \varphi_k = f_k - \sum_{n=0}^{n-1} \frac{\langle f_k, \varphi_n \rangle}{\|\varphi_n\|^{\tau}}, n = 1, 2, \dots$$

[5, 7, 21, 26]

Definition. We denote by

$$\mathcal{L}_{p,a,\varkappa,\tau}(G) \quad (1)$$

normed Lorentz–Morrey space of locally summability, measurable functions f, on G, with finite norm ($N^i > l^i > m^i \geq 0$, i=1,2,…,n)

$$\|f\|_{p,a,\varkappa,\tau:G} = \|f\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} =$$

$$\begin{cases} \left\{\int_0^{\infty} \left[\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}-1} \times \|f^*\|_{p,G_{t^{\varkappa}}(x)}\right]^{\tau} \prod_{k\in e_s} \frac{dt_k}{t_k}\right\}^{1/\tau}, \\ sup_{0<t<\infty}\left(\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \|f^*\|_{p,G_{t^{\varkappa}}(x)}\right) \end{cases} \quad (2)$$

where $|\varkappa_k|=\sum_{j=1}^{n_k} \varkappa_{k,j}$; $[t_k]_1 = \min\{1, t_k\}$ and $f^*(t)$ is the decreasing rearrangement of f [9].

The properties of this space are main objects of Analysis. Let us give some characterization of $\mathcal{L}_{p,a,\varkappa,\tau}(G)$:

1) $\|\cdot\|_{p,a,\varkappa,\tau:G}$ is a qiasi-norm.

2) We must note that, for every $\tau > 0$

$$\mathcal{L}_{p,a,\varkappa,p}(G) = \mathcal{L}_{p,a,\varkappa}(G)$$

3) The space $\mathcal{L}_{p,a,\varkappa,\tau}(G)$ is complete.

4) For c>0 we have

$$\|f\|_{p,a,c\varkappa,\tau:\,G} = \frac{1}{C^{\frac{s}{\tau}}}\|f\|_{p,a,\varkappa,\tau:\,G}.$$

5) For any $\varkappa=(\varkappa_1,\dots,\varkappa_n) > 0$ we get:

a)$\|f\|_{p,0,\varkappa,\infty:\,G} = \|f\|_{p,G}$;

b)$\|f\|_{p,1,\varkappa,\tau:\,G} \geq \|f\|_{\infty,G}$.

6) If $p \leq q, \frac{1-b}{q} \leq \frac{1-a}{p}$, $1 \leq \tau_1 \leq \tau_2 \leq \infty$ then

$$\mathcal{L}_{q,b,\varkappa,\tau_1}(G) \subset_{>} \mathcal{L}_{p,a,\varkappa,\tau_2}(G)$$

and

$$\|f\|_{p,a,\varkappa,\tau_2:\,G} \leq \|f\|_{q,b,\varkappa,\tau_1:\,G}. \tag{3}$$

[2, 4, 8, 15, 16, 18, 23]

Some relations between this norm and some corresponding sums of Fourier coefficients are introduced for the case with a general orthonormal bounded system.

Let us take following well-known inequalities for 1<p<∞

$$c_1\|\bar{f}\|^p_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} \leq \sum_{i\in Q}\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times$$

$$|r_k|_p \leq \frac{c_2}{\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}}}\left\|D^{\bar{l}^i}f\right\|^p_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} \quad (4).$$

In addition, here

$$\overline{f(t)} = \frac{1}{\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}}} \times \int_0^\infty f(s)\prod_{k\in s_k}\frac{ds_k}{s_k}$$

and $f'(t)$ is the derivative of the function f(t). That is, $\{a_k\}_{k=1}^\infty$ are the Fourier coefficients of the function f. Here, $\{a_k{}^*\}_{k=1}^\infty$ is the nonincreasing rearrangement of the sequence $\{|a_k|\}_{k=1}^\infty$.

Let us give generalized Lorentz-Morrey space such that

$$\|f\|_{\Lambda_{p,a,\varkappa,\tau}(\omega)} =$$

$$\begin{cases} \left\{\int_0^\infty \left[\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}-1} \times \|f^*\|_{p,G_{t^\varkappa}(x)}\omega(t)\right]^\tau \prod_{k\in e_s}\frac{dt_k}{t_k}\right\}^{1/\tau}, for\ 0<\tau<\infty \\ sup_{0<t<\infty}\left(\|f^*(t)\omega(t)\|_{p,G_{t^\varkappa}(x)}\right), for\ \tau=\infty \end{cases}$$

and where $\omega(t)$ positive, having some additional growth property and $\|f\|_{\Lambda_{p,a,\varkappa,\tau}(\omega)} < \infty$.

Therefore, in this problem, if $\sum_{n=1}^N c_n\,\varphi_n$ converges, then

$$\int_0^\infty \left(f(x) - \sum_{n=1}^N c_n\,\varphi_n\right)^\tau \to 0, N \to \infty.$$

Definition: Let $f(x) = f(x_1, \ldots, x_s)$ be integrability function with s variables $x_1, \ldots, x_s$ defining on the R^n. The Fourier series expansion of the function f is following

$$f(\sigma) = f(\sigma_1, \ldots, \sigma_s) = \int\int\cdots\int e^{-i(x_1\sigma_1+\cdots+x_s\sigma_s)}\prod_{k\in e_s} x_k.$$

Then we hold

$$f(x) = \frac{1}{2\pi}\int_0^\infty (\cdots\frac{1}{2\pi}\left\{\int_0^\infty f(x_1, \ldots, x_s)\,e^{ix_s\cdot\sigma_s}d\sigma_s\right\}\times$$

$$e^{ix_{s-1}\cdot\sigma_{s-1}}d\sigma_{s-1}\cdots)e^{ix_1\cdot\sigma_1}\frac{d\sigma_1}{\sigma_1}. \qquad (5)$$

We can imagine writing the Fourier series as following

$$\sum_{i\epsilon Q} c_{\sigma_1,\cdots,\sigma_s}\prod_{k\in e_s} e^{2\pi\sigma_k\cdot x_k}.$$

The Fourier series expansion in n dimensional is approximated following

$$f(x) = \sum_{i \in Z^n} c_i \prod_{k \in e_s} e^{2\pi\sigma_k \cdot x_k}.$$

The Fourier coefficients ($\hat{f} = c_n$) can be defined by the integral

$$\hat{f} = \int_0^\infty \int_0^\infty e^{-2\pi i\sigma_1 x_1} e^{-2\pi i\sigma_2 x_2} \cdots e^{-2\pi i\sigma_s x_s} f(x_1, \cdots x_s) \prod_{k \in e_s} x_k$$

$$r_k = r_k(f) = \int_0^\infty f(x)\varphi_k(x)dx, k \in N.$$

[11, 12, 22]

II. SOME MAIN RESULTS

Theorem (Generalized Parseval): Let $f \in \mathcal{L}_{p,a,\varkappa,\tau}(G)$. Then

$$\|f\|^\tau_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} = \sum_{k=1}^{\infty} |r_k|^\tau$$

where

$$r_k = r_k(f) = \int_0^\infty \left| \prod_{k \in e_s} [t_k]_1^{-\frac{|\varkappa_k| a}{p} - 1} \times f(x) \right| \times$$

$$\prod_{k \in e_s} e^{2\pi\sigma_k \cdot x_k} \prod_{k \in e_s} dx_k$$

are the Fourier coefficients of the functions f with respect to the trigonometric system.

Proof: Inverse of Fourier transformation is

$$f(x) = \sum_{k=1}^{\infty} r_k(\sigma) \prod_{k \in e_s} e^{2\pi\sigma_k \cdot x_k}$$

Use these two properties to rewrite the left-hand side of this theorem:

$$\int_0^\infty |f(x)|^\tau \prod_{k \in e_s} dx_k = \int_0^\infty |f(x)| \cdot |f(x)| \cdots |f(x)| \prod_{k \in e_s} x_k =$$

$$\sum_{k=1}^\infty |r_k| \{ \cdots \sum_{k=1}^\infty |r_k(\sigma)| \prod_{k \in e_s} e^{2\pi\sigma_k \cdot x_k} \}.$$

Then we hold

$$\int_0^\infty |f(x)|^\tau \prod_{k\in e_s} dx_k = \sum_{k=1}^\infty |r_k| \left\{ \cdots \sum_{k=1}^\infty |r_k(\sigma)| \prod_{k\in e_s} e^{2\pi\sigma_k\cdot x_k} \right\} =$$

$$= \sum_{k=1}^\infty |r_k| \cdots \sum_{k=1}^\infty |r_k^*|.$$

Taking generalized Cauchy-Schwarz and Holder inequalities we have

$$\int_0^\infty |f(x)|^\tau \prod_{k\in e_s} dx_k = \sum_{k=1}^\infty |r_k| \cdots \sum_{k=1}^\infty |r_k^*| = \sum_{k=1}^\infty |r_k|^\tau.$$

We must note that, Bessel inequality holds for any general orthonormal system. Let the function f be periodic with period 1 and integrable on $[0, \infty)$ and $\Phi = \{\varphi_n\}_{n=1}^\infty$ be an orthogonal system. The numbers

$$r_n = r_n(f) = \int_0^\infty |f(x)\varphi_k(x)| \prod_{k\in e_s} dx_k,\ n \in N,$$

are called the Fourier coefficients of the function f with respect to the system $\Phi = \{\varphi_n\}_{n=1}^\infty$.

Theorem (Bessel F.): Let $\Phi = \{\varphi_k\}_{k=1}^\infty$ are orthonormal system in $\mathcal{L}_{p,a,\varkappa,\tau}(G), f \in \mathcal{L}_{p,a,\varkappa,\tau}(G)$, and $r_k = r_k(f) = \int_0^\infty f(x)\varphi_k(x) \prod_{k\in e_s} dx_k$, $k \in \{1, \cdots, \infty\}$ are the Fourier coefficients of the function f. Then

$$\sum_{k=1}^\infty |r_k|^\tau \leq \int_0^\infty \left| \prod_{k\in e_s} [t_k]_1^{-\frac{|\varkappa_k|a}{p}-1} \times f(x) \right|^\tau \prod_{k\in e_s} \frac{dx_k}{x_k} = \|f\|^\tau_{\mathcal{L}_{p,a,\varkappa,\tau}(G)}.$$

Proof of theorem: Let us rewrite following

$$\sum_{k=1}^\infty |r_k|^\tau = \sum_{k=1}^\infty \left[\int_0^\infty f(x)\varphi_k(x) \prod_{k\in e_s} dx_k \right]^\tau.$$

If we introduce infinite sum

$$f \ = \sum_{k=1}^\infty \int_0^\infty (f(x)\varphi_k(x))\varphi_k(x) \prod_{k\in e_s} dx_k .$$

We know that, this series converges. With aid to Parseval's identity we have following

$$0 \le \left\| f - \sum_{k=1}^{\infty} \int_{0}^{\infty} \left(f(x)\varphi_k(x)\right)\varphi_k(x) \prod_{k \in e_s} dx_k \right\|^{\tau} =$$

$$\|f\|^{\tau} - C_{\tau}^{1} \sum_{k=1}^{\infty} \int_{0}^{\infty} f(x) \cdot \left(\left(f(x)\varphi_k(x)\right)\varphi_k(x)\right) \prod_{k \in e_s} dx_k +$$

$$\cdots + (-1)^{\tau+1} C_{\tau}^{\tau} \sum_{k=1}^{\infty} \int_{0}^{\infty} |f(x)\varphi_k(x)|^{\tau} \prod_{k \in e_s} dx_k =$$

$$\|f\|^{\tau} - C_{\tau}^{1} \sum_{k=1}^{\infty} \int_{0}^{\infty} f(x)\, |f(x)\varphi_k(x)|^{\tau} \prod_{k \in e_s} dx_k + \cdots$$

$$(-1)^{\tau+1} C_{\tau}^{\tau} \sum_{k=1}^{\infty} \int_{0}^{\infty} |f(x)\varphi_k(x)|^{\tau} \prod_{k \in e_s} dx_k =$$

$$\|f\|^{\tau} + \sum_{k=1}^{\infty} \int_{0}^{\infty} |f(x)\varphi_k(x)|^{\tau} \prod_{k \in e_s} dx_k$$

or $0 \le \|f\|^{\tau}$. (if τ is even)

Let us introduce Rietz F. and Ficher E. theorem for Lorentz-Morrey type spaces with many groups of variables, which is the result that given space is complete and that is, every Cauchy sequence of function in $\mathcal{L}_{p,a,\varkappa,\tau}(G)$ convergence to a function in $\mathcal{L}_{p,a,\varkappa,\tau}(G)$.

Theorem (Rietz F. and Ficher E.): Let $\Phi = \{\varphi_k\}_{k=1}^{\infty}$ are orthonormal system in $\mathcal{L}_{p,a,\varkappa,\tau}(G)$ and $\{a_k\}_{k=1}^{\infty}$ be an arbitrary sequence of $\sum_{k=1}^{\infty}|a_k|^{\tau} < \infty$ Then there exists a function $f \in \mathcal{L}_{p,a,\varkappa,\tau}(G)$ for which the numbers a_n are its Fourier coefficients in this system and following inequality exits

$$\|f\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)}^{\tau} \le \sum_{k=1}^{\infty} |r_k|^{\tau}.$$

Proof of the theorem: In order to proof this theorem I have to proof that, given space is complete. It has been proved in [19].

From this theorem we hold following theorem.

Teorem: (F. Hausdorf and W. Yong)

1) If $1 < \tau \leq 2, f \in \mathcal{L}_{p,a,\varkappa,\tau}(G)$ and

$$r_k = r_k(f) = \int_0^\infty \left| \prod_{k \in e_s} [t_k]_1^{-\frac{|\varkappa_k| a}{p} - 1} \times f(x) \right| \prod_{k \in e_s} e^{2\pi\sigma_k \cdot x_k} \prod_{k \in e_s} dx_k$$

then we get

$$\left(\sum_{k \in Z^n} |r_k|^\rho \right)^{1/\rho} \leq \|f\|_{\mathcal{L}_{p,a,\varkappa,\rho}(G)}.$$

2) If $\rho \geq 2$ and $\{r_k\}_{k \in Z} \in \mathcal{L}_{p,a,\varkappa,\rho}(G)$ then the trigonometric series $\sum_{k \in Z} r_k e^{2\pi i k x}$ converges in the metric $\mathcal{L}_{p,a,\varkappa,\rho}(G)$ to some function f and it holds that

$$\|f\|_{\mathcal{L}_{p,a,\varkappa,\rho}(G)} \leq \left(\sum_{k \in Z^n} |r_k|^\rho \right)^{1/\rho}.$$

Where $\rho = \frac{\tau}{\tau - 1}$.

Teorem (Paley R.): Let $\{\varphi_k\}_{k=1}^\infty$ be the orthonormal system on R^n such that $|\varphi_k(t)| \leq M$ for all $k \in N$ and $x \in R^n$ and $r_i = r_i(f) = \int_0^\infty f(x) \varphi_i(x) \prod_{k \in e_s} dx_k$, $k \in N$. Then we have

1) $\left(\sum_{i=1}^\infty |r_i|^\tau k^{\tau-2}\right)^{1/\tau} \leq c_3 M^{\frac{2-\tau}{\tau}} \|f\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)}$,

where $1 < \tau \leq 2$ and $f \in \mathcal{L}_{p,a,\varkappa,\tau}$.

