IMAGE: A MAP OF THE STARS OF THE ORION CONSTELLATION

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I. INTRODUCTION

Computerized adaptive testing implies the test adaptation to the level of knowledge of the test. During the testing process the system analyzes the answers and uses them to choose each following question based on the best correspondence to the level of examinee so that the questions gradually become complicated for a well-prepared examinee and simpler for a poorly prepared person. The process of test adaptation for an individual user is mentioned.

This means that the tests must be pre-calibrated according to their level of difficulty.

The modern computerized adaptive testing (CAT) is based on item response theory (IRT). IRT is a family of mathematical models that describe how people interact with test items (Embretson & Reise, 2000). According to this theory test items are described by their characteristics of difficulty and discrimination. Discrimination is independent of difficulty and shows how the probability of a positive response is distributed between different levels of examination. In addition, they can have a so-called "pseudo-guessing" parameter that reflects the probability that an examinee with a very low trait level will correctly answer an item solely by guessing (Baker, 2001).

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The combination of these three parameters allows us to evaluate the knowledge of an examinee via the Maximum Likelihood Estimation (MLE) method. The MLE method is much more flexible than the so-called "The Number Correct" assessment, which implies the number of correct answers from the questions asked (perhaps, considering question weight). For example, number-correct scoring of a 10-item conventional test can result in at most 11 scores (0 to 10); MLE for the same test can result in 2^{10}=1024. MLE also provides an individualized standard error of measurement (SEM) for each examinee.

Despite the above and other advantages, the MLE method requires extensive preliminary work to determine with appropriate accuracy the difficulty, discrimination, and guessing parameter for each issue of the test. The most common method of determining these parameters is the preliminary testing. To get real results via the preliminary testing, it is necessary to examine hundreds and thousands of users, which is not easy.

In general, to obtain the advantages of the Item Response Theory (IRT), the tests should be designed, constructed, analyzed and interpreted within the framework of the given theory. Particularly, IRT implies that the ability of the particular examinee is known in advance, and based on these data, the parameters of the characteristic curve of items (difficulty, discrimination, guessing parameter) are determined (Baker, 2001).

In the considered model a set of items of the test is divided into several parts, depending on complexity. Subsequently, there is no other information available about the items on a test. In other words, the difficulty, discrimination and parameter of guessing for each item separately are not available. The model under discussion does not present the preliminary estimate parameter θ of an examinee's abilities. True, the lack of information decreases the accuracy of the result, but the big advantage of a simple model is that its practical application is easy.

We will try to create a test assessment system that makes it easy for the test creator to use a computer-adaptive method for creating one's own test. For this purpose, let us not discuss IRT but another traditional approach to testing—Stradaptive Testing. The term "Stradaptive" is derived from the "Stratified Adaptive", and it belongs to D. J. Weiss (Betz & Weiss, 1973; Betz & Weiss, 1974).

Stradaptive testing considers different strategies of the leveling, which were fundamentally discussed and studied earlier. These strategies are:

- Two-stage approach (Betz & Weiss, 1973; Betz & Weiss, 1974; Larkin & Weiss, 1975);
- Multi-Stage Approach:
  - Fixed Branching Models:
    - Pyramidal Strategy (Larkin & Weiss, 1975);
    - Flexilevel (Lord, 1970; Betz & Weiss, 1975; Pyper, Lilley, Wernick, Jefferies, 2014);
    - Stradaptive Testing (Weiss 1973; Weiss, 1974; Waters, 1977);
  - Variable Branching Models:
    - Bayesian (Weiss, 1974; McBridge & Weiss, 1976);
    - Maximum likelihood approach (Weiss, 1974).

In the given paper we consider multistage testing.

II. MULTISTAGE ADAPTIVE TESTING

"Recently, multistage testing (MST) has been adopted by several important large-scale testing programs and become popular among practitioners and researchers" (Zheng & Chang, 2015, p. 104).

"MST is a balanced compromise between linear test forms (i.e., paper-and-pencil testing and computer-based testing) and traditional item-level computer-adaptive testing (CAT)" (Zheng, Nozawa, Gao & Chang, 2012, p. ii).

The multistage adaptive test represents a structured adaptive test, which uses pre-designed subtests as the main unit of testing control.

"In contrast to item-level CAT designs, which result in different test forms for each test taker, MST designs use a modularized configuration of pre-designed subtests and embedded score- routing schemes to prepackage validated test forms" (Melican, Breithaupt & Zhang, 2010, p. 171).

The "stage" in multistage testing is an administrative division of the test that facilitates the adapting of the test to the examinee. Each examinee is administered modules for a minimum of two stages, where the exact number of stages is a test design decision affected by the extent of desired content coverage and measurement precision. In each stage, an examinee receives a module that is targeted in difficulty at the examinee's provisional ability estimated, computed from the latter's performance on modules administered during the previous stage(s). Within a stage, there are typically two or more modules that vary from one another based on average difficulty. Because the modules vary this way, the particular sequence of item sets that any examinee is presented with is adaptively chosen based on the examinee's temporary assessment. After an examinee finishes each item set, his or her ability estimate is updated to reflect the new measurement information obtained about his ability. The next module is chosen to provide an optimal level of measurement information for a person at that computed proficiency level. High-performing examinees receive modules of higher average difficulty, while less able examinees are presented with modules that are comparatively easier (Zenisky, Hambleton & Luecht, 2010).

Thus, traditional CAT selects items for a test adaptively, while a multistage testing (MST) is an analogous approach that uses sets of items (modules, testlets) as the "building blocks" for a test. In MST terminology, these sets of items have come to be termed modules (Luecht & Nungester, 1998; Crotts, Sireci & Zenisky, 2012; Kim & Moses, 2014) or testlets (Wainer & Kiely, 1987; Wang, Bradlow & Wainer, 2002) and can be characterized as short versions of linear test forms where some specified number of individual items are administered together to meet particular test specifications and provide a certain proportion of the total test information.

III. ORDERING METHOD OF OUTCOME SET

The initial article Razmadze et al. (2017) considers an original method of CAT result estimation for multistage testing strategy.

In contrast to the classical item response theory (IRT) concepts (Embretson & Reise, 2000; Van der Linden & Hambleton, 1997; Baker, 2001), Rasch's model (Rasch, 1960/1980) or non-IRT (i.e. the Measurement Decision Theory) of CAT (Rudner, 2009), the model under discussion, does not present the preliminary estimate parameter θ of an examinee's abilities and the items of the same level have the same difficulty.

The method considers all possible variants of results, which is named an outcome set. The outcome set represents a non-typical unity of different dimensional elements. At Razmadze et al. (2017), article comparison criteria for these elements are defined, and principles of ordering of the set are described. The article shows how to receive the final score after ordering the outcome set. The ordered criteria of outcomes set may not be singular; this is confirmed by a comparative review of two examples presented in this work.

In multistage testing, to build a panel using modules, an author of a test uses a linear programming or heuristic methods. Apart from this, Fisher's Maximum Information Method is used for obtaining the classification cut-points for the optimization of the information of a module (Zheng et al., 2012). All the methods mentioned above requires specific knowledge. Our model does not have such limitations for a test author because such specific work is performed by an "automatic system of testing" compiler, while the author of a test has only to divide the testing items into several levels according to difficulty. This procedure should not be complicated because we assume that the author of this test is a professional in the field for which the appropriate test is created.

To express the ordering method of outcomes set, a specific procedure for testing is used in Razmadze et al.'s (2017) article. This procedure has an illustrative purpose for the evaluation method. The method described can be used for other similar strategies as well as for multistage testing, one of the models was discussed in the article „The Set Ordering Method for Scoring the Outcomes of 1-2-4 Multistage Model of Computerized Adaptive Testing" (Razmadze, 2019).

The current article discusses similar model, although unlike the three-stage 1-2-4 model, described in previous paper, there is the four-stage 1-2-3-4 model.

Thus, the paper presented is devoted to the realization of an ordering method of the outcome set, in particular on the example of a four-stage 1-2-3-4 model.

IV. THE FOUR-STAGE 1-2-3-4 ADAPTIVE MODEL

4.1 The scheme of 1-2-3-4 model

Now let us consider the usage of the ordering of testing result scores in case of multistage adaptive testing. For this purpose, we will discuss the four-stage 1-2-3-4 model, which is presented in the following scheme (Zheng et al., 2012):

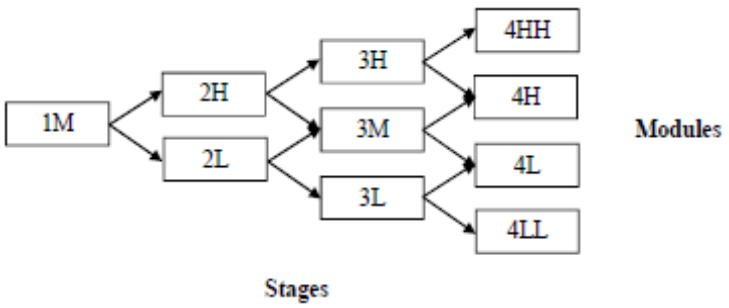

Figure 1: The 1-2-3-4 MST model

The number indicated in the rectangle of the module corresponds to the stage; the letters correspond to comparative difficulty (H: high; M: medium; L: low; HH: higher than H; LL: lower than L). Let us number the medium difficulties of modules. Each of these numbers can be considered as the weight of a corresponding module item:

Table 1: Comparative difficulty

Difficulty	LL	L	M	H	HH
Weight	1	2	3	4	5

In this case comparative difficulties are numbered although it is possible to assign different weights for modules at different stages with the same comparative difficulties:

Table 2: The item weights of the four-stage 1-2-3-4 model

#	1	2	3	4	5	6	7	8	9	10
Difficulty	4LL	3L	4L	2L	1M	3M	2H	4H	3H	4HH
Weight	1	2	3	3	4	4	5	5	6	7

The displayed classification can be considered as an analogy to the one used in the item response theory (IRT) (-3; 3) range, where the examinees' abilities are measured (Baker, 2001). But in this case instead of (-3; 3) range we use the weights provided in Table 2. This does not distort the achievement of the initial task. By considering the weights, the scheme from Figure 1 will transform into the following:

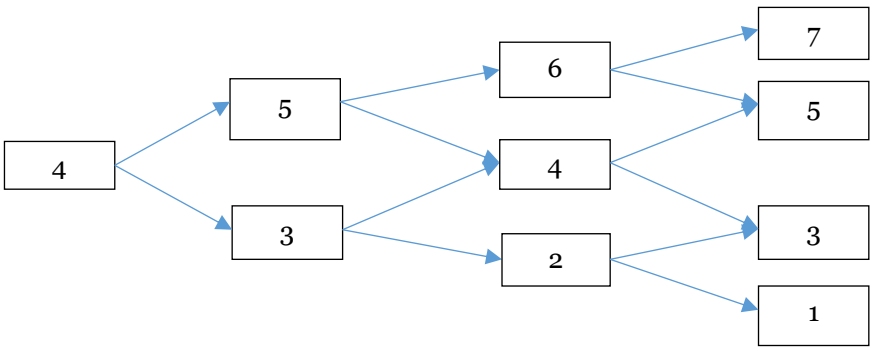

Figure 2: The 1-2-3-4 MST model with weights

4.2 Outcome of 1-2-3-4 model

In the first row of Table 2, all modules are numbered from 1 to 10. We will be using the given numbering for defining the test outcome. Taking into account the complexity levels of the modules, the outcome is expressed as a ten-dimensional vector: $n = \{ c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10} \}$, where c_i represents the number of correct answers of i module, $i = 1, 2, \dots, 10$. Due to the fact each testee performs only one item on each stage, there can be only 4 components out of a given 10 that are different from 0 in each test outcome. In addition, each c_i component, $i = 1, 2, \dots, 10$, has a weight, predefined according to Table 2.