2) $\|f\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} \leq c_4 M^{\frac{\tau-2}{\tau}} \left(\sum_{i=1}^\infty |r_k|^\tau \times k^{\tau-2}\right)^{1/\tau} < \infty$

where $2 \leq \tau < \infty$ and the sequence $\{r_k\}_{k=1}^\infty$ satisfies the following condition

$$\left(\sum_{i=1}^\infty r_i \times k^{\tau-2} \right)^{1/\tau} < \infty$$

and the function f is given by the formula $f = \lim_{i \to \infty} \left[\sum_{i=1}^\infty r_i \varphi_k\right]$.

Proof: Taking inequality (2.7.3) in [17] we hold

$$\left(\sum_{k=1}^{\infty}|r_k|^{\tau}\,k^{\tau-2}\right)^{1/\tau} \leq \sum_{k=1}^{\infty}\{|r_k|^{\tau}\}^{1/\tau}\sum_{k=1}^{\infty}\{k^{\tau-2}\}^{1/\tau} =$$

$$= \sum_{k=1}^{\infty}|r_k|\sum_{k=1}^{\infty}\{k^{\tau-2}\}^{1/\tau}.$$

Then taking Bessel theorem we get

$$\sum_{k=1}^{\infty}|r_k| = \|f\|_p$$

and using Hardy-Littlewood inequality follows

$$\sum_{k=1}^{\infty}\{k^{\tau-2}\}^{1/\tau} = \sum_{k=1}^{\infty} k^{\frac{\tau-2}{\tau}} =$$

$$\sum_{k=1}^{\infty}\left(k^{\frac{2-\tau}{\tau}}\right)^{-1/\tau} \leq c_3 M^{\frac{2-\tau}{\tau}}.$$

Then we get given first assumption.

2) Taking the Hardy-Littlewood inequality and Minkowski's inequality we hold

$$c_4 M^{\frac{\tau-2}{\tau}}\left(\sum_{i=1}^{\infty}|r_k|^{\tau}\times k^{\tau-2}\right)^{1/\tau} \geq c_4 M^{\frac{\tau-2}{\tau}}\left(\sum_{i=1}^{\infty}|r_k|^{\tau}\times(|k|+1)^{\tau-2}\right)^{1/\tau} \geq$$

$$\left(\sum_{i=1}^{\infty}|r_k^*|^{\tau}\times(|k|+1)^{\tau-2}\right)^{1/\tau} \geq \int_0^{\infty}|f(x)|^{\tau}\prod_{k\in e_s}dx_k.$$

Theorem A: Let us suppose that the function f satisfying following conditions

$$|f(x_1+t_1,x_2,\cdots x_s)-f(x_1,x_2,\cdots x_s)| \leq C|t_1|^{\alpha}$$

$$|f(x_1,x_2+t_2,\cdots x_s)-f(x_1,x_2,\cdots x_s)| \leq C(x_1)|t_2|^{\alpha}$$

....

$$|f(x_1,x_2,\cdots x_s+t_s)-f(x_1,x_2,\cdots x_s)| \leq C(x_1,\cdots x_{s-1})|t_s|^{\alpha}$$

and

$$\int_{0}^{\infty} C(x_1) \prod_{k=1,\cdots n_k} dx_{1,k}\,, k\epsilon e_s < \infty$$

$$\int_{0}^{\infty} C(x_1,\cdots,x_{s-1}) \prod_{k\epsilon e_s} \prod_{k=1,\cdots n_k} dx_{1,k} < \infty, 0 < \alpha \leq 1.$$

Then (5) holds, if we take limit for $N_s \to \infty, \ldots, N_1 \to \infty$:

$$f(x) = \frac{1}{2\pi} \lim_{N_1\to\infty} \int_{-N_1}^{N_1} (\cdots \frac{1}{2\pi} \lim_{N_{s-1}\to\infty} \left\{ \int_{-N_{s-1}}^{N_{s-1}} \lim_{N_s\to\infty} \int_{-N_1}^{N_1} f(x_1,\ldots,x_s)\, e^{ix_s\cdot\sigma_s} d\sigma_s \right\} \times$$

$$e^{ix_{s-1}\cdot\sigma_{s-1}} d\sigma_{s-1} \cdots) e^{ix_1\cdot\sigma_1} d\sigma_1. \qquad (6)$$

Proof: Taking

$$f_1(\sigma_1, x_1,\cdots,x_s) = \int_{0}^{\infty} f(x_1,\cdots,x_s)\, e^{ix_1\cdot\sigma_1} \prod_{k=1,\cdots n_k} dx_{1,k}$$

and with aid of Fubini's theorem the function $f(x_1,\cdots,x_s)$ is summarized for all $x_2,\cdots,x_s$. Following taking first condition we get

$$f(x_1,\cdots,x_s) = \lim_{N_1\to\infty} \frac{1}{2\pi} \int_{0}^{N_1} f_1(\sigma_1, x_1,\cdots,x_s) e^{i\sigma_1 x_1} \prod_{k=1,\cdots n_k} dx_{1,k}.$$

Indeed, the function $f_1(\sigma_1, x_1,\cdots,x_s)$ is summarized for all $x_3,\cdots,x_s$. In addition, with aid of given condition we have

$$|f(\sigma_1, x_2 + t_2,\cdots x_s) - f(x_1, x_2,\cdots x_s)| \leq$$

$$\int_0^\infty |f(\sigma_1, x_2 + t_2,\cdots x_s) - f(x_1, x_2,\cdots x_s)| \leq$$

$$|t_\alpha|^\alpha \int_0^\infty C(x_1) \prod_{k=1,\cdots n_k} dx_{1,k}.$$

Then we hold following

$$f_2(\sigma_1, \sigma_2, x_1, \cdots, x_s) = \int_0^{\infty} f_1(x_1, \cdots, x_s)\, e^{ix_2 \cdot \sigma_2} \prod_{k=1,\cdots n_k} dx_{2,k}.$$

Then next expression is real

$$f_1(\sigma_1, x_1, \cdots, x_s) = \lim_{N_2 \to \infty} \frac{1}{2\pi} \int_{N_2}^{N_2} f_2(\sigma_1, \sigma_2, x_1, \cdots, x_s)\, e^{ix_2 \cdot \sigma_2} \prod_{k=1,\cdots n_k} dx_{2,k}.$$

Where

$$f(x_1, \cdots, x_s)$$

$$= \lim_{N_1 \to \infty} \frac{1}{2\pi} \int_0^{N_1} \{ \lim_{N_2 \to \infty} \frac{1}{2\pi} \int_0^{N_2} f_2(\sigma_1, \sigma_2, x_3, \cdots, x_s)\, e^{i\sigma_2 x_2} \prod_{k=1,\cdots n_k} d\sigma_{2,k} \prod_{k=1,\cdots n_k} d\sigma_{1,k}.$$

Then continuing such way we get our assumption. [3, 6, 13, 14, 20, 28]

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ABSTRACT

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I. INTRODUCTION

Nitrogen is known to be one of the main essential elements forming a part of amino acids, DNA, ATP and other major organic substances [1]. In general, live organisms are not capable of fixing molecular nitrogen from atmospheric air though Earth's atmosphere, which is composed of about 78% nitrogen gas. Nitrogen-fixing bacteria, in particular, Azotobacter chroococcum, are able to obtain this gas from the atmosphere and are the first known type of such microorganisms. They can live in conditions of increased content of carbon dioxide and fix nitrogen in aerobic conditions [2]. The application of biostimulators, containing these bacteria and other useful microorganisms, in crop production contributes protein (consisting of amino acids) in agricultural crops and is very important for human adequate nutrition.

Recently the application of plant biostimulators containing beneficial microorganisms has become more urgent due to organic agricultural development.

Biostimulators differ from other fertilizers as they can contribute to plant growth when kept in small amounts. The term "biostimulator" was first applied by Kauffman et al. [3], has become more popular in the following years, embracing a broader range of elements [4-6]. A plant biostimulator is any substance or microorganism applied to plants with the purpose to enhance nutrition efficiency, abiotic stress tolerance and to improve crop quality [7-8]. Biostimulators promote root and fruit formation in plants, facilitate nutritional and water taking, their transportation and utilization from the soil, increase plant disease resistance and recovery after biotic and abiotic stresses, improve metabolic processes in plants, contributing to crop output and quality.

It is well-known an important role that play effective soil microorganisms in creation of optimal environment for soil nutrition of plants [9]. Plant growth promoting rhizobacteria are multifunctional and affect plant lives in all aspects: nutrition and growth, morphogenesis and development, biotic and abiotic stress tolerance, interaction with other organisms in agroecosystems [10-17].

Mycorrhizal fungi form symbiotic associations with 90% of all plants. An arbuscular mycorrhizal fungus (AMF) is a widespread type of endomycorrhizal fungi [18-19]. There is a growing concern of mycorrhiza utilization for plant nutrition, water balance provision, plant protection against biotic and abiotic stresses [20-25]. Gifalny networks can connect not only fungi and plant partners, but also some plants in the plant community [26-27].

The utilization of nonpathogenic soil bacteria, living in plant roots, is very perspective and opens considerable opportunities for organic farming [28-30]. The bacteria belonging to Bacillus type, especially Bac. subtilis, can be used for effective biocontrol of different plant diseases caused by soil pathogens [31-33]. Fungal entophytes, such as Trichoderma, are capable of colonizing plant roots transporting nutrients to them [34].

The biostimulant application on the basis of beneficial microorganisms (bacteria and fungi) for producing ecologically safe crop products with the optimum composition of amino acids, vitamins, antioxidants, vital chemical elements and other substances, essential to human health, is one of the most important problems of agriculture.

In the study we conduct the research on how biostimulants with beneficial microorganisms influence amino acids concentrations in red beetroots.

II. EXPERIMENTAL

The investigated biostimulator applied for red beet planting contains: nitrogen-fixing Azotobacter chroococcum bacterium, Trichoderma harzianum fungus and Bacillus subtilis bacterium as antagonists of phytopathogenes, and Bacillus megaterium bacterium as an immunomodulator. Except for purging phytopathogenes in the rhizosphere of plant roots, the spore-forming Bacillus subtilis bacterium has high enzymatic activity and decomposes different organic substances in the soil. The Bacillus megaterium bacterium triggers the natural defense response in plants and induces their immunity, it also regulates nutritious elements received by plants from the soil. Trichoderma harzianum fungus is an effective bioagent aimed at fighting against root rots, it suppresses the development of disease excitants in plants. Except for useful microorganisms, the investigated biostimulant contains the organic and mineral structure.

In our research we use red beet of the short-season «Gleb» variety. The period between seedling and harvesting stages lasts 80-100 days. The average weight of this beetroot is 200-300 g, it has intensive-purple coloring, is smooth-skinned, has flattened shape and rounded oval form. This variety is versatile in use, resistant to premature seeding and can be kept well.

The experiment was conducted on the sod-podzolic moderately clayey soil of the Moscow region. The field experience laid out on the following scheme: 1) control group; 2) biostimulant group. Red beet was sowed on sample plots 50 sq. m in area, with 4-fold frequency. The biostimulant was applied twice during vegetation with a time interval of a month. It was previously dissolved in water in the quantity of 10 g of stimulator per one sample plot and then plants were watered with the subsequent soil loosening. In 90 days after seeding the beetroots were removed and measured on the content of amino acids.

The amino-acid composition of beetroots was estimated by means of capillary electrophoresis method on analyzer Kapel -105 M.

III. RESULTS AND DISCUSSION

Red beet is the most widespread and major vegetable culture with the high content of essential vitamins for humans (groups B, C, PP, etc.), betaine, sugars, protein, folic acid, vital macro -

microelements (magnesium, potassium, calcium, iron, iodine and especially silicon and chrome), bioflavonoids. Its root crops contain both the irreplaceable, and replaceable amino acids which are part of protein. This vegetable culture renders general tonic influence on human organisms, as well as digestion and metabolism improving effects.

The lack of protein in human nutrition breaks normal organism life activity and leads to serious negative effects. Amino acids, irreplaceable for the majority of animals and human beings, are valine, isoleucine, leucine, threonine, methionine, lysine, phenylalanine, tryptophane, arginine, histidine. Replaceable amino acids include glycine, alanine, proline, serine, cysteine, aspartic acid, glutamine acid, tyrosine.

In our field experience we determined the contents of 17 amino acids (Table) in red beet of short-season «Gleb» variety cultivated with the innovative biostimulant.

Table: Amino-acid composition of «Gleb» variety red beet (g/kg of dry weight)

Indicators of amino-acid composition	Experimental group	Control group
Aspartic acid	4.72±0.40	2.67±0.21
Threonine	1.67±0.16	0.92±0.08
Glutamine acid	24.14±2.11	7.41±0.69
Serine	2.74±0.23	1.31±0.11
Proline	1.46±0.12	0.40±0.01
Glycine	0.94±0.08	0.47±0.04
Alanine	1.80±0.16	0.73±0.07
Cysteine	1.12±0.10	0.64±0.05
Valine	0.003±0.000	0.001±0.000
Isoleucine	1.20±0.13	0.20±0.00
Leucine	1.66±0.15	0.97±0.09
Tyrosine	1.05±0.09	0.80±0.07
Phenylalanine	0.67±0.06	0.46±0.04
Histidine	0.84±0.07	0.58±0.05
Lysine	1.51±0.13	0.70±0.07
Arginine	1.30±0.09	0.40±0.000
Tryptophane	0.43±0.04	0.38±0.03
Total amount of amino acids	47.25±4.21	19.04±1.56
Including irreplaceable	9.28±0.90	4.41±0.41
Crude protein, %	7.43±0.67	6.56±0.58

The total amount of amino acids was 2.5 times higher in the samples planted with the biostimulator, compared to the control ones, 47.25 g/kg and 19.04 g/kg of dry weight consequently. As far as the total amount of essential amino acids is concerned, it increased by 2.1 times (from 4.41 g/kg in the control group to 9.28 g/kg of dry weight in the experimental one).

Applied biostimulant contributed to the increased content of all investigated amino acids: tryptophane – by 1.1 times (from 0.38 to 0.43 g/kg of dry weight), tyrosine – by 1.3 times (from 0.80 to 1.05 g/kg of dry weight), histidine – by 1.4 times (from 0.58 to 0.84 g/kg of dry weight), phenylalanine - by 1.5 times (from 0.46 to 0.67 g/kg of dry weight), leucine – by 1.7 times (from 0.97 to 1.66 g/kg of dry weight), aspartic acid (from 2.67 to 4.72 g/kg of dry weight), threonine (from 0.92 to 1.67 g/kg of dry weight) and cystine (from 0.64 to 1.12 g/kg of dry weight) – by 1.8 time, glycine – twice (from 0.47 to

0.94 g/kg of dry weight), serine – by 2.1 times (from 1.31 to 2.74 g/kg of dry weight), lysine – by 2.2 times (from 0.70 to 1.51 g/kg of dry weight), alanine – by 2.5 times (from 0.73 to 1.80 g/kg of dry weight), valine – by 3 times (from 0.001 to 0.003 g/kg of dry weight), arginine (from 0.40 to 1.30 g/kg of dry weight) and glutamine acid (from 7.41 to 24.14 g/kg of dry weight) – by 3.3 time, proline – by 3.7 times (from 0.40 to 1.46 g/kg of dry weight), isoleucine – by 6 times (from 0.20 to 1.20 g/kg of dry weight).

Accumulated beneficial microorganisms in beetroots led to several times increase in the content of such essential amino acids as valine, isoleucine, lysine. Valine is an important amino acid which is associated with normal metabolism in brain muscle tissues, it also participates in body regeneration processes. The isoleucine has the immunostimulating properties, it is an important component of hormone and enzyme synthesis, it also promotes muscle gains. Lysine is necessary for synthesis of the major organism proteins – nucleoproteids. The lack of this amino acid delays formation of communicating tissues.

IV. CONCLUSION

Planting of «Gleb» variety red beet with the application of the biostimulant containing nonpathogenic microorganisms Azotobacter chroococcum, Bacillus subtilis, Bacillus megaterium and Trichoderma harzianum contributed to the increased contents of 17 amino acids in its beetroots, such as valine, isoleucine, leucine, threonine, lysine, phenylalanine, tryptophane, arginine, histidine, glycine, alanine, proline, serine, cysteine, asparaginic and glutamine acids, tyrosine, nine of which are irreplaceable.

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The mΘ Protocol F5 and Hamming mΘ Codes

Pemha Binyam Gabriel Cedric

ABSTRACT

The mΘ structure introduce pure and applied mathematics using the set FpZ ≡ Fp [fxpZ j e(x = 0(mod(p)))g, p prime, to then present mathematical structures resulting from the sets originally introduced F. Ayissi Eteme [6]. This work consists in defining on FpZ the notion of Hamming code according to mΘ set structure. We show a relation between mΘ protocol F5 and Hamming mΘ code. By using this relation, we get a new steganography based on the bit modalities of a code word.

Keywords: mΘ set, Hamming mΘ codes, steganography, mΘ protocol F5.

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ABSTRACT

The mΘ structure introduce pure and applied mathematics using the set FpZ ≡ Fp [fxpZ j e(x = 0(mod(p)))g, p prime, to then present mathematical structures resulting from the sets originally introduced F. Ayissi Eteme [6]. This work consists in defining on FpZ the notion of Hamming code according to mΘ set structure. We show a relation between mΘ protocol F5 and Hamming mΘ code. By using this relation, we get a new steganography based on the bit modalities of a code word.

Keywords: mΘ set, Hamming mΘ codes, steganography, mΘ protocol F5.

I. INTRODUCTION

Steganography [5, 7] can be comparable to protection of communication, as it is known as a technique being used in order to protect some information to be exchanged by hiding its original existence, onto some digital files, could it be a photographies or videograms. As we know of cryptography as the technique and science behind the protection of messages and information to be transmitted, the idea for steganography is actually to prevent a nasty observer to even detect the need for that protection firstly, and it is also depending on the situations, as for instance in places where cryptography cannot be used.

Sometimes, it is also possible to mix together both techniques for protection of communication and information. The classic example known to illustrate use of a steganographic scheme is the prisoners problem exchanging messages under the surveillance of a warden. Crandall [10] was the first to suggest it and later Westfeld [11] applied it.

The $m\Theta$ codes [2, 12, 13, 14] present an enrichment from the logical viewpoint and we can mathematically express that an information is lightly, partially or greatly damaged.

Let E be a finite $m\Theta$ set. A non-empty subset $\mathcal{C}$ of E is called a $m\Theta$ code. E is the $m\Theta$ set of n-tuples from a finite $m\Theta$ set $\mathbb{F}_{p\mathbb{Z}}$ with p^2 elements. Each element of E is called $m\Theta$ words and the elements of $\mathcal{C}$, $m\Theta$ codewords. E is an n-dimensional vector space over $\mathbb{F}_{p\mathbb{Z}}$, so $E = V(n, p\mathbb{Z})$.

Section 2 define firstly the modal Θ-valent set structure and the algebra of $(\mathbb{F}_{p\mathbb{Z}}, F_\alpha)$, secondly the linear $m\Theta$ codes and lastly the Hamming $m\Theta$-distance of $(\mathcal{C}, F^n_{\alpha|\mathcal{C}})$. Section 3 presents the Hamming codes on $V(n, 2\mathbb{Z})$. Section 4 is devoted to the $m\Theta$ steganographic protocol $F5$ and Hamming $m\Theta$ codes.

II. PRELIMINARIES

2.1 The modalΘ-valent set structure and the algebra of (FpZ; Fa)

Definition 0.1. *[15] Let* $E \neq \emptyset$, *I be a chain whose first and last elements are* 0 *and* 1 *respectively,* $(F_\alpha)_{\alpha\in I_*}$ *where* $I_* = I \setminus \{0\} = \{f : E \longrightarrow E \,|\, f \text{ application}\}$.

A $m\Theta$ *set is the pair* $\left(E, (F_\alpha)_{\alpha\in I_*}\right)$ *or* (E, F_α) *such that :*

- $\underset{\alpha}{\cap} F_\alpha(E) = \underset{\alpha\in I_*}{\cap} \{F_\alpha(x) : x \in E\} \neq \emptyset$;
- $\forall \alpha, \beta \in I_*$, *if* $\alpha \neq \beta$ *then* $F_\alpha \neq F_\beta$;
- $\forall \alpha, \beta \in I_*$, $F_\alpha \circ F_\beta = F_\beta$;
- $\forall x, y \in E$, *if* $\forall \alpha \in I_*, F_\alpha(x) = F_\alpha(y)$ *then* $x = y$.

Theorem 0.1. *[8]*

Let (E, F_α) *be a* $m\Theta$ *set.*

$$\forall x,\, y \in E,\, x =_\Theta y \iff \forall \alpha \in I_*,\, F_\alpha(x) = F_\alpha(y)\,.$$

Proof 0.1. *[8]*

Definition 0.2. *[12] Let* $\mathrm{C}(E, F_\alpha) = \underset{\alpha\in I_*}{\cap} F_\alpha(E)$. *We call* $\mathrm{C}(E, F_\alpha)$ *the set of* $m\Theta$ *invariant elements of the* $m\Theta$ *set* (E, F_α).

Proposition 0.1. *[8] Let* (E, F_α) *be a* $m\Theta$ *set. The following properties are equivalent:*

1. $x \in \underset{\alpha\in I_*}{\cap} F_\alpha(E)$;
2. $\forall \alpha \in I_*$, $F_\alpha(x) = x$;
3. $\forall \alpha, \beta \in I_*$, $F_\alpha(x) = F_\beta(x)$;
4. $\exists \mu \in I_*$, $x = F_\mu(x)$.

Proof 0.2. *[8]*

Definition 0.3. *[1]*
Let (E, F_α) and (E', F'_α) be two $m\Theta$ sets. We shall call

1. *(E', F'_α) a $m\Theta$ subset of (E, F_α) if the structure of $m\Theta$ set (E', F'_α) is the restriction to E' of the structure of the $m\Theta$ set (E, F_α), that means:*
   - *$E' \subseteq E$;*
   - *$\forall\alpha : \alpha \in I_*,\ F^{'}_\alpha = F_{\alpha|_{E'}}$.*
2. *Let X be a non-empty set. X a $m\Theta$ subset of (E, F_α) if:*
   - *$X \subseteq E$;*
   - *$(X, F_{\alpha|X})$ is a $m\Theta s$ which is a $m\Theta$ subset of (E, F_α).*

Let p be a prime number. Let us recall that if $a \in \mathbb{F}_{p\mathbb{Z}}$.

$$\mathbb{F}_{p\mathbb{Z}} = \mathbb{F}_p \cup \{x_{p\mathbb{Z}} : \neg (x \equiv 0\,(modp))\}\,; \quad \mathbb{F}_p = \{0,\,1,\,2,\,\cdots,p-1\}.$$

Let us define $s(a)$ the $m\Theta$ support of a as follows:

$$s\,(a) = \begin{cases} a & if\; a \in \mathbb{F}_p; \\ x & if\; a = x_{p\mathbb{Z}}\, with\, \rceil(x \equiv 0\,(mod\,p))\,. \end{cases}$$

Thus $s\,(a) \in \mathbb{F}_p$.

Definition 0.4. *[15] Let $\perp$ be a binary operation on $\mathbb{F}_p$. So, $\forall a,\, b \in \mathbb{F}_p$, $a \perp b \in \mathbb{F}_p$. Let $x,\, y \in \mathbb{F}_{p\mathbb{Z}}$. We define a binary operation $\perp^*$on $\mathbb{F}_{p\mathbb{Z}}$ as follows :*

$$x \perp^* y = \begin{cases} s(x) \perp s(y) & if \begin{cases} x, y \in \mathbb{F}_p \\ (s(x) \perp s(y)) \equiv 0\,(mod\,p) \end{cases} otherwise \\ (s(x) \perp s(y))_{p\mathbb{Z}} & otherwise. \end{cases}$$

$\perp^*$ as defined above on $\mathbb{F}_{p\mathbb{Z}}$ will be called a $m\Theta$ law on $\mathbb{F}_{p\mathbb{Z}}$ $\forall x, y \in \mathbb{F}_{p\mathbb{Z}}$.
So we can define $x + y \in \mathbb{F}_{p\mathbb{Z}}$ and $x \times y \in \mathbb{F}_{p\mathbb{Z}}$ $\forall x, y \in \mathbb{F}_{p\mathbb{Z}}$.

Theorem 0.2. *[1] $(\mathbb{F}_{p\mathbb{Z}}, F_\alpha, +, \times)$ is a $m\Theta$ ring of unity 1 and of $m\Theta$ unity $\frac{1}{p\mathbb{Z}}$.*

Proof 0.3. *[1]*

Remark 0.1. *Since p is prime, $(\mathbb{F}_{p\mathbb{Z}},\, F_\alpha)$ is a $m\Theta$ field.*

Definition 0.5. *[6] x is a divisor of zero in $(\mathbb{F}_{p\mathbb{Z}},\, F_\alpha)$ if $\exists y \in \mathbb{F}_{p\mathbb{Z}}$ verifying $x \times y = 0$*

Example 0.1. *[6] By this example, we show that $\mathbb{F}_{p\mathbb{Z}}$, p prime, is a $m\Theta$ field of p^2 elements.*
$p = 2$, we have $\mathbb{F}_{2\mathbb{Z}} = \{0,\, 1,\, 1_{2\mathbb{Z}},\, 3_{2\mathbb{Z}}\}$

The table of $m\Theta$ determination and tables laws of $\mathbb{F}_{2\mathbb{Z}}$.

$\mathbb{F}_{2\mathbb{Z}}$	0	1	$1_{2\mathbb{Z}}$	$3_{2\mathbb{Z}}$
F_1	0	1	1	*0*
F_2	0	1	0	*1*

$+^{\Theta}$	*0*	*1*	$1_{2\mathbb{Z}}$	$3_{2\mathbb{Z}}$
0	*0*	*1*	$1_{2\mathbb{Z}}$	$3_{2\mathbb{Z}}$
1	*1*	*0*	*0*	*0*
$1_{2\mathbb{Z}}$	$1_{2\mathbb{Z}}$	*0*	*0*	*0*
$3_{2\mathbb{Z}}$	$3_{2\mathbb{Z}}$	*0*	*0*	*0*

$\times^{\Theta}$	*0*	*1*	$1_{2\mathbb{Z}}$	$3_{2\mathbb{Z}}$
0	*0*	*0*	*0*	*0*
1	*0*	*1*	$1_{2\mathbb{Z}}$	$3_{2\mathbb{Z}}$
$1_{2\mathbb{Z}}$	*0*	$1_{2\mathbb{Z}}$	$1_{2\mathbb{Z}}$	$3_{2\mathbb{Z}}$
$3_{2\mathbb{Z}}$	*0*	$3_{2\mathbb{Z}}$	$3_{2\mathbb{Z}}$	$1_{2\mathbb{Z}}$

2.2 Linear mΘ codes

Let $(A,\, F_\alpha)$ be a finite $m\Theta$ set. $\forall n \in \mathbb{N}^*$, we will denote in what follows the $m\Theta$ set product of $(A,\, F_\alpha)$ by $(A^n,\, F_\alpha^n)$; where F_α^n is the product on A^n of F_α. By definition, we have:

$$F_\alpha^n : A^n \longrightarrow A^n; \quad (a_1, \cdots, a_n) \longmapsto F_\alpha^n(a_1, \cdots, a_n) = (F_\alpha(a_1), \cdots, F_\alpha(a_n))$$

$k,\, n \in \mathbb{N}^*$ such that $k \leq n$.

Definition 0.6. *[14] Let us set $\mathcal{C} = f(E)$ the image of f. As f is injective, f is a $m\Theta$ bijection from E to $\mathcal{C}$. $(\mathcal{C},\, F_\alpha^n|_{\mathcal{C}})$ is considered as the $m\Theta$ set of all possible $m\Theta$ messages.*

1. *A $m\Theta$ code of length n and of alphabet $(A,\, F_\alpha)$, the $m\Theta$ set $(\mathcal{C},\, F_\alpha^n|_{\mathcal{C}})$.*
2. *Elements of $\mathcal{C}$, $m\Theta$ messages or $m\Theta$ words of the $m\Theta$ code $(\mathcal{C},\, F_\alpha^n|_{\mathcal{C}})$.*
3. *Elements of $\mathcal{C}$, $(\mathcal{C},\, F_\alpha^n|_{\mathcal{C}}) = \cap_{\alpha \in I_*} F_\alpha^n|_{\mathcal{C}}(\mathcal{C})$, messages or words of the $m\Theta$ code $(\mathcal{C},\, F_\alpha^n|_{\mathcal{C}})$.*

Proposition 0.2. *[12] $(\mathcal{C},\, F_\alpha^n|_{\mathcal{C}})$ is a $m\Theta$ part of $(A^n,\, F_\alpha^n)$.*

Proof 0.4. *[12]*

Proposition 0.3. *[12] Let $(\mathcal{C},\, F_\alpha^n|_{\mathcal{C}})$ be a $m\Theta$ code of length n on $(A,\, F_\alpha)$. The set $C(\mathcal{C},\, F_\alpha^n|_{\mathcal{C}}) = \cap_{\alpha \in I_*} F_\alpha^n|_{\mathcal{C}}(\mathcal{C})$ is a classical code of length n on $\cap_{\alpha \in I_*} F_\alpha(A) = C(A,\, F_\alpha)$.*

Definition 0.7. *[13] Let $(\mathbb{F}_{2\mathbb{Z}},\, F_\alpha)$ be the $m\Theta$ field with $Card\,(\mathbb{F}_{2\mathbb{Z}},\, F_\alpha) = 4$ $\forall \alpha \in I_*$,*

1. *Hamming α-weight of $x = (x_1, \cdots, x_n) \in (V(n,\, 2\mathbb{Z}),\, F_\alpha^n)$ is the number $\omega_{H_\alpha}(x) = \omega(F_\alpha^n(x))$, of non zero coordinates of $F_\alpha^n(x)$.*

$$\omega_{H_\alpha}(x) = \omega(F_\alpha^n(x)) = Card\{i \mid F_\alpha(x_i) \neq 0;\, i = 1, \cdots, n\}.$$

2. *Hamming* $m\Theta$*-weight of* $x = (x_1, \cdots, x_n) \in (V(n, 2\mathbb{Z}), F_\alpha^n)$ *is the number* $\omega_{H_\Theta}(x)$ *and defined by:*

$$\omega_{H_\Theta}(x) = \begin{cases} \omega(x) & x \in \mathbb{F}_2^n; \\ \sum_{\alpha \in I_*} \omega_{H_\alpha}(x) = \sum_{\alpha \in I_*} \omega(F_\alpha^n(x)) & otherwise. \end{cases}$$

The alphabet used is the $m\Theta$ field $(\mathbb{F}_{p\mathbb{Z}} = (\frac{\mathbb{Z}_{p\mathbb{Z}}}{p\mathbb{Z}_{p\mathbb{Z}}}, F_\alpha))$.