In Razmadze et al.'s (2017, p. 1656) article, the outcome was defined as a vector drawn from the corresponding numbers of the levels of items obtained during the testing process. In this case, by definition, the outcome vector consists of the components that correspond to the number of correct answers in each module. This is more convenient for using the set ordering method for multistage adaptive tests.

Let us look at how many items there are per module. According to the module given by Zheng et al. (2012), the examinee is given 21 items that can be distributed among the stages differently:

Table 3: Amount of items according to stages

	1-2-3-4 model			
	Stage 1	Stage 2	Stage 3	Stage 4
Model condition A	6	5	5	5
Model condition B	7	6	4	4
Model condition C	4	6	6	5
Model condition D	4	4	6	7

Let us choose one of the model conditions, for example, model condition C. In $n = \{ c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10} \}$ only 4 components are able to obtain whole values different from 0 within the ranges [0–4], [0–6], [0–6] and [0–5]. These components may have 5, 7, 7 and 6 different answers, respectively; other components are always zero. The total amount of outcomes would be N = 5*7*7*6 = 1470.

4.3 Outcome route

Modules of the first, second and third stages have classification cut-points that define the route of the testing outcome; in other words, choosing the second, third and fourth stage modules. Classification cut-point is the amount of correct answers within the module that defines the branching — next stage module. Despite where the classification cut-points are chosen, the total amount of the testing outcomes is constant and N = 1470.

An example discussed in this article on the first stage of 1M module cut-point equals to 2. This means that in case of less than 2 correct answers (0 or 1) an examinee will be given the easier 2L module of the second stage, and in case of two or more correct answers (2, 3 or 4) the more difficult 2H module of the second stage.

In the second stage 2L module cut-point is 3, in 2H module it is 4. This means:

- In 2L module, if the number of correct answers is less than 3 (0, 1 or 2), an examinee will be provided with the 3rd stage easy 3L module and in case of 3 or more correct answers (3, 4, 5 or 6) - the 3rd stage medium 3M module items;
- In 2H module if the number of correct answers are less than 4 (0, 1, 2, or 3), an examinee will be provided with the 3rd stage medium 3M module and for more than 4 correct answers (4, 5, or 6) - the 3rd stage difficult 3H module items;

On the 3rd stage modules 3L and 3M, the cut-point is 3 and for 3H it is 4. This means the following:

- In 3L module if the number of correct answers is less than 3 (0, 1 or 2) an examinee will be provided with the 4th stage easiest 4LL module and in case of 3 or more correct answers (3, 4, 5, or 6) the 4th stage easy 4L module items;
- In 3M module if the number of correct answers is less than 3 (0, 1, or 2) an examinee will be provided with the 4th stage easy 4L module and in case of 3 or more correct answers (3, 4, 5, or 6) the 4th stage difficult 4H module items;

- In 3H module if the number of correct answers is less than 4 (0, 1, 2, or 3) an examinee will be provided with the 4th stage difficult 4H module and in case of 4 or more correct answer (4, 5, or 6) the 4th stage most difficult module 4HH items.

V. THE SET ORDERING METHOD FOR SCORING THE OUTCOMES OF THE 1-2-3-4 MODEL

5.1 Ordering according to the S(n) criterion

Let us discuss the first criterion from the initial article Razmadze et al. (2017, p. 1658, Formula (4)):

$$S(n) = \frac{R}{1+M}, \quad n \in N \qquad (1),$$

where R is a weighted sum of scores of correct answers and M is a weighted sum of scores of incorrect answers.

The corresponding formulas for calculating R and M are given in the article Razmadze et al. (2017, p. 1657, Formulas (1) and (2)). Based on these formulas, in case of the 1-2-3-4 MST model, we will obtain the following:

$$R = c_1 + 2 * c_2 + 3 * c_3 + 3 * c_4 + 4 * c_5 + 4 * c_6 + 5 * c_7 + 5 * c_8 + 6 * c_9 + 7 * c_{10},$$

$$M = 7 * d_1 + 6 * d_2 + 5 * d_3 + 5 * d_4 + 4 * d_5 + 4 * d_6 + 3 * d_7 + 3 * d_8 + 2 * d_9 + d_{10},$$

where d_i is a number of mistakes in i module, $i = \overline{1,10}$.

The Formula (1), which should be used for outcome estimation, is now used in the ten-module case. The structure of outcome set of the four-stage model discussed in this article is different from the one discussed in the initial article by Razmadze et al. (2017, p. 1656). This means that the domain of a function S(n) is different. Despite this, S(n) function will provide a complete ordering of set N in the given case too.

The result is provided in Table 4, where c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10} values are given in the columns B, C, D, E, F, G, H, I, J, K, respectively. The values calculated using Formula (1) are shown in column P. The data is sorted according to P column decreasing order. The table shows the first 10 (left half) and last 10 (right half) testing outcomes' estimation results.

Table 4: 1-2-3-4 Model's Outcome Estimation by S (n) Criterion

	B	C	D	E	F	G	H	I	J	K	P
1	4LL	3L	4L	2L	1M	3M	2H	4H	3H	4HH	S(n)
2	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	(4)
3					4		6		6	5	117.00
4					4		6		6	4	55.00
5					4		6		5	5	37.00
6					4		6		6	3	34.33
7					4		5		6	5	28.00
8					4		6		5	4	26.00
9					4		6		6	2	24.00
10					3		6		6	5	22.60
11					4		6		4	5	21.00
12					4		5		6	4	21.00

	B	C	D	E	F	G	H	I	J	K	P
1	4LL	3L	4L	2L	1M	3M	2H	4H	3H	4HH	S(n)
2	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	(4)
1463	1	0		1	0						0.04
1464	0	2		0	0						0.04
1465	0	0		0	1						0.04
1466	3	0		0	0						0.03
1467	1	1		0	0						0.03
1468	0	0		1	0						0.03
1469	2	0		0	0						0.02
1470	0	1		0	0						0.02
1471	1	0		0	0						0.01
1472	0	0		0	0						0.00

5.2 Ordering according to the F (n) criterion

Let us discuss the second criterion from the initial article Razmadze et al. (2017, p. 1658, Formula (9)):

$$F(n) = R * \frac{A}{\mu}, \quad n \in N \qquad (2),$$

where R is a weighted sum of scores of correct answers, A is an average complexity of incorrect answers and μ - the number of mistakes.

The corresponding formulas for calculating R and A are given in the initial article by

Razmadze et al. (2017, p. 1657, Formulas (1) and (3)). Based on these formulas, in the case of the 1-2-3-4 MST model, we will obtain the following:

$$R = c_1 + 2 * c_2 + 3 * c_3 + 3 * c_4 + 4 * c_5 + 4 * c_6 + 5 * c_7 + 5 * c_8 + 6 * c_9 + 7 * c_{10},$$

$$A = \frac{d_1 + 2 * d_2 + 3 * d_3 + 3 * d_4 + 4 * d_5 + 4 * d_6 + 5 * d_7 + 5 * d_8 + 6 * d_9 + 7 * d_{10}}{21 - (c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 + c_9 + c_{10})},$$

where d_i – is a number of mistakes in i module, $i = \overline{1,10}$.

$$\mu = 21 - (c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 + c_9 + c_{10}).$$

The Formula (2), which should be used for outcome estimation, is now used in the ten-module case. The structure of the outcome set of the four-stage model discussed in this article is different from the one discussed in the initial article by Razmadze et al. (2017, p. 1656). This means that the domain of a function $F(n)$ is different. Although it is easy to check that despite this, $F(n)$ function will provide a complete ordering of set N in the given case too.

The results obtained by using $F(n)$ criterion are shown in Table 5, where c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10} values are given in the columns B, C, D, E, F, G, H, I, J, K, respectively. The values calculated using Formula (2) are shown in column Q. The data is sorted according to Q Column decreasing order. Table 5 shows the first 10 (left half) and the last 10 (right half) testing outcomes' estimation results.

Table 5: 1-2-3-4 Model's outcome estimation by F(n) criterion

	B	C	D	E	F	G	H	I	J	K	Q
1	4LL	3L	4L	2L	1M	3M	2H	4H	3H	4HH	F(n)
2	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	(9)
3					4		6		6	5	819.00
4					4		6		6	4	770.00
5					4		6		5	5	666.00
6					4		5		6	5	560.00
7					3		6		6	5	452.00
8					4		6		6	3	360.50
9					4		6		5	4	338.00
10					4		6		4	5	315.00
11					4		5		6	4	315.00
12					4		5		5	5	291.50

	B	C	D	E	F	G	H	I	J	K	Q
1	4LL	3L	4L	2L	1M	3M	2H	4H	3H	4HH	F(n)
2	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	(9)
1463	1	0		1	0						0.52
1464	0	2		0	0						0.52
1465	0	0		0	1						0.47
1466	3	0		0	0						0.44
1467	1	1		0	0						0.40
1468	0	0		1	0						0.36
1469	2	0		0	0						0.27
1470	0	1		0	0						0.25
1471	1	0		0	0						0.13
1472	0	0		0	0						0.00

Comparative analysis of Tables 4 and 5 shows different sequences of outcome sets after ordering them. Thus, the creator of an automatized system of testing can choose the needed criterion on one's own. Furthermore, he can create a new, different criteria, which could be better suited to one's own requirements and assessments.