Proposition 0.4. *[6] We set* $E = V(k, p\mathbb{Z})$ *and* $\mathcal{C} = f(E)$*. Let* (E, F_α^k) *be the* $m\Theta$ *set of* $m\Theta$ *message and* f *a linear* $m\Theta$ *encoder of* (E, F_α^k) *in* $(V(n, p\mathbb{Z}), F_\alpha^n)$*.*
Then, the $m\Theta$ *code* $(\mathcal{C}, F_{\alpha|\mathcal{C}}^n)$ *is a* $m\Theta$ *vector subspace of* $(V(n, p\mathbb{Z}), F_\alpha^n)$ *over* $(\mathbb{F}_{p\mathbb{Z}}, F_\alpha)$*.*

Proof 0.6. *[6]*

Definition 0.8. *[14] A* $m\Theta$ *linear code of length* n *and of* $m\Theta$ *dimension* k *on* $(\mathbb{F}_{p\mathbb{Z}}, F_\alpha)$ *is a* $m\Theta$ *vector subspace of* $(V(n, p\mathbb{Z}), F_\alpha^n)$ *of* $m\Theta$ *dimension* k*. .*

Proposition 0.5. *[6] Let* $(\mathcal{C}, F_{\alpha|\mathcal{C}}^n)$ *be a linear* $m\Theta$ *code of length* n *and of* $m\Theta$ *dimension* k*.*
Then $C(\mathcal{C}, F_{\alpha|\mathcal{C}}^n) = \cap_{\alpha \in I_*} F_\alpha^n(\mathcal{C})$ *is a linear code of length* n *and of dimension* k*.*

Proof 0.7. *[6]*

As $C(V(k, p\mathbb{Z}), F_\alpha^k)$ is a $\mathbb{F}_p$-vector space of dimension k, so $C(\mathcal{C}, F_{\alpha|\mathcal{C}}^n)$ is a linear code of length n and $dim\,(C(V(k, p\mathbb{Z}), F_\alpha^k)) = dim\,(C(\mathcal{C}, F_{\alpha|\mathcal{C}}^n))$.

2.3 The Hamming mΘ-distance of (C; $F\frac{n}{a}$c)

Let $(\mathcal{C}, F_{\alpha|\mathcal{C}}^n)$ be a $m\Theta$ or a pseudo $m\Theta$ code on (A, F_α) of length n. In $(\mathcal{C}, F_{\alpha|\mathcal{C}}^n)$, let us define a notion compatible with the structure of $m\Theta$ code called $m\Theta$ distance.

$\forall \alpha \in I_*$, d_{H_α} on $A^n \times A^n$ is defined by:

$$\begin{aligned} d_{H_\alpha}(x, y) &= d_H(F_\alpha^n x, F_\alpha^n y) \\ &= card\{i : F_\alpha x_i \neq F_\alpha y_i\,;\, i = 1, \cdots, n\}. \end{aligned}$$

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Where $x = (x_1, \cdots, x_n)$; $y = (y_1, \cdots, y_n)$ and d_H is the Hamming distance on $(C(A, F_\alpha))^n$.

Proposition 0.6. *If (A, F_α) is a $m\Theta$ set and $(\mathcal{C}, F_\alpha)$ is a $m\Theta$ code on (A, F_α), then*
$\forall x, y \in A^n$, d_{H_Θ} on $A^n \times A^n$ is defined by:

$$d_{H_\Theta}(x, y) = \begin{cases} d_H(x, y) & \text{if } x \text{ and } y \in (C(A, F_\alpha))^n; \\ \sum_{\alpha \in I_*} d_{H_\alpha}(x, y) = \sum_{\alpha \in I_*} d_H(F_\alpha x, F_\alpha y) & \text{otherwise.} \end{cases}$$

$F_\alpha^n x = (F_\alpha x_1, \cdots, F_\alpha x_n)$; $F_\alpha^n y = (F_\alpha y_1, \cdots, F_\alpha y_n)$. Then d_{H_Θ} is a $m\Theta$ distance on (A^n, F_α^n).

Proof 0.8. *[12]*

Definition 0.9. *d_{H_Θ} will be called the Hamming $m\Theta$ distance on (A^n, F_α^n).*

Definition 0.10. *Let $(\mathcal{C}, F_\alpha)$ be a $m\Theta$ code; d_{H_Θ} is the $m\Theta$ Hamming distance. We define δ^Θ as follows:*

$$\delta^\Theta = \min \{d_{H_\Theta}(x, y) : x, y \in \mathcal{C}; x \neq y\}.$$

δ^Θ is the minimal $m\Theta$ distance of the $m\Theta$ code $(\mathcal{C}, F_\alpha)$.

III. THE HAMMING MΘ CODES

3.1 Dual mΘ codes

Let $(\mathcal{C}, F_\alpha)$, $\forall \alpha \in I_*$, be a linear $m\Theta$ code in $V(n, p\mathbb{Z})$. Let G be a $m\Theta$ matrix whose rows generate $(\mathcal{C}, F_\alpha)$. Let G be a generating $m\Theta$ matrix of $(\mathcal{C}, F_\alpha)$.

The dual $m\Theta$ code of $(\mathcal{C}, F_\alpha)$, denoted $\mathcal{C}^\perp$, is defined as follows

$$\mathcal{C}^\perp = \{x \in V(n, p\mathbb{Z});\ \forall \alpha \in I_*\ , \langle F_\alpha x, F_\alpha c\rangle = 0,\ \forall c \in (\mathcal{C}, F_\alpha)\}$$

$\forall u, v \in V(n, p\mathbb{Z})$ $\langle u, v\rangle := u_1v_1 + u_2v_2 + \cdots + u_nv_n$. $\mathcal{C}^\perp$ is clearly also a linear $m\Theta$ code, and has a generating $m\Theta$ matrix H. By the definition of $\mathcal{C}^\perp$,

$$\mathcal{C} = \{c \in V(n, p\mathbb{Z})/\ \forall \alpha \in I_*,\ F_\alpha(c)H^t = 0\}.$$

Where H is a parity check $m\Theta$ matrix for $(\mathcal{C}, F_\alpha)$. If a $m\Theta$ word u is received, then it can be verified that u is a $m\Theta$ codeword such that $uH^t = 0$, i.e, $\forall \alpha \in I_*, F_\alpha(u)H^t = 0$.

3.2 Hamming Codes on V (n; 2Z)

Hamming $m\Theta$ code is a linear $m\Theta$ code in $V(n, 2\mathbb{Z})$ for some $n \geq 2$. Let $\mathbb{F}_{2\mathbb{Z}}$ be the $m\Theta$ field of four elements and let H be the matrix whose columns are all the non-zero $m\Theta$ vectors of length k over $\mathbb{F}_{2\mathbb{Z}}$, $\forall k \leq n$. Note that there will be $2^k - 1$ of these. We define the Hamming $m\Theta$ code as follows:

Definition 0.11. *Let $k \geq 2$ and $n = 2^k - 1$. Let H denote the $k \times n$ $m\Theta$ matrix. The Hamming $m\Theta$ code, $Ham_{2\mathbb{Z}}(n)$, is the linear $m\Theta$ subspace of $V(n, 2\mathbb{Z})$ consisting of the set of all α-vectors, $\alpha \in I_*$, orthogonal to all the rows of H.*

$$Ham_{2\mathbb{Z}}(n) = \{v \in V(n, 2\mathbb{Z}) \,/\, \forall \alpha \in I_*,\, F_\alpha(v) \times H^t = 0\}.$$

Proposition 0.7. *The Hamming $m\Theta$ code $Ham_{2\mathbb{Z}}(n)$ is a $(2^k-1,\, 2^k-k-1,\, 3)$-code with $k \times (2^k - 1)$ parity check $m\Theta$ matrix .*

Proof 0.9. *[3]*

IV. THE MΘ STEGANOGRAPHIC PROTOCOL F5 AND HAMMING MΘ CODES

4.1 The mΘ protocol F5

$F5$ is a steganographic system developed by Westfeld in 2001 [11]. The $m\Theta$ protocol $F5$ over $\mathbb{F}_{2\mathbb{Z}}$ permits to hide $m\Theta$ messages of length k in cover $m\Theta$ words of length $n = 2^k - 1$ by partially or totally changing more than one of them .

Let $< F_\alpha^k m >_2$ be the α-binary word of m with k bits, $< m >_2 \in V(k, 2\mathbb{Z})$. Conversely, for $z \in V(k, 2\mathbb{Z})$, $\forall \alpha \in I_*$, let $< F_\alpha^k z >_{10}$ be an element of $\mathbb{F}_2^k$ which has $F_\alpha^k z$ as α-binary word, so $1 \leq < F_\alpha^k(z) >_{10} \leq 2^k - 1$.

Lastly, let e_i be the i^{th} α-vector of the canonical basis of $V(2^k-1, 2)$; $e_0 = 0_{V(2^k-1,\,2)}$.

Proposition 0.8. *The $m\Theta$ maps $\gamma_{2\mathbb{Z}}$, $e_{2\mathbb{Z}}$, and $r_{2\mathbb{Z}}$ as follows define:*

(i)
$$\begin{aligned} \gamma_{2\mathbb{Z}} : V(2^k-1, 2\mathbb{Z}) \times V(k, 2\mathbb{Z}) &\longrightarrow (\mathbb{N}_{2\mathbb{Z}}, F_\alpha') \\ (x, m) &\longmapsto (< F_\alpha^k(m) + \sum_{i=1}^{2^k-1} F_\alpha(x_i) < i >_2 >_{10})_{\alpha \in I_*} \end{aligned}$$

(ii)
$$\begin{aligned} e_{2\mathbb{Z}} : V(2^k-1, 2\mathbb{Z}) \times V(k, 2\mathbb{Z}) &\longrightarrow V(2^k-1, 2\mathbb{Z}) \\ (x, m) &\longmapsto (F_\alpha^{2^k-1}(u) + e_{F_\alpha'(\gamma_{2\mathbb{Z}}(x,\, m))})_{\alpha \in I_*} \end{aligned}$$

(iii)
$$\begin{aligned} r_{2\mathbb{Z}} : V(2^k-1, 2\mathbb{Z}) &\longrightarrow V(k, 2\mathbb{Z}) \\ x &\longmapsto (\sum_{i=1}^{2^k-1} F_\alpha(x_i) < i >_2)_{\alpha \in I_*} \end{aligned}$$

are well defined and $m\Theta$.

Proof 0.10.

(i) • *Let* $(x,\, m),\, (x',\, m') \in V(2^k - 1,\, 2\mathbb{Z}) \times V(k,\, 2\mathbb{Z})$

let us suppose that $(x,\, m) = (x',\, m')$ ($x = x'$ *and* $m = m'$) *and let us show that* $\gamma_{2\mathbb{Z}}(x,\, m) = \gamma_{2\mathbb{Z}}(x',\, m')$.

$$(x,\, m) = (x',\, m') \Longrightarrow \forall \alpha \in I_* \begin{cases} F_\alpha^{2^k-1}x = F_\alpha^{2^k-1}x' \\ F_\alpha^k m = F_\alpha^k m' \end{cases}$$

$\forall \alpha \in I_*$;

$$F_\alpha^k m + \sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2 = F_\alpha^k t + \sum_{i=1}^{2^k-1} F_\alpha x_i' < i >_2$$
$$\Longrightarrow < F_\alpha^k m + \sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2 >_{10} = < F_\alpha^k m' + \sum_{i=1}^{2^k-1} F_\alpha x_i' < i >_2 >_{10}$$
$$\Longrightarrow (< F_\alpha^k m + \sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2 >_{10})_{\alpha \in I_*} = (< F_\alpha^k m' + \sum_{i=1}^{2^k-1} F_\alpha x_i' < i >_2 >_{10})_{\alpha \in I_*}$$
$$\Longrightarrow \gamma_{2\mathbb{Z}}(x,\, m) = \gamma_{2\mathbb{Z}}(x',\, t)$$

Therefore the map $\gamma_{2\mathbb{Z}}$ *is well defined.*

• *Let us verify* $\gamma_{2\mathbb{Z}}$ *is* $m\Theta$ *map.*

Let $(x,\, m),\, (x',\, m') \in V(2^k - 1,\, 2\mathbb{Z}) \times V(k,\, 2\mathbb{Z})$
$\forall \alpha \in I_*$,

$$\begin{aligned}
\gamma_{2\mathbb{Z}} \circ (F_\alpha^{2^k-1},\, F_\alpha^k)(x,\, m) &= \gamma_{2\mathbb{Z}}(F_\alpha^{2^k-1}x,\, F_\alpha^k m) \\
&= (< F_\alpha^k(F_\alpha^k m) + \sum_{i=1}^{2^k-1} F_\alpha((F_\alpha^{2^k-1}x)_i) < i >_2 >_{10})_{\alpha \in I_*} \\
&= (< F_\alpha^k m + \sum_{i=1}^{2^k-1} (F_\alpha^{2^k-1}x)_i < i >_2 >_{10})_{\alpha \in I_*} \\
&= (< F_\alpha^k m + \sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2 >_{10})_{\alpha \in I_*} \\
F_\alpha' \circ \gamma_{2\mathbb{Z}}(x,\, m) &= F_\alpha'(< F_\alpha^k m + \sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2 >_{10})_{\alpha \in I_*} \\
&= (< F_\alpha^k m + \sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2 >_{10})_{\alpha \in I_*}
\end{aligned}$$

Therefore $\gamma_{2\mathbb{Z}}$ *is a* $m\Theta$ *map.*

(ii) • $(x, m), (x', m') \in V(2^k-1, 2\mathbb{Z}) \times V(k, 2\mathbb{Z})$ *such that* $(x, m) = (x', m')$ ($x = x'$ *and* $m = m'$), *let's show that* $e_{2\mathbb{Z}}(x, m) = e_{2\mathbb{Z}}(x', m')$.

$$(x, m) = (x', m') \Longrightarrow \forall \alpha \in I_*, \begin{cases} F_\alpha^{2^k-1}x = F_\alpha^{2^k-1}x' \\ F_\alpha^k m = F_\alpha^k m' \end{cases}$$

$$\begin{aligned}
\forall \alpha \in I_*, \begin{cases} F_\alpha^{2^k-1}x = F_\alpha^{2^k-1}x' \\ F_\alpha^k m = F_\alpha^k m' \end{cases} \quad &\Rightarrow \quad \forall \alpha \in I_*, \begin{cases} F_\alpha^{2^k-1}x = F_\alpha^{2^k-1}x' \\ \gamma_{2\mathbb{Z}}(x, m) = \gamma_{2\mathbb{Z}}(x', m') \end{cases} \\
&\Rightarrow \quad \forall \alpha \in I_*, \begin{cases} F_\alpha^{2^k-1}x = F_\alpha^{2^k-1}x' \\ F'_\alpha\gamma_{2\mathbb{Z}}(x, m) = F'_\alpha\gamma_{2\mathbb{Z}}(x', m') \end{cases} \\
&\Rightarrow \quad \forall \alpha \in I_*, \begin{cases} F_\alpha^{2^k-1}x = F_\alpha^{2^k-1}x' \\ e_{F'_\alpha\gamma_{2\mathbb{Z}}(x, m)} = e_{F'_\alpha\gamma_{2\mathbb{Z}}(x', m')} \end{cases}
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \quad \forall_\alpha^{I_*};\ F_\alpha^{2^k-1}x + e_{F'_\alpha\gamma_{2\mathbb{Z}}(x, m)} = F_\alpha^{2^k-1}x' + e_{F'_\alpha\gamma_{2\mathbb{Z}}(x', m')} \\
&\Rightarrow \quad (F_\alpha^{2^k-1}x + e_{F'_\alpha\gamma_{2\mathbb{Z}}(x, m)} = F_\alpha^{2^k-1}x' + e_{F'_\alpha\gamma_{2\mathbb{Z}}(x', m')})_{\alpha \in I_*} \\
&\Rightarrow \quad e_{2\mathbb{Z}}(x, m) = e_{2\mathbb{Z}}(x', m').
\end{aligned}$$

Therefore $e_{2\mathbb{Z}}$ *is well defined.*

- *Let us verify* $e_{2\mathbb{Z}}$ *is a* $m\Theta$ *map.*

Let $(x, m) \in V(2^k-1, 2\mathbb{Z}) \times V(k, 2\mathbb{Z})$

$$\begin{aligned}
e_{2\mathbb{Z}} \circ (F_\alpha^{2^k-1}, F_\alpha^k)(x, m) &= e_{2\mathbb{Z}}(F_\alpha^{2^k-1}x, F_\alpha^k m) \\
&= (F_\alpha^{2^k-1}(F_\alpha^{2^k-1}x) + e_{F'_\alpha\gamma_{2\mathbb{Z}}(F_\alpha^{2^k-1}x, F_\alpha^k m)})_{\alpha \in I_*} \\
&= (F_\alpha^{2^k-1}x + e_{F'_\alpha\gamma_{2\mathbb{Z}}(x, m)})_{\alpha \in I_*} \quad (\ \gamma_{2\mathbb{Z}} \text{ is } m\Theta \text{ map}).
\end{aligned}$$

$$\begin{aligned}
F'_\alpha \circ e_{2\mathbb{Z}}(x, m) &= F'_\alpha(F_\alpha^{2^k-1}x + e_{F'_\alpha(\gamma_{2\mathbb{Z}}(x, m))})_{\alpha \in I_*} \\
&= (F_\alpha^{2^k-1}x + e_{F'_\alpha(\gamma_{2\mathbb{Z}}(x, m))})_{\alpha \in I_*}.
\end{aligned}$$

Therefore;

$$e_{2\mathbb{Z}} \circ (F_\alpha^{2^k-1}, F_\alpha^k) = F'_\alpha \circ e_{2\mathbb{Z}}.$$

(iii) • *Let us show that* $r_{2\mathbb{Z}}$ *is well defined.*

Let us suppose that $x = x'$ ($F_\alpha^{2^k-1}x = F_\alpha^{2^k-1}x'$) *and let us show that* $r_{2\mathbb{Z}}x = r_{2\mathbb{Z}}x'$.

Let $\alpha \in I_*$;

$$\begin{aligned}
F_\alpha^{2^k-1}(x) = F_\alpha^{2^k-1}(x') \quad &\Rightarrow \quad F_\alpha x_i = F_\alpha x'_i \\
&\Rightarrow \quad F_\alpha x_i < i >_2 = F_\alpha y_i < i >_2 \\
&\Rightarrow \quad \sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2 = \sum_{i=1}^{2^k-1} F_\alpha x'_i < i >_2 \\
&\Rightarrow \quad (\sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2)_{\alpha \in I_*} = (\sum_{i=1}^{2^k-1} F_\alpha x'_i < i >_2)_{\alpha \in I_*} \\
&\Rightarrow \quad r_{2\mathbb{Z}}(x) = r_{2\mathbb{Z}}(x')
\end{aligned}$$

Therefore $r_{2\mathbb{Z}}$ *is a* $m\Theta$ *map.*

- *Let us show that* $r_{2\mathbb{Z}}$ *is* $m\Theta$ *map.*

  Let $x \in V(2^k-1,\, 2\mathbb{Z})$*, let* $\alpha \in I_*$.

$$\begin{aligned} r_{2\mathbb{Z}} \circ F_\alpha^{2^k-1}(x) &= r_{2\mathbb{Z}}(F_\alpha^{2^k-1}x) \\ &= (\sum_{i=1}^{2^k-1} F_\alpha((F_\alpha^{2^k-1}x)_i) < i >_2)_{\alpha\in I_*} \\ &= (\sum_{i=1}^{2^k-1} F_\alpha(F_\alpha x_i) < i >_2)_{\alpha\in I_*} \\ &= (\sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2)_{\alpha\in I_*} \end{aligned}$$

$$\begin{aligned} F_\alpha^{'} \circ r_{2\mathbb{Z}}(x,\, m) &= F_\alpha^{'}((\sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2)_{\alpha\in I_*}) \\ &= (\sum_{i=1}^{2^k-1} F_\alpha x_i < i >_2)_{\alpha\in I_*} \end{aligned}$$

Therefore $r_{2\mathbb{Z}}$ *is a* $m\Theta$ *map.*

Proposition 0.9. $(e_{2\mathbb{Z}},\, r_{2\mathbb{Z}})$ *before define in the proposition* 0.8 *is a* $m\Theta$ *steganographic protocols.*

Proof 0.11. *Let's show that* $(e_{2\mathbb{Z}},\, r_{2\mathbb{Z}})$ *is a* $m\Theta$ *steganographic protocol. In other words,* $r_{2\mathbb{Z}}(e_{2\mathbb{Z}}(x,\, m)) = m$*, for any* $m \in \mathbb{F}_{2\mathbb{Z}}^k$ *and for any* $x \in V(2^k-1,\, 2\mathbb{Z})$.
So, $\forall\alpha \in I_*$, $F_\alpha^k(r_{2\mathbb{Z}}(e_{2\mathbb{Z}}(x,\, m))) = F_\alpha^k(m)$.

1.

$$\begin{aligned} F_\alpha^k(r_{2\mathbb{Z}}(e_{2\mathbb{Z}}(x,\, m))) &= r_{2\mathbb{Z}}(F_\alpha^{2^k-1} \circ e_{2\mathbb{Z}}(x,\, m)) \ (r_{2\mathbb{Z}} \text{ is } m\Theta \text{ map}) \\ &= r_{2\mathbb{Z}}(e_{2\mathbb{Z}} \circ (F_\alpha^{2^k-1},\, F_\alpha^k))(x,\, m) \ (e_{2\mathbb{Z}} \text{ is a } m\Theta \text{ map}) \\ &= r_{2\mathbb{Z}}(e_{2\mathbb{Z}}(F_\alpha^{2^k-1}x,\, F_\alpha^k m)) \\ &= r_{2\mathbb{Z}}(F_\alpha^{2^k-1}x + e_{F_\alpha^{'}(\gamma_{2\mathbb{Z}}(x,\, m))}). \end{aligned}$$

we put

$$
\begin{aligned}
j = F'_\alpha(\gamma_{2\mathbb{Z}}(x,\, m)) &= \gamma_{2\mathbb{Z}} \circ (F_\alpha^{2^k-1},\, F_\alpha^k)(x,\, m)\\
&= \gamma_{2\mathbb{Z}}(F_\alpha^{2^k-1}x,\, F_\alpha^k m)\\
&= < F_\alpha^k(F_\alpha^k m) + \sum_{i=1}^{2^k-1} F_\alpha((F_\alpha^{2^k-1}x)_i) < i >_2>_{10}\\
&= < F_\alpha^k m + \sum_{i=1}^{2^k-1} F_\alpha(F_\alpha x_i) < i >_2>_{10}\\
&= < F_\alpha^k m + \sum_{i=1}^{2^k-1} F_\alpha(x_i) < i >_2>_{10}
\end{aligned}
$$

then $< j >_2 = F_\alpha^k m + \sum_{i=1}^{2^k-1} F_\alpha(x) < i >_2 \qquad (*)$

2.

$$
\begin{aligned}
r_{2\mathbb{Z}}(F_\alpha^{2^k-1}x + e_j) &= r_{2\mathbb{Z}}(F_\alpha x_1,\, F_\alpha x_2, \cdots, F_\alpha x_j + 1, \cdots, F_\alpha x_n)\\
&= \sum_{i=1,\, i\neq j}^{2^k-1} \{F_\alpha(F_\alpha x_i) < i >_2 + (F_\alpha x_j + 1) < j >_2\}\\
&= \sum_{i=1,\, i\neq j}^{2^k-1} \{F_\alpha(x_i) < i >_2 + (F_\alpha x_j + 1) < j >_2\}
\end{aligned}
$$

changing $< j >_2$ *by expression given in* $(*)$ *we get:*
$r_{2\mathbb{Z}}(F_\alpha^{2^k-1}x + e_j) = F_\alpha^k m$*; so*

$$\forall \alpha \in I_*,\ F_\alpha^k(r_{2\mathbb{Z}}(e_{2\mathbb{Z}}(x,\, m))) = F_\alpha^k(x,\, m).$$

Therefore, $r_{2\mathbb{Z}}(e_{2\mathbb{Z}}(x,\, m)) = (x,\, m)$*. Thus m*$\Theta$ *protocol F5 is a m*Θ *steganographic protocol.*

Remark 0.2. *1. Embed a m*Θ *message s by the m*Θ *steganographic protocol F5 in a m*Θ *cover u consists to swap the m*Θ *coordinate number* $\gamma_{2\mathbb{Z}}(u,\, s)$*.*

2. *$m\Theta$ extraction consists to add all products of each α-component, $\forall \alpha \in I_*$, to the value of the $F_{2\mathbb{Z}}$ expression of the index. In other words,*

$$r_{2\mathbb{Z}}(u) = \sum_{i=1}^{2^k-1} F_\alpha u_i < i >_2 .$$

Example 0.2. *Let $[7, 4]_{2\mathbb{Z}}$ be a Hamming code, $k = 3$. We want to insert $m = 01_{2\mathbb{Z}}1_{2\mathbb{Z}}$ into $x = 1_{2\mathbb{Z}}1_{2\mathbb{Z}}003_{2\mathbb{Z}}01_{2\mathbb{Z}}$ by the $m\Theta$ steganographic protocol $F5$.*
$F_1^3 m = 011$, $F_2^3 m = 000$, $F_1^7 x = 1100001$, $F_2^7 x = 0000100$.
So, how to calculate $e_{2\mathbb{Z}}(01_{2\mathbb{Z}}1_{2\mathbb{Z}}, 1_{2\mathbb{Z}}1_{2\mathbb{Z}}003_{2\mathbb{Z}}01_{2\mathbb{Z}})$.

$$\begin{aligned}\gamma_{2\mathbb{Z}}(1_{2\mathbb{Z}}1_{2\mathbb{Z}}003_{2\mathbb{Z}}01_{2\mathbb{Z}}, 01_{2\mathbb{Z}}1_{2\mathbb{Z}}) &= (< F_1^3(01_{2\mathbb{Z}}1_{2\mathbb{Z}}) + \sum_{i=1}^{7} F_1 x_i < i >_2 >_{10}, \\ &\quad < F_2^3(01_{2\mathbb{Z}}1_{2\mathbb{Z}}) + \sum_{i=1}^{7} F_2 x_i < i >_2 >_{10})\end{aligned}$$

$$\begin{aligned}< F_1^3(01_{2\mathbb{Z}}1_{2\mathbb{Z}}) + \sum_{i=1}^{7} F_1 x_i < i >_2 >_{10} &= < 011 + 1(001) + 1(010) + 1(111) >_{10} \\ &= 7\end{aligned}$$

and

$$\begin{aligned}< F_2^3(01_{2\mathbb{Z}}1_{2\mathbb{Z}}) + \sum_{i=1}^{7} F_2 x_i < i >_2 >_{10} &= < 000 + 1(101) >_{10} \\ &= 5.\end{aligned}$$

$$\gamma_{2\mathbb{Z}}(x, m) = (7; 5) = (F_1^{'}(\gamma_{2\mathbb{Z}}(x, m)); F_2^{'}(\gamma_{2\mathbb{Z}}(x, m))).$$

$$e_{2\mathbb{Z}}(x, m) = (F_1^7 x + e_{F_1^{'}(\gamma_{2\mathbb{Z}}(x,m))}; F_2^7 x + e_{F_2^{'}(\gamma_{2\mathbb{Z}}(x,m))})$$

$$F_1^7 x + e_{F_1^{'}(\gamma_{2\mathbb{Z}}(x,m))} = 1100001 + e_7 = 1100001 + 0000001 = 1100000.$$

$$F_2^7 x + e_{F_2^{'}(\gamma_{2\mathbb{Z}}(x,m))} = 0000100 + e_5 = 0000100 + 0000100 = 0000000.$$

$$\begin{aligned}e_{2\mathbb{Z}}(x, m) &= (1100000, 0000000) \\ &= 1_{2\mathbb{Z}}1_{2\mathbb{Z}}00000 \\ &= v.\end{aligned}$$

Now, we will extract the $m\Theta$ message hidden m in the $m\Theta$ stego-word $y = 1_{2\mathbb{Z}}1_{2\mathbb{Z}}00000$.

In other words, how to calculate $r_{2\mathbb{Z}}(1_{2\mathbb{Z}}1_{2\mathbb{Z}}00000)$*? By definition of* $r_{2\mathbb{Z}}$ *given in the remark* 0.3.*:*

$$r_{2\mathbb{Z}}(y) = (\sum_{i=1}^{7} F_1 y_i < i >_2, \sum_{i=1}^{7} F_2 y_i < i >_2)$$

$$\begin{aligned} r_{2\mathbb{Z}}(y) &= (1(001) + 1(010);\, 1(000)) \\ &= (011;\, 000) \\ &= 01_{2\mathbb{Z}}1_{2\mathbb{Z}} \\ &= m. \end{aligned}$$

4.2 The F5 $m\Theta$ Algorithm

To increase embedding efficiency, the $F5$ algorithm introduces for the first time the concept of matrix embedding technique for embedding in the context of using Hamming codes.