5.3 The final score of outcome

Now let us transform the points obtained in Tables 4 and 5 into integer numbers [0; 100] segment. While ordering the data obtained by the first and the second criteria in Razmadze et al.'s (2017, pp. 1659, 1660) article, the point correction was performed. In case of the first criterion, the first 90 points, and in case of the second criterion, the first 30 points. This felt somewhat artificial.

Now let us act differently. The criteria $S(n)$ and $F(n)$, used in Tables 4 and 5, have fulfilled their mission and ordered the set of the testing outcomes N. The resulting points do not have essential importance. They can be substituted by any decreasing sequence of 1470 numbers. The decreasing order ensures to keep the ordering of the testing outcomes so that the better testing result corresponds to the higher point.

It will be natural if we distribute the scores within the whole number segment [0; 100] using the normal distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3}$$

Table 6 illustrates the testing outcome scores with normal distribution for 1-2-3-4 MST model ordered by $F(n)$ criterion, where $\mu = 65$; $\sigma = 25$. The table shows the first 25 and the last 25 testing outcomes' scores.

Table 6: Testing outcomes for 1-2-3-4 MST model scores with a normal distribution

	B	C	D	E	F	G	H	I	J	K	W
1	4LL	3L	4L	2L	1M	3M	2H	4H	3H	4HH	F(n)
2	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Normal
3					4		6		6	5	100
4					4		6		6	4	100
5					4		6		5	5	100
6					4		5		6	5	100
7					3		6		6	5	100
8					4		6		6	3	100
9					4		6		5	4	100
10					4		6		4	5	100
11					4		5		6	4	100
12					4		5		5	5	100
13					3		6		6	4	99
14					4		4		6	5	99
15					3		6		5	5	99
16					3		5		6	5	99
17					4		6		6	2	99
18					2		6		6	5	99
19					4		6		5	3	99
20					4		6		4	4	99
21					4		5		6	3	99
22					4		5		5	4	99
23					3		6		6	3	98
24					4		5		4	5	98
25					4		4		6	4	98
26					3		6		5	4	98
27					4		4		5	5	98

	B	C	D	E	F	G	H	I	J	K	W
1	4LL	3L	4L	2L	1M	3M	2H	4H	3H	4HH	F(n)
2	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Normal
1448	3	0		1	0						9
1449	2	2		0	0						9
1450	5	0		0	0						9
1451	0	0		1	1						8
1452	2	0		0	1						8
1453	1	1		1	0						8
1454	3	1		0	0						7
1455	0	1		0	1						7
1456	0	0		2	0						7
1457	2	0		1	0						6
1458	1	2		0	0						6
1459	4	0		0	0						6
1460	1	0		0	1						5
1461	0	1		1	0						5
1462	2	1		0	0						5
1463	1	0		1	0						4
1464	0	2		0	0						4
1465	0	0		0	1						3
1466	3	0		0	0						3
1467	1	1		0	0						2
1468	0	0		1	0						2
1469	2	0		0	0						1
1470	0	1		0	0						1
1471	1	0		0	0						0
1472	0	0		0	0						0

The whole table graphically looks as follows (Fig. 3):

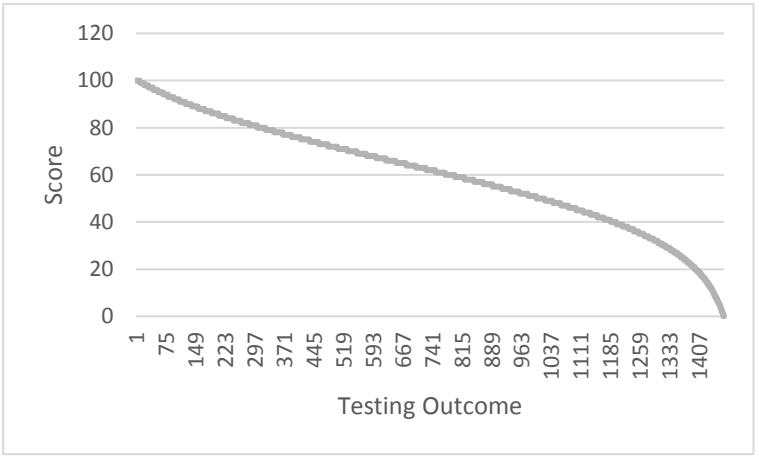

Figure 3: Graph of 1-2-3-4 MST model testing outcomes' score's normal distribution

The fact of the point and times of usage is visible from the following graph (Fig. 4):

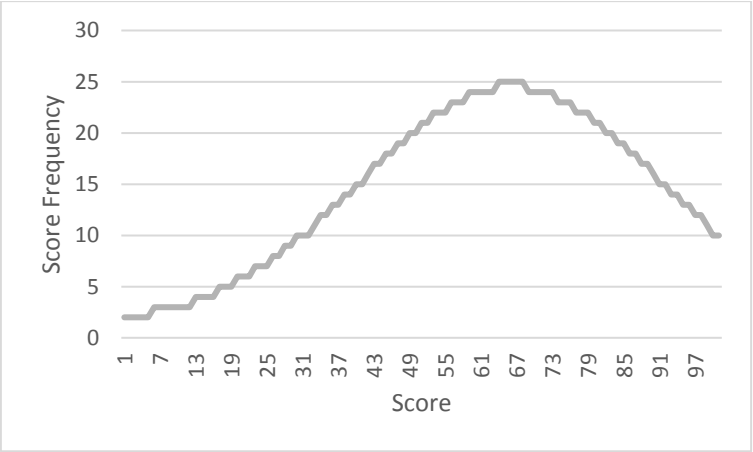

Figure 4: Normal distribution of the testing outcome points

VI. CONCLUSION

The ordering method of the outcome set can be used in case of different testing procedures. The obvious example of this is the realization of the method for multistage adaptive testing's (MST) 1-2-3-4 model, which is described in the presented paper.

The author of a test has no direct contact with this method and its specific nuances because the realization of the method is a one-time procedure carried out during the computerized adaptive testing portal formation.

The method does not require a detailed calibration of the item pool or preliminary testing of examinees to create a calibration sample. The ordering method of outcome set is oriented on the test author; it helps him avoid the problem of preliminary adaptation of test items for the examinee's knowledge level and simplifies the workload at maximum. Preliminary work for the test author might only include the division of test items into several difficulty levels based on expert assessment.

In the situation where there is a lack of information about test item's and examinee's level, the method maximally uses the existing information for an examinee estimation: it takes into account all the answers to the questions provided to the examinee, and the set of received answers is compared to all the possible variants and placed on a corresponding level in the estimation hierarchy.

The paper presents the usage of the ordering method of outcomes set for multistage adaptive testing (MST) model as a sample. The method can be used for different modern testing models, but it is the subject of further research.

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Dynamics of States with Non-Zero Moment in Own Field

A. S. Chikhachev

ABSTRACT

The paper studies a nonstationary self-consistent quantum system that intensively interacts with its own field. At a non-zero moment, the psi function cannot be independent of the angles of the spherical coordinate system. In this paper, a superposition of angular distributions is found, leading to a spherically symmetric charge distribution for the whole value of the moment (l=1).

The paper defines the conditions under which in the case of half-integer values of the moment $l = \frac{1}{2}$ the charge density distribution turns out to be spherically symmetric. In this case, a self-consistent system can be described by a system of ordinary differential equations.

In the final section, a classical collisionless one-component system of charged particles characterized by a nonzero moment is considered.

Keywords: schrodinger equation, orbital moment, nonstationary coordinates, kinetic equation.

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ДИНАМИКА СОСТОЯНИЙ С НЕНУЛЕВЫМ МОМЕНТОМ В СОБСТВЕННОМ ПОЛЕ

A. S. Chikhachev

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The paper studies a nonstationary self-consistent quantum system that intensively interacts with its own field. At a non-zero moment, the psi function cannot be independent of the angles of the spherical coordinate system. In this paper, a superposition of angular distributions is found, leading to a spherically symmetric charge distribution for the whole value of the moment (l=1).

The paper defines the conditions under which in the case of half-integer values of the moment $l = \frac{1}{2}$ *the charge density distribution turns out to be spherically symmetric. In this case, a self-consistent system can be described by a system of ordinary differential equations.*

In the final section, a classical collisionless one-component system of charged particles characterized by a nonzero moment is considered.

Keywords: schrodinger equation, orbital moment, nonstationary coordinates, kinetic equation.

Аннотация

В работе изучается нестационарная самосогласованная квантовая система, интенсивно взаимодействующая с собственным полем. При ненулевом моменте пси-функция не может быть независящей от углов сферической системы координат. В работе найдена суперпозиция угловых распределений, приводящая к сферически симметричному распределению заряда при целом значении момента ($l = 1$ *).*

В работе определены условия, при которых в случае полуцелых значениях момента $l = \pm\frac{1}{2}$ *распределение плотности заряда оказывается сферически симметричным. В этом случае самосогласованная система может быть описана системой обыкновенных дифференциальных уравнений.*

В заключительном разделе рассмотрена классическая бесстолкновительная однокомпонентная система заряженных частиц, характеризуемая ненулевым моментом.

Ключевыеслова: уравнение Шредингера, орбитальный момент, нестационарные координаты.

I. Введение

Изучение нестационарных систем, интенсивно взаимодействующих с собственным полем представляет большой интерес как с экспериментальной, так и с теоретической точек

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зрения. Особый интерес представляет квантовомеханическая система, использующая нестационарный гамильтониан. В настоящей работе будет использован нестационарный гамильтониан, следующий из работ Мещерского [1]. Этот гамильтониан использован в работе [2] для решения квантовомеханической задачи. В работах [3,4] решались задачи в одномерной конфигурации и сферически симметричная проблема при нулевом орбитальном моменте l=0.

В настоящей работе приведено решение уравнения Шредингера в сферических координатах при ненулевом орбитальном моменте, причем рассмотрены задачи с целым (l=1) и полуцелым орбитальным моментом. Так же, как и в работах [3,4] точное решение сводится к системе обыкновенных дифференциальных уравнений 4-го порядка.

Заметим здесь, что используемый нестационарный гамильтониан может быть использован как для квантовых, так идля классических систем. Кинетическое уравнение с использованием модельного нестационарного гамильтониана, впервые, по-видимому, сформулировано в работе [5]. В дальнейшем динамика нестационарных самосогласованных систем в плоской исферической геометрии изучались в работах [6],[7],[8].

Состояния с моментом l=1

Состояние квантовой системы с ненулевым моментом в центральном поле описывается уравнением Шредингера:

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial r^2}+\frac{2}{r}\frac{\partial\Psi}{\partial r}-\frac{\hat{L}}{r^2}\Psi\right)+U(r,t)\Psi(r,t), \qquad 1.1$$

здесь

$$\hat{L}=\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}.$$

Положим

$$\Psi\left(\vec{r},t\right)=\psi(r,t)Y(\varphi,\theta)$$

При этом

$$\hat{L}Y=-LY\ ,$$

где $L=l\left(l+1\right)$ -квадрат полного момента. При $L\neq 0$ в уравнении присутствуют производные по угловым переменным, что означает отсутствие сферической симметрии изучаемого состояния, что является существенным обстоятельством для систем с собственным полем.

Рассмотрим, далее, случай, когда $L=l\left(l+1\right)$ и введем функцию ψ_1 посредством равенства $\psi(r,t)=\psi_1(r,t)r^l$, причем будем изучать нестационарную систему, описываемую

потенциалом вида: $\frac{1}{\xi(t)^2}U\left(\frac{r}{\xi(t)}\right),$
получим:

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$$i\hbar\frac{\partial\psi_1}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_1}{\partial r^2}+\frac{2(l+1)}{r}\frac{\partial\psi_1}{\partial r}\right)+\frac{1}{\xi(t)^2}U\left(\frac{r}{\xi(t)}\right)\psi_1(r,t), \qquad 1.2$$

Здесь $\xi(t)$ функция, удовлетворяющая уравнению: $\ddot{\xi}=\frac{\lambda}{\xi^3}$,

U/ξ^2 -потенциал собственного поля, r,φ,θ - координаты сферической системы.

Введем новые переменные: $\tau=\int\frac{dt'}{\xi(t')^2}, \rho=\frac{r}{\xi}$.

Тогда (1.2) приводится к виду:

$$i\hbar\left(\frac{\partial\psi_1}{\partial\tau}-\rho\frac{\dot{\xi}}{\xi}\frac{\partial\psi_1}{\partial\rho}\right)=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_1}{\partial\rho^2}+\frac{2(l+1)}{\rho}\frac{\partial\psi_1}{\partial\rho}\right)+U(\rho)\psi_1(\rho,\tau). \qquad 1.3$$

Произведем следующее преобразование, положим $\psi_1=\Lambda\psi_2$, где $\Lambda=\exp\left(\frac{im}{2\hbar}\frac{\dot{\xi}}{\xi}\rho^2\right)$. Для ψ_2 следует уравнение:

$$i\hbar\frac{\partial\psi_2}{\partial\tau}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_2}{\partial\rho^2}+\frac{2(l+1)}{\rho}\frac{\partial\psi_2}{\partial\rho}\right)+U(\rho)\psi_2(\rho,\tau)-\frac{\lambda m\rho^2}{2}\psi_2-\frac{i\hbar}{2}\frac{\dot{\xi}}{\xi}(l+\frac{3}{2})\psi_2. \quad 1.4 \qquad 1.4$$

При решении самосогласованной задачи с $l=1$ удобно ввести функцию ψ_3 : $\psi_2=\psi_3/\xi^3$. Получим:

$$i\hbar\frac{\partial\psi_3}{\partial\tau}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_3}{\partial\rho^2}+\frac{4}{\rho}\frac{\partial\psi_3}{\partial\rho}\right)+U\ \rho\ \psi_3\ \rho\ \tau\ -\frac{\lambda m\rho^2}{2}\psi_3+\frac{i\hbar}{2}\frac{\dot{\xi}}{\xi}\psi_3 \qquad 1.5$$

В (1.5) положим

$$\psi_3=\exp\left(-\frac{iE\tau}{\hbar}\right)\psi(\rho),\ \frac{\dot{\xi}}{\xi}\equiv const=\frac{1}{2\tau_0},\ \lambda=-\frac{1}{4\tau_0^2}.$$

Тогда

$$E\psi=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial\rho^2}+\frac{4}{\rho}\frac{\partial\psi}{\partial\rho}\right)+U(\rho)\psi(\rho)-\frac{\lambda m\rho^2}{2}\psi+\frac{i\hbar}{4\tau_0}\psi(\rho). \qquad 1.6$$

При этом плотность заряда определяется выражением $|\Psi|^2=\rho^2|\psi|^2|Y|^2/\xi^4$.

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В случае $l=1$ угловую часть пси-функции можно рассматривать как состояние со спином единица, описываемое столбцом $:Y=\begin{Bmatrix}Y_1\\Y_2\\Y_3\end{Bmatrix}$, соответственно Y^+ -это строка $\left(Y_1^*,Y_2^*,Y_3^*\right)$. Положим

$Y_1=\frac{1}{\sqrt{2}}\sin\theta\exp(i\phi), Y_2=\cos\theta, Y_3=\frac{1}{\sqrt{2}}\sin\theta\exp(-i\phi).$ При таком выборе представления

$$\overline{M_z}=Y^+M_zY=0, \overline{M_x}=Y^+M_xY=\frac{1}{\sqrt{2}}\sin 2\theta\cos\phi, \overline{M_y}=Y^+M_yY=\frac{1}{\sqrt{2}}\sin 2\theta\sin\phi,$$

а плотность заряда, определяемая произведением $Y^+Y,$ не зависит от углов: $Y^+Y=1.$
В этом случае уравнение для потенциала можно записать в виде:

$$\frac{1}{\xi^4}\frac{1}{\rho^2}\frac{d}{d\rho}\rho^2\frac{dU}{d\rho}=-\kappa_*\rho^2|\psi|^2\frac{1}{\xi^4}. \quad 1.7$$

Здесь κ_* -константа связи. Положим, далее, $\psi=R(\rho)\exp(i\theta(\rho))$. Получим систему уравнений:

$$ER=-\frac{\hbar^2}{2m}\left(R''-R\theta'^2+\frac{4}{\rho}R'\right)+R\left(U-\frac{m\rho^2}{8\tau_0}\right), \quad 1.8$$

$$-\frac{\hbar^2}{2m}\left(R'\theta'+R\theta''+\frac{4}{\rho}R\theta'\right)+\frac{\hbar}{4\tau_0}= \quad 1.9$$

Вместо ρ введем переменную s: $\rho=l_0s, l_0^2=\hbar\tau_0/m$. Обозначим

$y=\theta' l_0, V(s)=\frac{4\tau_0}{\square}(U-E).$ Тогда система принимает вид:

$$R''-Ry^2+\frac{4R'}{s}=V(s)R-s^2R, \quad 1.10$$

$$2R'y+Ry'+\frac{4}{s}Ry=R \quad 1.11$$

$$\frac{d^2V}{ds^2}+\frac{2}{s}\frac{dV}{ds}=-\kappa_0s^2R^2. \quad 1.12$$

Можно переписать (1.11) в виде: $(R^2ys^4)'=R^2s^4,$ а (1.12)-в виде
$\frac{d}{ds}s^2\frac{d}{ds}V(s)=\kappa_0R^2s^4=-\kappa_0(R^2s^4y)'.$ Тогда $s^2\frac{dV}{ds}=-\kappa_0R^2ys^4+C_*.$ Полагая $C_*=0$
получим уравнение:

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$$\frac{dV}{ds} = -\kappa_0 R^2 y s^2. \qquad 1.13$$

Уравнение(1.10) удобно переписать в виде:

$$\frac{R''}{R} + \frac{4R'}{sR} = V + y^2 - s^2, \qquad 1.14$$

а (1.11)-

$$y' = -2y\frac{R'}{R} - \frac{4}{s}Ry + R. \qquad 1.15$$

Т.е. имеем систему (1.13)-(1.15).

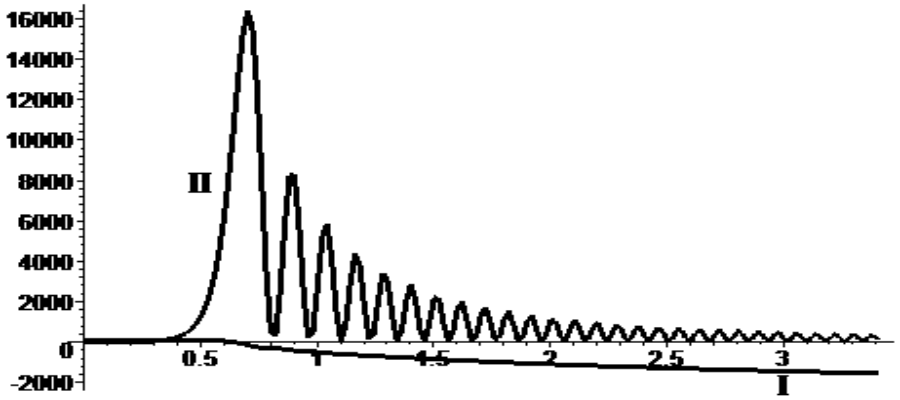

Рис. 1: Зависимость потенциала (I) и плотности заряда (II) от автомодельной переменной s.

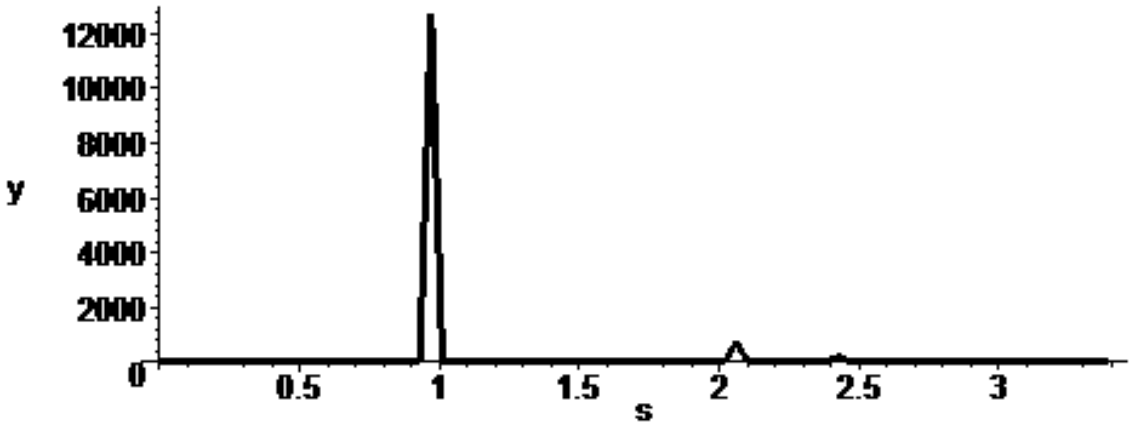

Рис. 2: Зависимость y (s).