More formally, the desired purpose of the matrix $m\Theta$ embedding technique is to communicate a $m\Theta$ message $m \in V(n-k,\, p\mathbb{Z})$ through the cover $m\Theta$ vector $x \in V(n,\, p\mathbb{Z})$, modifying it as little as possible.
The principle is to change the cover $m\Theta$ vector x to stego $m\Theta$ vector y, such that:

$$H(F_\alpha y)_{\alpha \in I_*} = (F_\alpha m)_{\alpha \in I_*},$$

with $H \in \mathcal{M}_{n-k,n}$ the parity check matrix of Hamming $m\Theta$ code. The $m\Theta$ transformation of the cover $m\Theta$ vector x into y is then carried out by seeking the $m\Theta$ vector of modification $e \in V(n,\, p\mathbb{Z})$:

$$(F_\alpha y)_{\alpha \in I_*} = (F_\alpha(x+e))_{\alpha \in I_*};$$

$$H(F_\alpha(x+e))_{\alpha \in I_*} = (F_\alpha m)_{\alpha \in I_*} \Longleftrightarrow H(F_\alpha e)_{\alpha \in I_*} = (F_\alpha m)_{\alpha \in I_*} - H(F_\alpha x)_{\alpha \in I_*}.$$

Example 0.3. *Taking* $[7, 4]$ *Hamming $m\Theta$ code, we explain how to embed* 3 *$m\Theta$ bits of* $\mathbb{F}_{2\mathbb{Z}}$ *into* 7 *pixels. Let* $m = 01_{2\mathbb{Z}}1_{2\mathbb{Z}}$ *be the $m\Theta$ message that we want to insert in the cover $m\Theta$ vector* $x = 1_{2\mathbb{Z}}1_{2\mathbb{Z}}003_{2\mathbb{Z}}01_{2\mathbb{Z}}$. *The parity check matrix is therefore in the following form:*

$$H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

The purpose is to find the α-vector $e = (e_1, e_2, e_3, e_4, e_5, e_6, e_7)$ *such that* $H(x+e) = m$.
Otherwise,

$$\begin{cases} F_1(m) = 0\,1\,1, & F_2(m) = 0\,0\,0. \\ F_1(x) = 1\,1\,0\,0\,0\,0\,1, & F_2(x) = 0\,0\,0\,0\,1\,0\,1. \end{cases}$$

So,

$$\begin{aligned} F_1(m) - H \times F_1(x) &= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \end{aligned}$$

Thus, the modification α-vector is $F_1(e) = (0, 0, 0, 0, 0, 0, 1)$.

$$\begin{aligned} F_2(m) - H \times F_2(x) &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \end{aligned}$$

Thus, $F_2(e) = (0, 1, 0, 0, 0, 0, 0)$.

$$e = (F_1(e), F_2(e)) = (0, 3_{2\mathbb{Z}}, 0, 0, 0, 0, 1_{2\mathbb{Z}}).$$

The cover $m\Theta$ vector x is then transformed into

$$\begin{aligned} y = x + e &= 1_{2\mathbb{Z}}1_{2\mathbb{Z}}003_{2\mathbb{Z}}01_{2\mathbb{Z}} + 03_{2\mathbb{Z}}00001_{2\mathbb{Z}} \\ &= 1_{2\mathbb{Z}}0003_{2\mathbb{Z}}00 \end{aligned}$$

We have the cover $m\Theta$ vector $x = 1_{2\mathbb{Z}}1_{2\mathbb{Z}}003_{2\mathbb{Z}}01_{2\mathbb{Z}}$ and the stego $m\Theta$ vector $y = 1_{2\mathbb{Z}}0003_{2\mathbb{Z}}00$. When embedding m into x, it appears that 2 pixels of x have been partially damaged, namely the second and the last component of x. Indeed,

$$\begin{cases} 1_{2\mathbb{Z}} = (F_\alpha 1_{2\mathbb{Z}})_{\alpha \in I_*} = (F_1 1_{2\mathbb{Z}},\, F_2 1_{2\mathbb{Z}}) = (1,\, 0) \\ 0 = (F_\alpha 0)_{\alpha \in I_*} = (F_1 0,\, F_2 0) = (0,\, 0) \end{cases}$$

The passage from $1_{2\mathbb{Z}}$ to 0 shows that the pixels has been partially damaged.

V. CONCLUSION

This note shows that the Hamming $m\Theta$ code is a $\mathbb{F}_{2\mathbb{Z}}$-vector subspace of $V(n,\, 2\mathbb{Z})$ of dimension n. It appears that there exists a close relation between the $m\Theta$ protocols $F5$ and the Hamming $m\Theta$ code. The embedding of a $m\Theta$ message of k bits into the cover $m\Theta$ vector of n pixels changes at the level of the α-modalities because it partially or totally damages at most one pixel of the cover $m\Theta$ vector.

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The Mythos of Gravity

Dr. Javad Fardaei

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The material that you are about to read is unprecedented and, therefore, cannot be peer-reviewed by today's mainstream perceptions due to the unambiguous empirical evidence. It never uses assumption, postulation, prediction, or modeling systems as in the past or biased opinions as our current science dictates to us. Cogitating on an intrinsic universal truth of law in science that would apply to billions of galaxies and their residence of this most accurate entity of our universe. The law of nature has never been tested or questioned for centuries yet has been taught for over a hundred years. The truth is, the nature of our three-dimensional universe that is changing constantly, cannot be described by a one-dimensional static equation that never recognizes temperature and pressure, such as Sir Isaac Newton or Mr. Einstein have stated with convoluted, esoteric pseudoscience that only a few people think understand them, while science should be easy to know.

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Newtonian and Einsteinian Gravity is a Myth

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I. WHAT IS GRAVITY? MYTH

If you ask this question to any prestigious scientist who teaches physics and who specializes in gravity, he or she would probably pick up an object and then drop it to the ground and comment, "This is gravity." He or she might verbally say a larger mass in motion (like Earth) attracts smaller objects (smaller masses) to itself based on the gravitational force that it creates around itself. This is the foundation and platform of Sir Isaac Newton's theory of gravity, which he formulated and whose principle we have been practicing for centuries. We assume the force that is holding us down to the ground and allowing us to walk freely on Earth is gravity; that is all we know.

We think the center of a planet (center of a mass) is holding this force. How this force of gravity is created on a planet in general is a mystery. In any case, gravity is not the only thing, but it is one of the major theories that has been integrated into our brain and blood for centuries that consider gravity as an external force. No one knows the origin of gravity or how this force is working or where and how a mass is creating this force; furthermore, it doesn't give us any clue to follow. Therefore, no one has ever attempted to find out where gravity originated from. No one knows for sure or has brought any convincing suggestions to the scientific community of reconciliation to match all the experiments that we are conducting. Historically, up until the 1600s, no one said much about gravity until Galileo convinced authorities that gravity is real and was affecting Earth's tides by the moon's gravitational force. After him, Sir Isaac Newton (1643–1727) hypothesized gravity through his famous scenario while he was resting under a tree and having an apple hit his head. He wrote a formula for it, and he became the father of gravity.

Scientists do not know what gravitational force is or what generates gravity on Earth or any planet, but the strong postulation is that the motion (rotation) of Earth (planets) is forming this gravity phenomenon. As the spinning gets faster in space, gravity gets stronger. Even though some don't call gravity a "force," it acts as a force. For instance, if Earth stopped spinning, gravity would disappear or die, and everything on Earth would repel into space; or if the spin increased, matter would be more packed and denser. So theoretically, we are very lucky that Earth has this "Goldilocks" rotation that is holding us firmly on the ground and at a safe distance from the sun.

For centuries, the subject of gravity has been the biggest mystery in the science of physics. The scientists who think they know everything about this force are more confused than anyone else about it.

They place this force as one of the primary forces in creation, but they do not know where this force is coming from before creation. Gravity's force is much more complicated than any other force because it is not measurable in a sense that can be applied to all the experimental conditions in a vacuum or a nonvacuum space. In the charting to investigate the source of gravity through the big bang theory, gravity is supposedly one of the four forces in the creation of the universe. It theorizes that gravitational force was in existence before creation occurred (questionable). On the other hand, gravity was involved in creating mass inside the space of the universe (questionable). Then gravity became reliant on mass to create gravity (again, questionable). Gravitational force became a force that has a direct relationship with velocity (questionable). Furthermore, gravity can show itself as a wave (questionable) and a force to hold galaxies as one unit (questionable), bending light (questionable), slowing down time (questionable), and placing planets in their orbit (questionable). This list can go on. This scenario is like someone being the father of his/her own father. It existed before creation. Now, the universe's matter is making this force. This setup of principles is not reflected as a science that we should practice, but it seems everyone has confidence in this evolutionary sequence regarding gravity.

Lately, some scientists are saying that gravity is a myth and that it does not exist due to some experiments and cosmological observations in space. They are saying that it is something that was made up. In recent years, some young scientists think that there is no gravity at all in the universe due to the empirical expansion evidence of the universe and the separation of galaxies with many other factors and some other indications on Earth and space. They claim that there must be some other forces that we don't know about. But the big question arises: What other force exists that we don't know about, or how do we walk on Earth if there is no gravity?

The answer for Earth is that the electromagnetic energy field that is holding up all the planets and galaxies in the universe is also holding Earth's atmosphere and allows us to walk on Earth. The force that is holding us down on Earth is coming from this electromagnetic force from above. This force is pushing us down to the ground versus a force from the center of Earth, or gravity, pulling us down to the ground. An example of this phenomenon is that the pressure at sea level is higher than at a higher elevation.

The Newtonian gravitational laws have been taught for more than three hundred years. The clarification of this force is mechanics as Newton shows in his formulas, but by recent observation, teaching Newtonian gravity has been slashed by teachers as they are bombarded by many questions that cannot be answered. On the other hand, this force has never been studied systematically from a scientific aspect. No one or any organization has ever attempted to reject it or study it thoroughly. The simplicity and complexity of Newtonian gravity have brought many inconsistencies to the table until Albert Einstein promoted it without further testing or having knowledge of the universe.

He extended Newton's mechanics of gravity to his space-time theory and called it general theory of relativity. General relativity (GR) is a geometric universal dimensional property of gravity that is called space-time. This space-time's curvature has a direct physical correlation with energy, radiation, matter, and momentum of matter (planets); and he wrote an equation for this phenomenon. This new science blended with classical physics and was confirmed in all observations and exercises to date through mechanical gravity in space and it became a solid science in mainstream society.

The most remarkable observation that Einstein made through GR on Earth is that time is running slower closer to a gravitational field (i.e., time ticks slower in a basement than in an attic). This remark is accepted by all scientists around the world. Another remark is that the gravity of planets can affect the sunlight (photoelectric effect). For this theory, Albert Einstein got his Nobel Prize for his prediction that gravity bends light, which was confirmed during an eclipse in 1919 (but it was later rejected).This is the general idea of GTR, but it has more details that are esoteric science that a few people are barely familiar with (and not understanding it totally), and Mr. Einstein himself couldn't explain it; therefore, many unsolved questions remain on the table to present time. But Einstein's gravity, time delay, and the bending of light are more commonly known.

To clarify for our readers, to comprehend the mysteries of our modern science and apply it to the universe is that GTR is talking about mass (a sun) distorting space-time to make a gravitational force to hold planets. Einstein's theory brought a few new things to the picture as well, like the prediction of the existence of a black hole and the propagation of gravitational waves, which tens of thousands of scientists are working on in this field through the big bang theory principle. Einstein and Newtonian gravity are following the same principle, but Einstein extended the structure to the whole universe, modeled as a flat universe in his space-time of four dimensions. So, from there onward, gravity became a universal force, the force that is holding our universe together.

The problem is this: Due to the flat horizon of Mr. Einstein's space-time model, it does not let us observe the hundred billion galaxies in all directions that exist. Therefore, the actual three-dimensional space of the universe is a hard indication that our universe is not flat. If our sun, or any sun, is holding its planets and moons by this gravitational field, then how can this force intelligently do multiple tasks and hold all its planets at a perfect speed without delay of rotation by holding them or hold hundreds of moons within billions of miles of space with different distances and momentum?

The principle of each solar system, based on GTR, is the reason Earth or other planets go around a sun. It is systematic of a mechanical phenomenon that the sun's gravity and the centrifugal force of Earth or other planets give them a circular motion. But the question arises: Doesn't a mechanical force have friction? And would not that friction be exhausted over time? This means, therefore, that all the planets and moons should slow down after billions of years from this friction.

Here are some problematic issues from Sir Isaac Newton's to Einstein's eras, which had a few things in common. The perception of the world was that it was a one-galaxy universe (it is not clear if Newton was aware of other solar systems besides our solar system), and the smallest element was an indivisible atom. The big bang (primordial) theory originated for a one-galaxy universe, then Einstein came up with GTR, and two decades later we find that the universe has billions of galaxies. Also, the building blocks of the universe are from the indivisible element of the atom. which became a quantum phenomenon. These two major changes have never considered, adjusted, or revised classic physics theory to a quantum physics (QP) theory, nor accounted for the size of the universe to adjust to a new version of theory to match with QP.

On the other hand, the four mechanical forces of the big bang are not capable of creating this massive universe with billions of galaxies and placing them at precise distances from each other and placing billions of solar systems in each galaxy. The reality is the universe does not show us any mechanical function to follow any modeling theory from our icons.

Sir Newton neither had much knowledge of space in his era nor many of the tools to prove his theory and, in fact, didn't know anything about quantum science. Nevertheless, Mr. Einstein had much more opportunity to correct Newtonian gravity, but he didn't and continued his mechanical theory of GR and SR (special relativity) for modeling the universe, and he did not revise his theories even when he found out the universe has billions of galaxies or when quantum theory was turned into reality as science claimed. Not to mention his GTR was designed for one sun, a few planets, and one moon, not the whole universe.

The meaning of science is to find the truth to match empirical evidence of nature through examination. Unfortunately, we extend our classical physics to quantum mechanics (QM) without further examination of these two sciences. As a result, the whole science of the universe and QM becomes convoluted, and both are in a static stage in the real world because the classic physics (CP) that we know does not show it can be broken down to a QM level without friction.

All empirical evidence of ontology strongly demonstrates that this force has been misrepresented for centuries because this force of gravity has never been experimented on thoroughly by the observer. It should be mentioned and accepted that the fact is that our universe should be recognized as a complete, well-organized entity that knowingly places all objects in the right place with the right force, the right equilibrium energy, and on the right natural evolutionary journey. Unfortunately, scientists have never documented these major principles, yet they follow the last centuries' pseudoscience mathematics.

The unambiguous empirical evidence shows that the universe rules all its elements from the smallest to the largest units to follow their task(s). Thus, everything is connected in a three-dimensional phenomenon in which this magnificent structure strongly discards all the one-dimensional, flat, static formulation theorem that has been written in the past, because mathematics is not capable of describing nature that is constantly changing in fractional moments by pressure and temperature with its interaction expedition; yet it is undeniable that mathematics is the greatest tool known to advance mankind's technology.

II. NEWTONIAN EARTH GRAVITY

Earth's gravity is following *temperature, pressure, and mass of an object* while Newtonian gravity ignores these three elements and only follows *weight*. By today's standard or modeling of the atom, the building blocks of the universe are a quantum phenomenon. This fact didn't exist in Newton's era or in Einstein's or when the primordial theory was pronounced. In addition, when science transformed the indivisible atom to quantum science in the mid-1920s, the observation of the universe was also reformed from a one-galaxy universe to a several billion-galaxy universe. These two major discoveries never led to the revision of the theory of Newtonian mechanical gravity or Einsteinian mechanical gravity or the concept of the one galaxy big bang theory.

The difference between quantum physics and classical physics is day and night. To unify these two is impossible, it is choosing between apples and oranges. Simply, CP cannot collapse to QP. Why?

- By today's standard modeling of the atom, it takes a very large number of elementary particles in nature to make one atom while science does not have any explanation for the particle itself. Also, it is hard to believe that each particle must be smart to create a complete, organized entity such as an atom while science does not believe an atom is a sentient entity. In addition, there is no evidence of the creation of an atom.
- QP follows its nature by consuming energy without friction while CP must be motivated by some force outside of its body to move mechanics' function with friction.

Analyzing these two major facts is bringing tremendous difficulty to all QM theories that we are practicing up to this date with subsiding classical or modern physics principles. Simply, the QMs that we think exist do not exist. But the manifestation of Newtonian and modern physics is that any mass creates gravity, no matter how small or big the object is (this subject matter has its own detail).

Newtonian theory has stated that the center of a mass (like Earth) is holding gravitational force and that the mass by its motion creates a gravity field. If an object comes into this field, it will be grasped by the heavier object where proportionally both meet somewhere in between. Similarly, this mindset was confirmed by Albert Einstein, stating as a comet reaches Earth's surface, the body of Earth will move fractionally toward the comet as well, by gravitational attraction of 9.81 Newtonian weight.