На рисунках 1 и 2 изображены результаты решения системы (1.13), (1.14) и (1.15). В качестве начальных условий использовались соотношения: R(0)=10,R'(0)=0,V(0)=100,y(0)=0.

Полагалось $\kappa_0 = 1$ Зависимость y(s) характеризуется наличием резких и узких

максимумов, плотность заряда быстро осциллирует и стремится к нулю, а потенциал убывает монотонно.

Состояния с моментом $l = 1/2$

В отличие от прелылущего раздела рассмотрим, далее, случай, когда $L = 3/4$ (l=1/2). Как и в предыдущем случае положим $\Psi(\vec{r},t) = \psi(r,t)Y(\theta,\varphi).$

Введем функцию ψ_1 посредством равенства $\psi(r,t) = \psi_1(r,t)\sqrt{r},$ Представим также и потенциал в виде произведения функции от углов на функцию от радиуса: $U(\vec{r},t) = \Delta(\theta,\varphi)U(r,t).$ В дальнейшем будут определены условия, при которых $\Delta(\theta,\varphi) \equiv const.$

Получим:

$$i\hbar\frac{\partial\psi_1}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_1}{\partial r^2} + \frac{3}{r}\frac{\partial\psi_1}{\partial r}\right) + \frac{\Delta}{\xi(t)^2}U\left(\frac{r}{\xi(t)}\right)\psi_1(r,t), \qquad 2.1$$

Введем новые переменные: $\tau = \int\frac{dt'}{\xi(t')^2}, \rho = \frac{r}{\xi}$,

тогда (2.1) приводится к виду:

$$i\hbar\left(\frac{\partial\psi_1}{\partial\tau} \quad \rho\frac{\dot{\xi}}{\xi}\frac{\partial\psi_1}{\partial\rho}\right) \quad \frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_1}{\partial\rho^2} \quad \frac{3}{\rho}\frac{\partial\psi_1}{\partial\rho}\right) \quad U\ \rho\ \psi_1\ \rho\ \tau \qquad 2.2$$

Введем, длее, ψ_2, положив $\psi_1 = \psi_2\Lambda/\xi^{5/2}$, где $\Lambda = \exp\left(\frac{im}{2\hbar}\frac{\dot{\xi}}{\xi}\rho^2\right).$

Получим:

$$i\hbar\frac{\partial\psi_2}{\partial\tau} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_2}{\partial\rho^2} + \frac{3}{\rho}\frac{\partial\psi_2}{\partial\rho}\right) + U\ \rho\ \psi_2\ \rho\ \tau\ \Delta - \frac{m\rho^2}{8\tau_0^2}\psi_2 + \frac{i\hbar}{2}\frac{\dot{\xi}}{\xi}\psi_2 \qquad 2.3$$

Плотность заряда имеет вид: $Q = |\Psi|^2 = |\psi_2|^2\ \rho\,|Y|^2\ /\xi^4,$

где Y удовлетворяет уравнению:

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial Y}{\partial\theta} + \frac{3}{4}Y + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\varphi^2} = 0. \qquad 2.4$$

Вместо переменной θ введем $\eta = \ln\left(\tan\left(\frac{\theta}{2}\right)\right).$ тогда уравнение принимает вид:

$$ch^2\eta\left(\frac{\partial^2 Y}{\partial\eta^2} + \frac{\partial^2 Y}{\partial\varphi^2}\right) = -\frac{3}{4}Y. \qquad 2.5$$

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Далее будем учитывать спинорный характер Ψ -функции и представлять Y в виде столбца $: Y = \sqrt{\Omega(\eta)/2}\begin{pmatrix} \exp(i\varphi/2) \\ \exp(-i\varphi/2) \end{pmatrix}$,

соответственно Y^+ -это строка $Y^+ = \sqrt{\Omega(\eta)/2}\left(\exp(-i\varphi/2), \exp(i\varphi/2)\right)$. Проекции момента на оси координат имеют вид:

$$M_x = Y^+\sigma_x Y = \Omega\cos\varphi, M_y = Y^+\sigma_y Y = -\Omega\sin\varphi, M_z = Y^+\sigma_z Y = 0.$$

Зависимость плотности от углов определяется произведением Y^+Y и при выбранном представлении не зависит от угла φ: $Y^+Y = \Omega(\eta).$

Представим потенциал в виде произведения функции от радиуса на функцию от угловых переменных, а поскольку плотность заряда не зависит от φ, потенциал также не зависит от $\varphi : V = \Delta(\eta)U(\rho)$. Используя переменную η вместо θ получим:

$$\Delta(\eta)\frac{1}{\rho^2}\frac{d}{d\rho}\rho^2\frac{dU(\rho)}{d\rho} + \frac{U(\rho)}{\rho^2}\mathrm{ch}^2(\eta)\left(\frac{d^2\Delta}{d\eta^2}\right) = Q. \qquad 2,6$$

Здесь Q -плотность заряда. В случае взаимодействия с собственным полем $Q = -\mu|\Psi|^2, \mu$ - константа связи.

Полное отделение функции радиуса от функции угла может быть достигнуто, если выполнены условия: $\mathrm{ch}^2(\eta)\frac{d^2\Delta}{d\eta^2}) = \nu\Delta$ (ν – -константа) и $\Omega \equiv \Delta$. Из (2.5) можно получить:

$$\mathrm{ch}^2(\eta)\left(2\Omega''\Omega - \Omega'^2 - \Omega^2\right) = -3\Omega^2. \qquad 2.7$$

Исключая из этих соотношений $ch^2(\eta)$ получим уравнение:

$$\Omega''\Omega(2\nu+3) = \nu(\Omega'^2 + \Omega^2).$$

Это уравнение имеет интеграл:

$$C_1 = \Omega^{-\frac{2\nu}{2\nu+3}}\left(\Omega'^2 - \frac{\nu}{\nu+3}\Omega^2\right) \qquad (2.8)$$

Введем функцию $S(\eta) = \Omega^{\frac{\nu+3}{2\nu+3}}$. Тогда выражение для интеграла принимает вид:

$$C_1 = \left(\frac{2\nu+3}{\nu+3}S'\right)^2 - \frac{\nu}{\nu+3}S^2.$$

Из этого соотношения следует, что при $\nu = -\frac{3}{2}$ $S \equiv const.$ В то же время при $\nu = -3/2 \quad \Omega \equiv 1.$ Из (2.6) следует уравнение:

$$\frac{1}{\rho^2}\frac{d}{d\rho}\rho^2\frac{dU}{d\rho} - \frac{3}{2}\frac{U}{\rho^2} = -\kappa_0 |\psi_2|^2 \rho. \qquad 2.9$$

В уравнении (2.3) сделаем замену:

$$\psi_2 = \exp\left(-\frac{iE\tau}{\hbar}\right)R(\rho)\exp(i\theta(\rho)), \qquad 2.10$$

где E - действительная величина, $R(\rho), \theta(\rho)$, -действительные функции. Положив $\frac{\dot{\xi}}{\xi} \equiv const = \frac{1}{2\tau_0}$, получим:

$$ER = -\frac{\hbar^2}{2m}\left(R'' - R\theta'^2 + \frac{3}{\rho}R'\right) + R\left(U + \frac{\lambda m\rho^2}{2}\right), \qquad 2.11$$

$$-\frac{\hbar^2}{2m}\left(2R'\theta' + R\theta'' + \frac{3}{\rho}R\theta'\right) + \frac{\hbar}{4\tau}R = 0. \qquad 2.12$$

Уравнение (2.9) (с заменой $|\psi_2|^2$ на R^2 и (2.11) и (2.12) образуют замкнутую систему, описывающую нестационарную динамику. Вместо ρ введем безразмерную переменную $s: \rho = l_0 s, l_0^2 = \frac{2\hbar\tau_0}{m}$. Обозначим

$$y = l_0\theta', V = \frac{4\tau_0(U-E)}{\hbar}, \kappa_1 = \kappa_0 = \frac{4\tau_0}{\hbar}\left(\frac{2\hbar\tau_0}{m}\right)^2.$$

Тогда система принимает вид:

$$R'' - Ry^2 + \frac{3R'}{s} = (V(s) - s^2)R, \qquad 2.13$$

$$2R'y + Ry' + \frac{3}{s}Ry = R, \qquad 2.14$$

$$\frac{d^2V}{ds^2} + \frac{2}{s}\frac{dV}{ds} - \frac{3}{2}\frac{V(s)}{s^2} = -\kappa_1 sR^2. \qquad 2.15.$$

Удобно, далее, ввести функцию $W(s) = \frac{dV}{ds.}$. Уравнение (2.14) можно представить в виде: $(R^2 s^3 y)' = R^2 s^3$. Тогда (2.15) приводится к виду:

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$$s^2\frac{dV}{ds}-\frac{3}{2}W(s)=-\kappa_1 R^2 s^3 y(s)+C_*. \qquad 2.16$$

По-видимому, система (2.13), (2.14), (2.16) не имеет решения, регулярного в нуле. На Рис.3. приведены решения, использующие краевые условия при $s=1$. Полагалось $C_*=0, W(1)=-100, W'(1)=0, R'(0)=0, y(1)=0, R(1)=10, \kappa_1=1$. Решение характеризуется колебательным поведением функции y(s) и монотонным убыванием потенциала и ростом полного заряда внутри сферы радиуса s. Начальная точка счета полагалась $s_0=0.9$. При $s<0.002$ решение не определено.

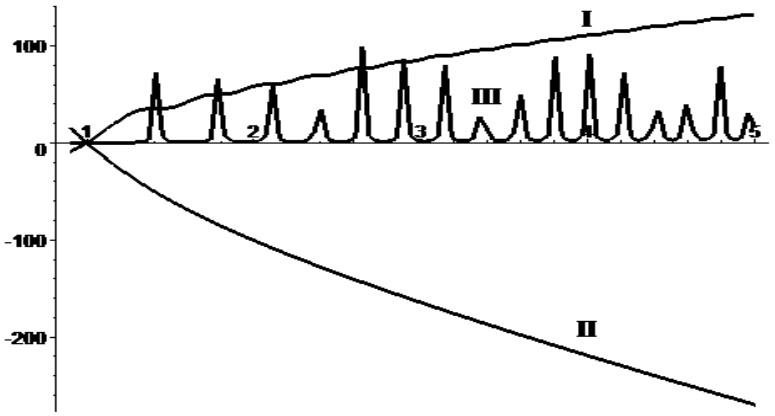

Рис.3: Кривая I-зависимость полного заряда внутри сферы радиуса s от s, кривая II-потенциал, кривая III-зависимость y(s).

Состояния с моментом $l=-\frac{1}{2}$

В отличие от предыдущего раздела рассмотрим, далее, случай, когда $L=-\frac{1}{4}\quad (l=-\frac{1}{2}).$

Как и в предыдущем случае положим $\Psi(\vec{r},t)=\psi(r,t)Y(\theta,\varphi).$ Введем функцию ψ_1 посредством равенства $\psi(r,t)=\psi_1(r,t)/\sqrt{r},$ причем потенциал также представим в виде произведения функции от углов на функцию от r,t: $\Delta(\theta,\varphi)U\left(\frac{r}{\xi}\right)\frac{1}{\xi^2}.$
Получим:

$$i\hbar\frac{\partial\psi_1}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_1}{\partial r}\right)+\frac{\Delta}{\xi(t)^2}U\left(\frac{r}{\xi(t)}\right)\psi_1(r,t), \qquad 3.1$$

Введем новые переменные: $\tau=\int\frac{dt'}{\xi(t')^2}, \rho=\frac{r}{\xi}.$

Тогда (3.1) приводится к виду:

$$i\hbar\left(\frac{\partial\psi_1}{\partial\tau}-\rho\frac{\dot{\xi}}{\xi}\frac{\partial\psi_1}{\partial\rho}\right)=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_1}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\psi_1}{\partial\rho}\right)+U(\rho)\psi_1(\rho,\tau)\Delta. \qquad 3.2$$

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Введем, длее, ψ_2, , положив $\psi_1 = \psi_2 \Lambda / \xi^{3/2}$. Получим:
\begin{equation}

$$i\hbar \frac{\partial \psi_2}{\partial \tau} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 \psi_2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \psi_2}{\partial \rho}\right) + U(\rho)\psi_2(\rho,\tau)\Delta - \frac{m\rho^2}{8\tau_0^2}\psi_2 + \frac{i\hbar}{2}\frac{\dot{\xi}}{\xi}\psi_2. \qquad 3.3$$

Плотность заряда имеет вид: $Q = |\Psi|^2 = \frac{|\psi_2|^2 |Y|^2}{\rho \xi^4}$, где Y удовлетворяет уравнению:
\

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial Y}{\partial\theta} - \frac{1}{4}Y + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\varphi^2} = 0. \qquad 3.4$$

Вместо переменной θ введем $\eta = \ln\left(\tan\left(\frac{\theta}{2}\right)\right)$, тогда уравнение принимает вид:\

$$\mathrm{ch}^2\eta\left(\frac{\partial^2 Y}{\partial\eta^2} + \frac{\partial^2 Y}{\partial\varphi^2}\right) = \frac{1}{4}Y. \qquad 3.5$$

Далее будем учитывать спинорный характер Ψ – функции и представлять Y в виде столбца: $Y = \sqrt{\Omega(\eta)/2}\begin{pmatrix} \exp(i\varphi/2) \\ \exp(-i\varphi/2) \end{pmatrix}$,

соответственно Y^+ это строка $Y^+ = \sqrt{\Omega(\eta)/2}\left(\exp(-i\varphi/2), \exp(i\varphi/2)\right)$. Зависимость плотности от углов определяется произведением Y^+Y и при выбранном представлении не зависит от угла φ: $Y^+Y = \Omega(\eta)$. Представим потенциал в виде произведения функции от радиуса на функцию от угловых переменных, а поскольку плотность заряда не зависит от φ, потенциал также не зависит от $\varphi: V = \Delta(\eta)U(\rho)$. Используя переменную η вместо θ получим:

$$\Delta(\eta)\frac{1}{\rho^2}\frac{d}{d\rho}\rho^2\frac{dU(\rho)}{d\rho} + \frac{U(\rho)}{\rho^2}\mathrm{ch}^2(\eta)\left(\frac{d^2\Delta}{d\eta^2}\right) = Q. \qquad 3,6$$

Здесь Q -плотность заряда. В случае взаимодействия с собственным полем $Q = -\mu|\Psi|^2, \mu$ - константа связи. Полное отделение функции радиуса от функции угла может быть достигнуто, если выполнены условия: $\mathrm{ch}^2(\eta)\frac{d^2\Delta}{d\eta^2}) = \nu\Delta$ ($\nu\$$ – константа) и $\Omega \equiv \Delta$. Из (3.5) можно получить:

$$\mathrm{ch}^2(\eta)\left(2\Omega''\Omega - \Omega'^2 - \Omega^2\right) = \Omega^2. \qquad 3.7$$

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Исключая из этих соотношений $\mathrm{ch}^2(\eta)$ получим уравнение:

$$\Omega''\Omega(2\nu-1)=\nu(\Omega'^2+\Omega^2). \quad 3.8$$

Это уравнение имеет интеграл: $C_1=\Omega^{-\frac{2\nu}{2\nu-1}}\left(\Omega'^2-\frac{\nu}{\nu-1}\Omega^2\right)$. Введем определение $S=\Omega^{\frac{\nu+1}{2\nu-1}}$.

Тогда $C_1=S'^2\left(\frac{-2\nu+1}{\nu-1}\right)-\frac{\nu}{\nu-1}S^2$. Из этого соотношения при $\nu=1/2$ следует,что

$S=const,$ а $\Omega=S^0\equiv 1,$ также отсюда следует $\Delta\equiv 1$ и что распределение плотности заряда является сферически симметричным.

Из (3.6) следует уравнение:

$$\frac{1}{\rho^2}\frac{d}{d\rho}\rho^2\frac{dU}{d\rho}+\frac{1}{2}\frac{U}{\rho^2}=-\kappa_0\frac{|\psi_2|^2}{\rho}. \quad 3.9$$

В уравнении (3.3) сделаем замену:

$$\psi_2=\quad\left(-\frac{iE\tau}{\hbar}\right)R\ \rho\qquad i\theta\ \rho \quad 3.10$$

Где E - действительная величина, $R(\rho),\theta(\rho)$ -действительные функции. Положив

$\frac{\xi}{\xi}\equiv const=\frac{1}{2\tau_0}$, получим систему:

$$ER=-\frac{\hbar^2}{2m}\left(R''-R\theta'^2+\frac{1}{\rho}R'\right)+R\left(U+\frac{\lambda m\rho^2}{2}\right) \quad 3.11$$

$$-\frac{\hbar^2}{2m}\left(\ R'\theta'+R\theta''+\frac{1}{\rho}R\theta'\right)+\frac{\hbar}{4\tau_0}R= \quad 3.12$$

Уравнеие (3.9) (с заменой $|\psi_2|^2$ на R^2 и (3.11) и (3.12) образуют замкнутую систему, описывающую нестационарную динамику. Вместо ρ ведем безразмерную переменную $s:\rho=l_0 s, l_0^2=\frac{2\hbar\tau_0}{m}$. Обозначим

$$y=l_0\theta', V=\frac{4\tau_0(U-E)}{\hbar}, \kappa_1=\kappa_0=\frac{4\tau_0}{\hbar}\left(\frac{2\hbar\tau_0}{m}\right)^2.$$

Тогда система принимает вид:

$$R''-Ry^2+\frac{R'}{s}=(V(s)-s^2)R, \quad 3.13$$

$$2R'y+Ry'+\frac{1}{s}Ry=R, \quad 3.14$$

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$$\frac{d^2V}{ds^2}+\frac{2}{s}\frac{dV}{ds}+\frac{1}{2}\frac{V(s)}{s^2}=-\kappa_1\frac{R^2}{s}. \qquad 3.15$$

При решении этой системы полагалось $V(s)=sV_1(s)$ и использованы граничные условия: $R(0)=1, R'(0)=0, y(0)=0, V_1'(0)=0, V_1(0)=-2/5.$ Для получения регулярного в нуле решения необходимо использовать равенство $5/2V_1(0)=-\kappa_1 R^2(0)$ Отметим, что на рис.4 можно видеть, что потенциал (кривая I) и полный заряд (кривая II) медленно изменяются, тогда как y(s) (кривая III) колеблется относительно линейно растущей функции.

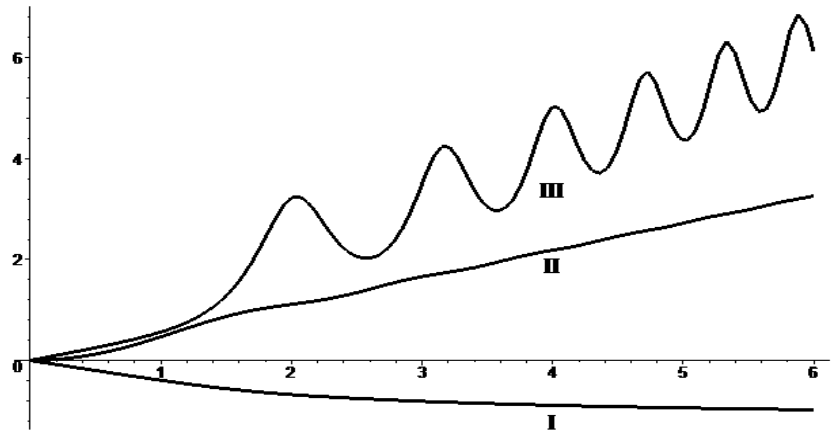

Рис.4: Зависимости потенциала (I), полного заряда (II) и функции y от s

Кинетическая модель сферически симметричной системы зарядов

Нестационарный гамильтониан сферически симметричной системы точечных зарядов имеет вид:

$$H=\frac{p_r^2}{2m}+\frac{L}{2mr^2}+\frac{1}{\xi^2(t)}U\left(\frac{r}{\xi(t)}\right) \qquad 4.1$$

Здесь $p_r=\dot{m}r, L-$ квадрат полного момента количества движения, в классической задаче $L>0,\ \xi(t)$ - вспомогательная функция, удовлетворяющая уравнению $\ddot{\xi}=\frac{\lambda}{\xi^3}, \lambda-$ -константа. Используя выражение для гамильтониана, можно получить выражение для инварианта:

$$I=\frac{m}{2}(r\xi-\dot{r}\xi)^2+U\left(\frac{r}{\xi(t)}\right)+\frac{\lambda m}{2}\frac{r^2}{\xi^2}+\frac{L}{2m}\frac{\xi^2}{r^2}. \qquad 4.2$$

Рассмотрим уравнение для $\xi(t)$ подробнее. Из него следует:

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$\dot{\xi}^2 - \dot{\xi}_0^2 = -\frac{\lambda}{\xi^2} + \frac{\lambda}{\xi_0^2}.$ Далее будем полагать $\lambda = -\frac{1}{4\tau_0^2}$, а вместо t введем переменную

$$\tau = \int_0^t \frac{dt;}{\xi(t')^2}. \quad \text{Получим} \quad \frac{d\xi}{d\tau} = \pm\xi\sqrt{(\dot{\xi}_0^2 - \frac{1}{4\tau_0^2\xi_0^2})\xi^2 + \frac{1}{4\tau_0^2}}$$

В дальнейшем рассматривается случай, когда $\dot{\xi}_0^2 - \frac{1}{4\tau_0^2\xi_0^2} = 0.$ При этом

$\xi(t) = \sqrt{\xi_0^2 \pm \frac{t}{\tau_0}}, \xi(\tau) = \xi_0 e^{\pm\frac{\tau}{2\tau_0}}$. Подставим в (4.2) переменную τ вместо t и введем переменную $\rho = \frac{r}{\xi}$. Тогда инвариант I приводится к виду, аналогичному гамильтониану:

$$I = \frac{m}{2}\dot{\rho}^2 + U(\rho) - \frac{m}{8\tau_0^2}\rho^2 + \frac{L}{2m\rho^2}. \qquad 4.3$$

В (4.3) точка означает производную по τ .