This constant, 9.81, is referred to as object weight, like if a mass is 1 kilogram × 9.81, it equals Newton's weight on the surface of Earth. So, this constant is changing on different planets. In other words, the pulling force of Earth is 9.81 × mass. This is also considered a Goldilocks force on Earth. So, if the rotation of Earth becomes greater, then our weight would be more, which would disturb our size and behavior of walking or would even pull us to a point that we would be crawling. Or if Earth stopped spinning, we would be repelled into space.

The Newtonian formula that was produced more than three hundred years ago probably was perfect for that era's perception due to their limited knowledge. However, since then, science has relied on this old testimonial with no additional experimental evaluation; yet it has been accepted from its debut through all scientific fields without further examination to this date.

Unfortunately, the Newtonian gravity theory does not apply to a variety of experiments that have been conducted by today's standards due to the distinguishing feature of Earth, which has oxygen over its surface compared to other planets which we know do not. This mass of oxygen is playing a great role to discard the Newtonian law.

The force of gravity is so convoluted that schoolteachers are slashing the teaching of gravity because of many unanswered questions, such as the variation of testing and pulling objects differently in different conditions. Here we are going to bring enough comparative examples to understand the faultiness of our teachings and the transparent truth of this force, and maybe it will evolve our science in the future.

When gravity is investigated, it is not actual gravity that is pulling an object to the ground. It is that object's mass falling to the ground. To be clear, every object is fighting with the mass of oxygen or atmosphere to reach the ground, nothing else. To give an example, cogitate a raindrop and a snowflake. Both are water in different forms, but both are holding the same mass. The velocity of a raindrop is much faster than a snowflake. The axiom of the raindrop's condensed and aerodynamic shape is the reason.

Newton's law is stating that the gravitational force of Earth is pulling two objects at the same rate, but hard evidence does not show it that way due to the mass of the atmosphere and the mass of each object. Another angle to examine is the raindrop. If Earth is pulling a raindrop, then why isn't the raindrop pointing to the ground at a rate of 9.81 times the g-force of its mass of water? The answer would be that the raindrop is fighting with the mass of oxygen as it is falling to the ground. That is why the bottom of a raindrop is rounded, not pointed.

This remark is common knowledge, but we have ignored it for centuries for no reason. Theoretically, since our atmosphere is carrying mass, the atmosphere is considered to have the texture and structure of oxygen or atoms; therefore, heavier elements can break through the atmosphere's mass easier than lighter objects, or the shape of an object affects the falling. This phenomenon has been confirmed by all geologists. For the same reason, anything that has less mass than oxygen rises in the air, like a helium-filled balloon, and anything that has more mass than the atmosphere lays on the ground, like an air-inflated balloon.

Absurdly, science has indirectly discarded Newtonian gravity since its debut by using a **buoyancy** principle where the real meaning of it is that Earth is ignoring the mass of an object if it is lighter than the atmosphere or lighter than oxygen per volume, such as a helium balloon and an air-inflated balloon. This eye-opening principle makes a strong statement to confirm that Earth is a complete entity that is following its nature very accurately through quantum physics or a universal-law phenomenon.

Earth as we know has a very delicate system that can distinguish the fractional mass of an object (i.e., we are able to put ten layers of different liquids on top of each other without thinking how sensitive Earth is to manage it). Or water and ice are made of the same components, yet ice always floats on top of water because of the fractional mass differential per volume that is recognized by Earth. It is the same principle that the mass of water can evaporate to the sky because the nature of hydrogen's angular momentum is pulling the mass of oxygen up. Again, it is not Newtonian or buoyancy law; it is the nature of QP.

The best evidence that we never pay attention to is earthquakes and volcanoes. The reason that planets in the universe have seismic activity is because of the nature of their developing evolution by sending heavier elements down to the center and pushing lighter elements (life ingredients) up to the surface. The function of all the trillions of planets must be the same: heavier elements tend to go deep, and lighter ones come up.

We as a species are breathing oxygen and creating carbon dioxide, and all vegetation is taking this carbon dioxide, which is heavier than oxygen and sinks just beneath the surface of Earth for roots to take carbon dioxide as a surviving element and then release oxygen from above the ground. This mass differential might not be much for our apparatus, but it is a lot on a quantum scale for Earth to distinguish these differentials. It is the same for all species even under the water. They take oxygen out of the water and release carbon dioxide, which goes to the seabed for vegetation there to survive. This circulation of life components has nothing to do with the gravity that we practice.

If Earth was following Newtonian gravity, the oxygen and carbon dioxide should both be pulled at the same rate and stay at the same level or the roots of all trees all around the world would be aiming and growing down in the same direction to the center of Earth, not spreading out near the surface of Earth for breathing.

Making this experiment at home is simple. Put an egg in a glass of water, and it sinks under the water. If salt is added to the water, the egg will float on top of the water. If there were gravity, we wouldn't have these conditions. The egg would still sink.

Newton's 9.81 × mass is a strong force that would affect everything that is growing. If this force existed on our Earth, nothing would grow spherical, like all the fruit that we know. The apple that hit Sir Newton's head should be like an icicle and pointing down to Earth. This strong force would not let trees grow tall or never grow at all while we have no evidence of such force under our feet either.

Another theory that we inherited from Galileo Galilei (1564-1642) is tidal movement, which is stating that high/low tide is the result of the gravitational force of the moon. To view it from a Newtonian perspective, it is the same mechanical principle. It states that the reason that the moon has rotation around Earth is because of the centrifugal force of the moon's movement, and the gravity of Earth is pulling the moon in its orbit. Therefore, this mechanical phenomenon exerts tides on Earth. We are carrying this theory since the 1500s and have forgotten that any mechanical remark and pulling function must have friction while the moon's closest point to Earth is 363,104 kilometers and the farthest point of its rotation is 405,694 kilometers, which means it is not a round orbit and therefore should affect the tides differently in the course of a month.

When we study the motion of low/high tides on Earth, it is happening every twelve hours, or half a day to be precise, and is actually due to the rotation of Earth on its axis, which is moving the ocean water and creating the tides. One side is low tide, and the other side of Earth is high tide. This is common sense, and we should be reconsidering our perspective of the natural movement of Earth and the natural positioning of Earth in our solar system.

Furthermore, we should consider the internal gravity of a molecule of water that is holding itself while the mechanical gravitational force from the moon to Earth has no foundation for this remark. To extend this thought, the effect of the rotation of Earth and the positioning of Earth causes the circulation of hurricanes and tropical cyclones to run in two different directions (duality) while both reflect each other.

The moon's orbital period is about 27.32 days, and its gravity is a constant 1.62 m/s2 (Earth 9.81) (these constants are an estimation of Newtonian gravity) and an average of 238,900 miles from Earth. Shouldn't we ask ourselves how Galileo did this estimation with his old apparatus that we cannot do with all the technology that we have today? If Galileo's theory is correct, it should change the tides differently every month due to the positioning of the moon in relation to Earth, but the tides are consistent. In addition, in the last few decades, we have never experienced any of the moon's gravitational force of 1.62 m/s2 in our space station or space shuttles while our ocean is much farther away to be affected by such gravity. Surprisingly, no one ever thought in this regard. Picture this and cogitate.

It is obvious that a heavier object supposedly should be attracted to Earth's mass faster than a lighter object because it has a stronger gravitational field force due to Newtonian law or Galileo's experiment. To visualize these two objects better, give an object a weight of 1 kilogram or 1 pound; and for the second one, give it a hundred times greater mass. In the famous work of Galileo's experiment of dropping balls from the Tower of Pisa, he discovered that gravity accelerates all objects at the same rate based on their weight. The heavier objects accelerate faster, the lighter objects fall more slowly. In his observation, he related to the gravitational force of Earth that is pulling a heavier object more strongly.

Newton theorized that if two objects with two different weights are released from a tower at the same time, both will reach the ground at the same time because the gravitational field will pull both at the same rate, but the formula shows that the heavier object reaches the ground before the lighter one as Galileo stated. Also, our observation agrees with the formula and Galileo that a heavier object reaches the ground first.

Furthermore, if two objects, such as the feather of a bird and a heavier object such as a rock, were dropped at the same time in a vacuum (no oxygen), both objects do reach the ground at the same time. This fact has been proven in a vacuum chamber where there is neither friction nor atmosphere. This hard evidence does not match the formula of Newtonian gravity of more mass, more gravity but does match Newton's verbal statement.

There is also the famous experiment of releasing a feather and a hammer from the same height on the moon. Both reached the surface of the moon at the same time. Again, if there were any gravitational force in the moon's center, the heavier object should reach the surface before the feather based on the theory of more mass, more gravity. But it does not react that way because there is no friction of oxygen or atmosphere on the moon. Virtually, it is the same as a vacuum chamber.

As you see, the atmosphere is playing a big role in objects. Heavier bodies accelerate faster and come down first, and lighter bodies fight harder with the mass of oxygen and come down slower. If the volume of an object is lighter than air like light gas or if a mass weighs less than oxygen like helium, it will not come down at all.

This axiom of evidence is clear to us: Earth is following its nature. The nature of all planets must be the same by shifting heavier elements to the center and lighter ones to the surface. Newton's estimation is that a strong gravitational force of 9.81 should hold any heavy object to the ground firmly due to a force that is about ten times its mass.

There is a public site where they conducted an experiment with a feather with a bowling ball in a vacuum chamber. Both were released from the same height at the same time; both hit the ground at the same time. At the end of this experiment, the bowling ball bounced back up. If Earth had gravity, then it should not let the bowling ball bounce up because a gravitational force of 9.81 should act similarly to magnetism.

This hard evidence shows that Earth is not following Newtonian gravity. Don't be mistaken, gravity is a one-way force, and the action/reaction does apply to gravity. Yet the perception of gravity works like magnetism.

In recent years, NASA scientists believe that the moon has diverse gravity in different sections. Again, if Newtonian gravity considers the moon as one unit of mass, then it should generate the same gravity all over the mass and not have some patches with more or less gravitational force.

Inflated with oxygen, a balloon has weight and stays on the ground; but when we inject a mass of helium into it, theoretically, with the Newtonian concept and a constant of 9.81 × mass, the balloon should have more mass or Newtonian weight and stay on the ground. Instead, it goes up because helium is lighter than oxygen.

To explain this scenario, science created a buoyancy law. When we delve into the buoyancy law, it means Earth does not have Newtonian gravity. Indirectly, we are aware of this fact, but we do not practice or teach it.

As an airplane runs for takeoff, it does not fight gravity to get off the ground. It is fighting with oxygen or the atmosphere to lift its own weight off the ground. The speed of the airplane makes the airplane's wings have friction with the air at precisely the right angle to fly. For designing airplanes, engineers concentrated on a few things, aerodynamic shape, and the propulsion force of the engine for an airplane to reach the correct speed for takeoff. Again, car designers consider an aerodynamic shape for more speed. Helicopters rotate their blades to the maximum speed to generate friction with the atmosphere to lift off. Birds can fly because they flap their wings against the mass of the atmosphere. Their speed forward, aerodynamic structure, and light weight make them able to fly. If it was just fighting gravity, nothing could fly at all.

When a nautical engineer wants to design a ship, he or she never calculates the gravitational aspect of it. The most important factor is how the ship floats over the water by a calculation of the angle of resistance of the water against the body of the ship. If gravity was the factor, then the ship's mass, with the weight of thousands of large heavy containers, would never float over the water. It would sink to the bottom due to the Newtonian formula of gravity, $g = 9.81$.

A submarine is made of heavy thick metal. When it wants to dive into a deeper level of water, the law of physics dictates that the same volume of water must be pumped into the hefty submarine for it to sink more. The nautical and aviation industries do not have any gauge to show Earth's gravitational force if gravity was a factor.

A hot air balloon is designed to heat (give temperature to) the air inside of a balloon, giving it lighter pressure than the outside air for liftoff while Newtonian gravity does not recognize pressure and temperature in any condition.

A hundred pins or metal scraps can be picked up with the force of a little magnet by a crane. You would think that Earth, with its immense mass and strong gravity, should have more effect than the magnet; but it does not.

Heated air is much lighter than cold air, such as hot oxygen, which stays over cold air, which is proven by the science of physics. But this significant evidence is not following Newton's gravity. Boiled water expands and needs more space than cold water, but the fact is that the steam of water is carrying the same mass of the water, which is much heavier than the atmosphere in any condition. The question is this: How is the same mass of the water going up? There is no scientific answer to it (if you think the steam of water has less mass of oxygen per volume, it is incorrect).

Deep inside us, we know that we do not feel any pull of gravity in our system from the ground, but we all assume that if Earth did not have gravity to pull us down, then we would start to float through the air. But we don't due to our mass or our weight that is heavier than the atmosphere.

Thus, it is impossible to find evidence to work completely with Newtonian gravity's formula or his verbal explanation. The fact is, it is impossible to generalize gravity and write a formula for it. Also, we shouldn't forget that our mathematics is the greatest tool to advance our technology, but unfortunately it isn't the right tool to write any formula for nature where all the objects are acting differently in different pressure and temperature situations and is changing all the time.

III. EINSTEINIAN EARTH GRAVITY

Traditionally, science requires mathematical equations for each theory. This mindset goes back to the 1500s or maybe much farther, but to write something on the universe, it started with Galileo saying that the moon affects the tides. The problem with an equation on the nature of the universe is that all the elements of the equation would be based on a prediction probability outcome and the absence of numbers with no empirical evidence. Thus, it brings a series of arguments. On the other hand, all the equations are presenting one dimension with two flat points or a static phenomenon. While the nature of the universe is three-dimensional and non-static, it works with multipoint through pressure and temperature that are continuously changing through intermingling interaction and growth. Simply, there is no single equation that exists to describe nature. Science must define truth resolution through unambiguous empirical evidence, not equations. The most powerful of Newton's testimonials, non-equation is "for every action, there is an equal and opposite reaction." This general theory or fact applies to everything in all the sciences. This duality of the action-reaction phenomenon can be traced in all science fields. As a matter of fact, this action-reaction can be outlined as complementary duality in the foundation of the universe as well.

The duality in nature or complementary partner of duality in nature is oneness. It means one cannot exist without the other (i.e., left, and right; up and down; male and female; and the famous Einstein's platform of his theory, space-time or that time created space).

Albert Einstein redefined Newtonian gravity. He also extended Galileo's remark of the moon's gravity affecting the tides and applied it to a vast space of the universe that we know today as his general theory of relativity. Einstein's manifesto is that everything is connected to each other: force, energy (photon), and mass. Einstein's declaration is this: more mass, more gravity, or the reason that the moon goes around Earth is that the mass of Earth generates enough force to hold the moon in its orbit; otherwise, the centrifugal force of the moon would escape from Earth. Due to this same reasoning, all planets go around the sun.

Every planetary body like Earth is surrounded by its own gravitational field as he stated, which can be hypothesized with Newtonian physics as exerting an attractive force on all mass objects. For example, if a big object falls to Earth, even Earth moves toward that object, and this amount of movement is dependent on the size and mass of that object. For instance, if a comet is approaching Earth,

Earth in space would move toward this object when it reaches each other's gravitational field. However, this phenomenon has never been proven but is accepted due to Mr. Einstein's remark.

When we evaluate Newtonian-Einsteinian law, it seems they are approaching the same thing. More mass is creating more gravity. These two modeling theories have been practiced for centuries, and up to this date, no one ever contributed strong evidence to prove or reject the old fashion notion of these two theories. Yet there are so many contradictions out there that one side of the equation does not match the other side.

Unfortunately, in the last three centuries, no one has paid attention to the details of the law of gravity discrepancy. Even the legendary Mr. Albert Einstein did not detect the inconsistency of the results of Newton's law of gravity in the twentieth century with all the blunders, yet he extended Newtonian mechanical gravity to his theory of creation and blended it with his time, space, mass, and energy (photon) platform.