Можно, далее, построить интеграл $J_I^{\pm}$, сопряженный с I Рассмотрим выражение:

$$J_I = -\tau + \int_{\rho_0}^{\rho} \frac{d\rho'\sigma(\frac{2}{m}(I - U(\rho') + \frac{m\rho'^2}{8\tau_0^2} - \frac{L}{m^2\rho'^2})}{\sqrt{\frac{2}{m}(I - U(\rho') + \frac{m\rho'^2}{8\tau_0^2} - \frac{L}{m^2\rho'^2})}} \qquad 4.4$$

здесь $\sigma(x)$ - функция Хевисайда, $\rho > \rho_0, \rho_0$ -стартовая точка частицы. Выполнение равенства $\frac{dJ_I}{d\tau} \equiv 0$ является очевидным, если частицы движутся от центра. При движении к центру знак перед интегралом должен быть изменен.

Плотность частиц выражается интегралом в фазовом пространстве:

$$n = \int d\vec{q} f(I, J_I, L). \qquad 4.5$$

Элемент фазового пространства представим в виде:

$$d\vec{q} = dq_r dq_\theta dq_\phi, \quad dq_\phi = \frac{dM_\phi}{r\sin\theta}, dq_r = m d\dot{r} = \frac{dI}{\xi\sqrt{\frac{2}{m}(I - U) - \lambda\frac{r^2}{\xi^2} - \frac{L\xi^2}{m^2r^2}}}, dq_\theta = \frac{dL}{2r\sqrt{L - \frac{M_\phi^2}{\sin^2\theta}}}.$$

Усреднение по M_ϕ приводит к выражению:

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$$n=\frac{\pi}{2r^2}\int\frac{dIdLf(I,L,J_1)\sigma(\frac{2}{m}(I-U)-\lambda\frac{r^2}{\xi^2})}{\xi\sqrt{\frac{2}{m}(I-U)-\lambda\frac{r^2}{\xi^2}-\frac{L\xi^2}{m^2r^2}}}.$$

В переменных: ρ,τ

уравнение Пуассона принимает вид:

$$\frac{1}{\xi^4(\tau)}\frac{1}{\rho^2}\frac{d}{d\rho}\rho^2\frac{dU}{d\rho}=-\frac{4\pi e^2}{\xi^3(\tau)\rho^2}\int\frac{dIdLf(I,L,J_1)\sigma(\frac{2}{m}(I-U)-\lambda\frac{r^2}{\xi^2})}{\sqrt{\frac{2}{m}(I-U(\rho))-\lambda\rho^2-\frac{L}{m^2\rho^3}}}. \quad 4.6$$

Функция распределения должна содержать множитель, экспоненциально зависящий от J_1 . Положим:

$$f=\kappa_*\delta(I-I_0)\delta(L-L_0)\exp\{-\frac{1}{2\tau_0}J_i\}.$$

Если выполнено условие $\xi\exp\left(\frac{\tau}{2\tau_0}\right)\equiv\xi_0$, то в уравнение Пуассона в качестве независимой

переменной входит только ρ. Таким образом, $\xi(t)=\sqrt{\xi_0^2-\frac{t}{\tau}}$. .Обозначим, далее,

$$\upsilon_0^2=\frac{2I_0}{m},s=\frac{\rho}{2\tau_0\upsilon_0},y=\frac{2U}{m\upsilon_0^2},l=\frac{L}{4m^2\tau_0^2\upsilon_0^4},u(s)=\int_0^s\frac{ds'\sigma(1-y(s')+s'^2-l/s'^2)}{\sqrt{1-y(s')-l/s'^2}}.$$

Тогда из уравнения Пуассона следует:

$$\frac{d}{ds}s^2\frac{d}{ds}y(s)=-\vartheta u'e^{-u(s)},u'(s)=\frac{\sigma(1-y(s)+s^2-l/s^2)}{\sqrt{1-y(s)+s^2-l/s^2}}. \quad 4.7$$

Константа ϑ определяется параметрами задачи -- $\kappa_*,m,\upsilon_0,\tau_0$ и зарядом e :

$\vartheta=\frac{8\pi e^2\kappa_*}{m\upsilon_0^3}\xi_0$. В уравнении (4.7) плотность заряда определяется только уходящими частицами. Система интегрируется:

$$s^2\frac{d}{ds}y(s)=\vartheta e^{-u(s)}+C_1,u'(s)=\frac{\sigma(1-y(s)+s^2-l/s^2)}{\sqrt{1-y(s)+s^2-l/s^2}}. \quad 4.8$$

Эта система решалась при следующих условиях: $C_1=-1,y(0)=0,u(0)=0,l=1,\vartheta=1.$

Следует отметить, что полный заряд сгустка при $s\to\infty$ остается конечным $(e^{-u}-1\to-1).$

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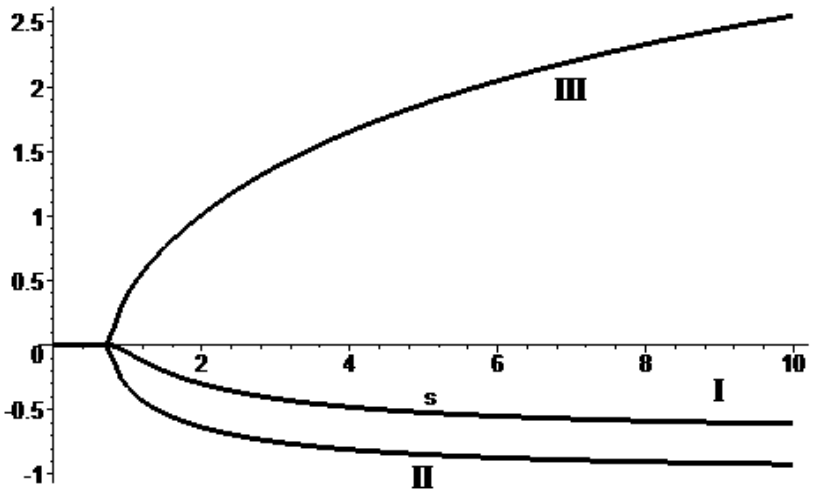

Рис. 5: З ависимость потенциала y(s) (I), полного заряда внутри сферы радиуса s (II) и функции u(s) (III)

II. ЗАКЛЮЧЕНИЕ

Таким образом, в работе найдены частные решения модельных нестационарных квантовомеханических задач, характеризуемых ненулевым моментом. Показана возможность построения пси-функции, приводящей к сферически симметричной плотности заряда при l=1. При $l = \pm\frac{1}{2}$ получены частные решения, при которых также плотность заряда не зависит от углов. Решение кинетической задачи показывает возможность существования сгустка с конечным значением заряда при больших значениях автомодельной переменной.

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Seth Amoako Atta, Arthur Yarhands Dissou, Simon Kojo Appiah & Vivian Maanu

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ABSTRACT

The motive of the study was to find out if students' mathematics interest mediates other variables such as Instructor Quality, Teacher Self-Efficacy and Student Teacher Relationship Towards Student Mathematics Performance. The study was a quantitative research design that uses a structural equation model. A simple random sampling procedure was used to select 497 Senior High School students from five SHS across the Ashanti region of Ghana. The data collection was mainly based on administered questionnaire to the participants which was analyzed using the Structural Equation Model (SEM) with the help of SPSS and AMOS. It was found that teacher student relationship had no significant effect on students' mathematics performance. Instructional quality and teacher self-efficacy on the other hand had significant positive effects on students' mathematics performance. Also, it was found that student mathematics interest did not mediate the relationship between teacher student relationship and mathematics performance. Similarly, mathematics interest played no mediating role in the relationship between instructional quality and mathematics performance. Finally, it was concluded that student mathematics interest partially mediated the relationship between teacher self-efficacy and students' mathematics interest.

Keywords: instructor quality, teacher self-efficacy, teacher student relationship.

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ABSTRACT

The motive of the study was to find out if students' mathematics interest mediates other variables such as Instructor Quality, Teacher Self-Efficacy and Student Teacher Relationship Towards Student Mathematics Performance. The study was a quantitative research design that uses a structural equation model. A simple random sampling procedure was used to select 497 Senior High School students from five SHS across the Ashanti region of Ghana. The data collection was mainly based on administered questionnaire to the participants which was analyzed using the Structural Equation Model (SEM) with the help of SPSS and AMOS. It was found that teacher student relationship had no significant effect on students' mathematics performance. Instructional quality and teacher self-efficacy on the other hand had significant positive effects on students' mathematics performance. Also, it was found that student mathematics interest did not mediate the relationship between teacher student relationship and mathematics performance. Similarly, mathematics interest played no mediating role in the relationship between instructional quality and mathematics performance. Finally, it was concluded that student mathematics interest partially mediated the relationship between teacher self-efficacy and students' mathematics interest.

Keywords: instructor quality, teacher self-efficacy, teacher student relationship.

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I. INTRODUCTION

In the last decade or two, research in the area of student mathematics performance has intensified across the globe. Prominent among them are the Trends in Science and Mathematics Studies (TIMSS) and Program for International Student Assessment (PISA). Studies have revealed that mathematics application is in all facets of human endeavors and the knowledge of mathematics has the impetus to make an individual critical in thinking, innovative and problem solving oriented. Modern studies have revealed that school mathematics can improve one's communication skills, liberate learners from oppression towards social justice and promote democratic values like leadership qualities, loyalty, and tolerance aimed at promoting peace and tranquility for national development. However, achieving this feat depends on many other factors like, Instructor quality, teacher-student relationship, teacher self-efficacy and student interest.