Einstein's general relativity, published in 1915, proposed that gravity, as well as motion, can affect the intervals of time and space. Einstein's earlier theory of time and space and special relativity (SR) proposed that distance and time are not absolute. The ticking rate of a clock depends on the motion of the observer of that clock. The key idea of GR is that gravity pulling in one direction is completely equivalent to acceleration in the opposite direction. A car accelerating forward feels just like gravity pushing you back against your seat. An elevator accelerating upward feels just like gravity pushing you down into the floor. These two are exact statements of Mr. Einstein in textbooks.

In both cases, it is not the result of gravity pushing us that we feel in our bodies. What we feel is the result of speed versus our body mass resisting the mass of the atmospheric pressure. In a simple form, it is the replacement of our body mass against the mass of the atmosphere (i.e., a child runs against the wind to raise his kite). The mass of the atmosphere holds the kite in the air and the child runs harder against the wind.

The acceleration upward in an elevator is the same; the mass of atmospheric gases pushes our body down for a few moments till the elevator speed becomes steady and the air pressure becomes stable. Basically, it is not g-force (gravitational force) as Mr. Einstein proclaimed. It is atmospheric force or a-force. It is happening all the time to a jet pilot in an airplane upon takeoff, on a roller coaster ride in an amusement park, the forward and backward motion of a swing, riding a motorcycle, and the air pressure against your body and many more interactions of speed that causes the mass of air pressure that pushes our body's mass. But it is not the result of any gravity. If we apply the same principle underwater, the resistance of the mass of water would be even more effective than the pressure of the atmosphere. I am afraid to say that it has nothing to do with gravity as stated by Mr. Einstein.

Another good example is in space. When astronauts are repairing instruments outside the space shuttle, they are moving at 28,000 kilometers per hour (17,500 m/h), but they do not feel any gravity resistance against their bodies because there is no atmospheric mass pushing against their bodies at that speed in space. This perception of gravity, based on action-reaction in these cases, has been misjudged.

Albert Einstein at one point predicted that gravity would bend conventional light, and he wrote a formula for it. In November of 1919, at the age of forty, he became an overnight celebrity thanks to an eclipse that showed the exact amount of bending of light that he had calculated and was found on the planet Mercury. The scientific community in that era confirmed that light rays from a distance were deflected by the gravity of Mercury in just the amount Einstein had forecast in his theory. So, Einstein got the Nobel Prize for it, and he became the second person who had a theory on gravity since Sir Isaac Newton's theory more than three hundred years before.

Unfortunately, this phenomenal theory of Einstein's bending of light by gravity has been rejected by today's technology. Scientists took more than a thousand pictures of similar scenarios of our sun and the planet Mercury in our galaxy simulating this same phenomenon, but they could not find any gravity from these planets that would pull the sunlight. This announcement is the result of many scientists who have worked in this field for more than twenty years. This news jeopardizes many other theories, including gravity in space and sunlight behavior.

The theory that *time* is going slower closer to the ground versus it going faster at a higher elevation on Earth due to gravity has been tested by the most accurate atomic clocks in the world. The results show that the same atomic clock does indeed run slower in the basement than in the attic. This difference in time was a very minor difference, like 1/1,000nths of a second over a long period of time.

The fact is that an atomic clock should work differently in two different environments with diverse temperatures and pressure, but it has nothing to do with gravity. It is very clear that pressure at sea level affects all molecules. For example, water will boil at 100°C at sea level, but the same water will boil at less than 100°C at a higher elevation because of the difference in atmospheric pressure. Elevation and pressure are changing the atom's behavior, and hard evidence also shows that it is harder to breathe at a higher elevation.

Let's assume that the result is not the discrepancy between these two measurements, but at the end of the day, these two are under one umbrella of time. How could they possibly both go at different speeds in the same direction in the same mass of Earth based on the same time dimension?

Cogitate for a moment: If GTR's principle is correct, the top of a mountain is closer to the space-time curvature than a basement on Earth. It should, therefore, have the opposite result.

The principles of mechanical gravity of Newton, Einstein, and Galileo are thought to be on the same page; but none of these legends could explain it in detail because of erroneous fabrications of gravity theories that have been a mystery through mechanical perception.

The fact is the universe is a cosmic consciousness that follows its natural destiny. It consists of several billion individual units (galaxies) with different provenances and different purposes, but all function as a single entity where gravity is one of its spectacular functions to be explained. Furthermore, all its chemical elements are cognizant of its nature for its action-reaction. These phenomena must be the same all over the universe due to nature's performance.

IV. SPACE GRAVITY

Albert Einstein's GTR is formulated inside the framework of classical physics. By the late nineteenth century, the laws of physics were founded on mechanics, electricity, and magnetism and based on a large collection of matter. The classic physics demonstrates difficulties from the beginning in the portrayal of the universe's structure. These laws of physics described nature very well under most conditions; however, some measurements of the late nineteenth and early twentieth centuries could not be understood due to esoteric prediction structure. Thus, classical physics delved into the nature of the source of the origination of mass or the atom. As a result, quantum mechanics theory was introduced. Perhaps the discovery of knowledge that the atom has internal construction is considered a high leap in science for mankind. Thus, it has revolutionized the building block of the universe from a single indivisible atom to internally knowing atoms. Since then, the effort started more deeply to unify CP with quantum physics for many reasons, and most scientists must find the origination of everything through QM principle, not through CP. Consequently, arguments became much more convoluted where it is making it even harder to understand QMs based on a CPs foundation where gravitational force is one of them. (This section of collapsing CP to QM has its own discussion).

Albert Einstein's theories have more diverse meanings that, up to this date, scientists are challenged to grasp these theories and apply them to QM while it is hard to understand his original manuscript theories on CP itself. He places mass (or matter), space, time, gravity, force, light, and energy all together on one plane of space-time, not to mention mixing all these elements with the big bang theory and then putting them side by side in four dimensions, then calling it a flat space-time plane as a portrayal of the universe.

As you may have noticed, everything in his manifesto of the universe is CP-based, not QM-based, yet all the above elements must work mechanically together to run over several billion galaxies and place several billion solar systems in a limited space of a galaxy under a mechanical energy force spectacle.

The most accurate and best description of Einstein's GTR about gravity in space-time, assumedly, is as follows: Gravity is not a force but is a consequence of the curvature of space-time caused by the uneven distribution of mass and energy. The result is that time lapses more slowly in lower elevations (stronger gravity) that are closer to Earth's gravitational force. This manifesto of time going slower in Earth's gravity is mentioned in a previous Earth's gravity.

But in space, his assumption is that suns have heavier mass and put more dent in space-time and create more gravity, which causes all the planets of a solar system to go around the sun involuntarily. This is the manifestation of Einstein's theory on space-time, resembling a trampoline, so it is called the trampoline factor. The trampoline factor is when a heavier object (a sun) sinks more into the trampoline (or fabric of space) and creates a gravitational field for other planets to go around the sun. This mechanical view of the universe is deeply unsettling to the general public because in a wider range, as we view the universe with over several (hundred) billion galaxies, where each galaxy is carrying billions of solar systems like ours where none of them is working the same way as ours, it is hard to accept as an ultimate theory. To continue this discussion with this philosophy, it is not even working with our own solar system due to the size of the planets.

The theory of the trampoline factor, that the sun is millions of times greater and heavier than all the planets in the solar system together, was hypothesized over a century ago without having any knowledge of the contents of the sun and the understanding was that the universe was just the Milky Way Galaxy. Honestly, the postulation of the sun's mass was tailored for the space-time theory to work in a proper manner only. However, by knowing the contents of the sun, based on today's knowledge, this calculation is incorrect by far.

Just hypothetically, if we calculate the mass of our sun with Earth's mass, they are not much different based on atomic mass, in which each oxygen has eight times the mass of hydrogen whereas our sun has more than 98% of the lightest elements in the periodic table, hydrogen and helium. In addition, there is no evidence of the sun's strong gravity.

One, any object (comet, asteroid) that is aiming at the sun is repelled by the sun's strong cosmic heatwave. Suns are made of sun-hydrogen and sun-helium. If any other chemical element reaches a sun, it destroys some of its purity and energy. For instance, our sun has over 1% of contaminated spots of chemical elements.

To investigate Einstein's model of GTR, it is not even working with our solar system due to the size and mass of planets. With some planets having over fifty moons and being a long distance from the sun, holding them mechanically does not make any scientific sense. Frankly, the whole GTR is about one sun, a few planets, and one moon, not the whole solar system or the entire universe.

Einsteinium's model of space-time does not extend to the universal level or long distances. Simply, it's limited to our solar system with one sun, Earth, and moon. It does not ever reference other planets in the solar system with tens of moons or with massive distances of billions of miles from our sun. Not to mention it calls the universe a flat universe while the universe is observed in all directions in three dimensions.

A few planets have a large system of moons that are in orbit around them. Jupiter has more than sixty moons, Saturn has more than thirty, and Uranus has more than twenty. And honestly, there must be some other reason for planets to go around a sun. In addition, the space-time of GTR does not bring any motivation for trillions of planets to go around suns unconsciously without friction or that gravity affects the "timeline" is not an adequate statement to even give it credit to stand up as a science declaration. These two are the fundamentals of GR in a flat universe. Also, a flat universe does not allow us to see the unlimited galaxies in its limited horizon while we can observe the universe's three dimensions in all four directions.

Furthermore, a mass in space is weightless as we have experienced for decades by sending astronauts into space. For the weightless scenario, we literally use the expression of free fall or zero gravity, but still, we are following Einstein's theories on gravity. Therefore, the theory of gravity does not work with classic physics in space. We have been working with this same scenario for decades using our modern technology in space, and we have never observed any indication of gravity in the weightlessness of space. It just does not follow the modeling of the universe's physics as stated by Albert Einstein. Not to mention the modeling of nature is not science to follow.

To claim the statement that gravity does not exist in both conditions of Newton's and Einstein's' theories, we are obligated to prove scientifically that gravity does not exist externally yet must be found internally in atoms due to the intrinsic nature of the building blocks of the universe (the atom) with the reality that it manages all its belongings through its complex natural behavior that can place and control everything to detail.

We live in the Milky Way Galaxy, which has more than a billion solar systems like ours. Some are much bigger, and some are smaller, yet none of them are the same in general due to the size and number of planets. But all have one thing in common: all the planets go around a sun. Why?

To investigate this wonder, we should understand how the physical space of a galaxy works and what makes a galaxy a galaxy.

First, the difference between the inside and outside space of a galaxy is temperature and wave. As we have observed, sunlight cannot travel outside of its galaxy. For this same reason, we are able to see individual galaxies in the space of the universe. Additionally, we have experienced receiving pictures through the wave of space from several billion miles away from the inside of a galaxy through a space telescope. These two magnificent properties of a galaxy distinguish a galaxy from the outside because wave and temperature are complementary oneness of duality that exist only inside the galaxies. Thus, one of the characteristics of space is that when it receives temperature, it creates waves to transfer energy all over the space of the galaxy. In other words, these two cannot exist without each other. To extend this thought, nothing exists without temperature and wave. An atom or mass would vanish without temperature and wave.

Moreover, more facts are as follows:

- The flat shape of a galaxy is the result of an unprecedented statement of the trajectory of sunlight (photons) from the equatorial widow of the sun(s); also, galaxies are born flat due to the expansion of the universe.
- Each sun is made of clouds of the lightest elements of hydrogen and helium where mass-wise, at the end of the day, it is equivalent to Earth's mass (i.e., each oxygen has eight times the mass of hydrogen).
- We must put in our mind that nothing in the universe is working by force because any involuntary event would slow down after a while; therefore, there is no resistance due to the structure of the universe. Also, there is no friction in a cosmic world; therefore, there is no pull or push involved.
- When we study the planets of our solar system, all are following their own instinctive path without any friction or delay. Also, it seems they are situated in such a manner to protect the sun by rotating around it similar to the moons protecting the planets.

For example, our moon is protecting Earth, evidenced by the back of the moon where it has been bombarded by comets during the course of its journey. The same scenario goes for the back of the planet Mercury.

Since humans never had experience in space until the mid1950s, they never knew better until today. We know astronauts never feel gravity in free fall or zero gravity as the name implies, nor is a space station pulled by the sun's gravity; therefore, no gravity of the sun can be felt. The assumption of gravity that the sun is theoretically producing with its weight in space-time is not sufficient to create this trampoline effect (reaching hundreds of light-years) through the solar system even with the calculations of a hundred years ago. So, gravity in this regard is not working with the sun's mass at all. The result of the trampoline issue is completely irrelevant, and it is incorrect.

Consider that some of the planets that are far away are much bigger than Earth with tens of moons. Also, as we are accustomed to knowing that gravitational force is a one-way force, it cannot be multifunctional for planets with tens of moons and billions of miles away. Again, a lot of scientists admitted and started to question the trampoline theory and ask how it could be possible that our entire solar system, with its massive diameter (tens of billions of miles or km) around the sun, would go around the sun using the trampoline dynamic factor.

We have strong evidence by sending shuttles into space for the last few decades (right after Einstein died), and we are now witnessing that there is no gravity pull or weight in space. All the astronauts float in the space shuttle, and they say that there is no gravity and that the body is weightless. There is no other explanation for this phenomenon other than stating that there is no gravity in space and that the mass of an object is weightless in space. This example of an astronaut's body in a space shuttle symbolizes the whole universe.

Another example is when the astronauts in the space shuttle are sleeping in a sleeping bag. If there was any gravity, then they would be pulled to one side by that gravity. However, that does not happen. They simply float, weightless, due to the universe's natural function. Also, if the sun's gravity or moon's gravity, which supposedly affects Earth's ocean waters, was so strong, then why would a small space shuttle not be pulled in the sun's or moon's direction when it is only ten minutes in light-years away from the sun or even closer to the moon's gravity. Why wouldn't the astronauts be pulled in either direction while sleeping?

Simply, gravity does not exist in space and the expansion of the universe is in spherical volume in all directions versus the theory of an expansion of a flat universe with infinite space. The observation shows that the universe is expanding in all directions. So, if gravity did exist in space or space-time, then all the galaxy's movements would be pulled by this gravity and tend to shift toward that one path or one direction. However, that is not the case. We are experiencing that the universe is expanding in all directions, not just one direction.

Since we are at a point where gravity does not exist on a galactic scale in space, then Einstein's theory of gravity, which states that gravity bends light, is false due to twenty-five years of investigation by a group of scientists at NASA that found no gravity exists in space.

The recent results of NASA scientists' research on this theory where they studied thousands of pictures of the light effects on stars and planets (like our sun and the planet Mercury), showing that the light did not bend around the planets, proves that gravity does not bend light and is correct and their results are legitimate.

Without a doubt, gravity does not exist in the space-time of Albert Einstein or in Earth gravity of Isaac Newton. Thus, it would be unscientific and impossible to write any equation.

For the last thought, we all are residents of this magnificent universe; thus, all laws would apply to everything inside this universe. All the elements of the periodic table must be the same universally, and all forces act and follow the structure of the universe. Also, all the conservation of energy must be the same as well.

Having said this, the big question arises: Do we know any law of the universe? If we truly know gravitational force, then how are UFOs or UAPs flying in our space without showing any propulsion?

The question remains, what is gravity? What is an atom? Why do we think the atom is working mechanically when it does not act mechanically, due to its use of energy without friction. And much more can be explained in my next article or read my book "The Universe Revealed" for more details.