II. INSTRUCTIONAL QUALITY AND STUDENTS' PERFORMANCE

Instructor quality refers to tutoring strategies that meet students' demands for autonomy (a sense of self-determination and freedom from control), competence, and relatedness - a sense of belongingness (Williams & Sembiante, 2022). According to Dupuis et al., (2020) highly effective instruction enable both long-term and short-term intellectual learning changes in learners. Consequently, in every educational context, teacher quality is the most contentious aspect of advancing student achievement and eliminating achievement gaps (Hachfeld & Lazarides, 2021). Quality instruction motivate students to learn and students who are motivated to learn mathematics attend all mathematics lessons, follow lessons attentively, participate in group discussions, learn extremely to achieve their goals, receive higher grades, are eager to learn at school, recognize the importance of mathematics in other subjects, and are eager to learn (Affuso et al., 2022).

Multiple studies have indicated that characteristics such as instructors' cognitive ability, subject matter expertise, teaching and learning knowledge, and classroom teaching behaviours are associated with improved student accomplishment (Khodarahmi et al., 2022). In fact, some data suggests that teacher instructional approaches influence student mathematical achievement more than any other variable (McKinney & Frazier, 2008). Unlike some other variables, such as gender, race, and socioeconomic status (SES), teacher instructional practices are controlled at the local school level and can thus be modified.

III. TEACHER STUDENT RELATIONSHIP AND STUDENTS' PERFORMANCE

Personal characteristics related to responsiveness to student requests and the teaching environment, resilience at school, self-reflection, resourceful ness, and the development of positive relationships are regarded as essential components of the emotional work required of teachers in their classroom practice (Amorim Neto et al., 2022). Since, in a conducive classroom where students are encouraged to ask questions and take risks, they are more inclined to seek assistance when they experience challenges (Awoniyi & Butakor, 2021). This social contact, which is an essential component of student- centered teaching techniques, plays a crucial role in problem solving. Through the implementation of student-centered teaching approaches that foster inquiry-based learning, students are afforded opportunity to share their own opinions and strategies with their peers when confronted with difficulties located in real-world contexts (Amoako Atta & Asiedu-Addo, 2021). Since, student-centred instructional practises do not pit students against one another in a competitive environment, student-to-student and student- initiated student-to-teacher interactions become the norm in the classroom (Lerkkanen et al., 2016; Muhonen et al., 2016). In student- centered classrooms, students are more likely to share their ideas and learning practises to both their classmates and teachers since they do not fear being embarrassed or criticized for making a mistake (Muhonen et al., 2016).

According to Vygotsky (1978) cited in (Lasmawan & Budiarta, 2020), there are learners who can perform thinking and problem-solving abilities independently, learners who can perform thinking and problem-solving abilities with assistance, and learners who cannot perform thinking and problem-solving abilities with assistance. A classroom environment that encourages students to collaborate with their peers and exposes them to multiple approaches during problem-solving situations is associated with higher mathematics achievement (Atta & Brantuo, 2021). The interaction process in class cannot be separated from the learning process. Interaction is part of a person's endeavour to comprehend something. In light of the significance of teacher-student interactions in mathematics learning, it is vital to identify the relationships that must be constructed between teachers and students. The kinds of healthy teacher-student contact must be understood and fostered by the instructor in order to improve student learning.

Huinker, (2018) observed that the NCTM in 2014 suggested in the United States that teachers should assist students in mastering mathematics through dialogue, creative tools, and past knowledge and experience. The national curriculum of Ghana is centered on fostering an atmosphere conducive to problem-solving, communication, and mathematical thinking among students (NaCCA, MOE, 2019). The connection between teachers and students is the focal point of classroom instruction and a crucial component in enhancing teaching quality and student progress. Therefore, a good teacher- student relationship provides an environment conducive to learning.

IV. TEACHER SELF-EFFICACY AND STUDENTS' PERFORMANCE

Self-efficacy is the confidence in one's ability to attain the necessary levels of learning and behaviour (Bandura, 1977) cited in (Abdullah & Ahmed, 2022). Self-efficacy is the belief that one is capable of doing a task; it is considerably distinct from knowing what to do. It is the evaluation of an individual's self-efficacy, skills, and capacities, as well as their transformation into behaviour. Self-efficacy therefore is an individual's belief in their ability to implement a set of actions and accomplish a set of objectives. The actions include managing physical behaviour and cognitive and emotional states (Bandura, 1997). In student-centered education, teachers share authority with students, and students are expected to participate as co-investigators in mathematics learning communities. Innovative classrooms can be intimidating for teachers accustomed to teacher-centered practices (Khodarahmi et al., 2022), however, teachers with a strong sense of self-efficacy are more likely to use innovative approaches.

Numerous studies have found that teachers with high self-efficacy beliefs exert more effort in the classroom, are more willing to engage in the learning-teaching process, are more successful in the selection of methods and techniques, and, in general, are more effective in implementing the curriculum (Affuso et al., 2022; Ahmed et al., 2022; Akman, 2021; Woodcock & Faith, 2021). Therefore, a teacher with a high self-efficacy belief is more enthusiastic in the classroom, plans more efficiently.

V. MEDIATING ROLE OF STUDENT MATHEMATICAL INTEREST

According to Ainley (2006), as cited in (Wong & Wong, 2019a), interest is an emotional state that describes how a person feels when they are learning. In this study, interest is operationally defined as the way students feel when they are engaged in learning math and enjoy the process of learning. It's important to look at the student's interest, attitude, and motivation to learn mathematics because these are the things that might make students want to do better in mathematics classes. Hashim et al., (2021) found that a student's interest in mathematics affected how they felt about learning. Asante (2012) for instance says that male and female students are interested in mathematics in different ways, and the school, the environment, the beliefs and attitudes of the teachers, the way they teach, and the opinions of the parents were all looked as factors that affect how students feel about mathematics. It is collaborative learning, learning through games, and other strategic methods that can be used to get learners interested in and excited about learning mathematics.

(Langat, 2015) for instance, conducted a survey in Kenya and discovered that the majority of students who perform well had positive attitudes toward mathematics. Similarly, (de Vera et al., 2022; Peteros, 2022) concluded that students' views toward mathematics have a greater influence on their academic performance. Attitude toward mathematics has been broken down into multiple components, according to the research. For instance, Davadas and Lay (2017) posited motivation, enjoyment, and self-confidence. Researchers in the Trends in International Mathematics and Science Study (TIMSS) did a similar analysis, focusing on three aspects of students' mindsets toward mathematics (Martin et al., 2020) students' love of mathematics, the importance of mathematics, and their confidence in mathematics. In conclusion, while many researchers may have used various names, three components

have consistently been employed to measure mathematics-related attitudes: like mathematics (LM), value mathematics (VM), and trust in mathematics (CM).

VI. THE OBJECTIVE OF THE STUDY

Many studies on students' mathematics performance have tried to look at how individual constructs like teacher self-efficacy, and student mathematics interests among others enhance students' mathematics performance. Especially in Ghana much is not known about the mediating role of students' interest in promoting student mathematics performance so far as these constructs are concerned. The motive of this student was to assess the mediating role of student mathematical interest in instructor quality, teacher self-efficacy and student-teacher relationship towards student mathematics performance. To achieve this objective, the following hypotheses were formulated.

H1: Instructional Quality has a direct positive effect on Student Performance.

H2: Teacher Student Relationship has a direct positive effect on Student Performance.

H3: "Teacher Self-Efficacy has a direct positive effect on Student Performance.

H4: "Student Mathematics Interest mediates the relationship between teacher student relationship and student performance

H5: "Student Mathematics Interest mediates the relationship between instructional quality and student performance.

H6: "Student Mathematics Interest mediates the relationship between teacher self-efficacy and student performance.

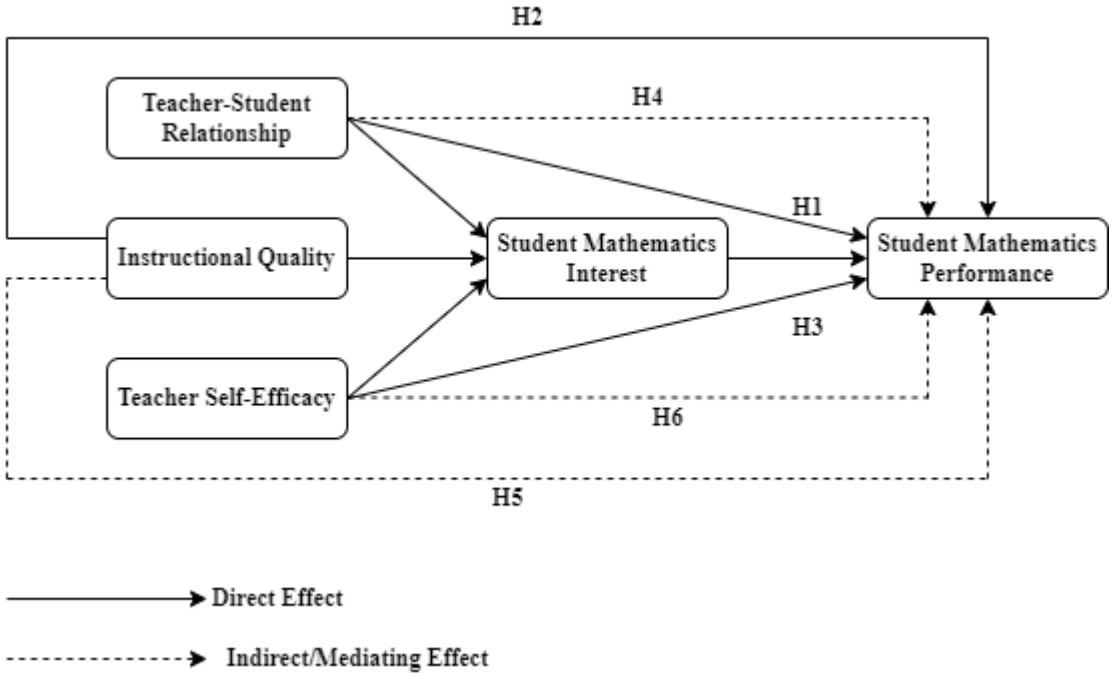

Figure 1: Conceptual framework

VII. METHODOLOGY

7.1 Sample and Data Collection

The study was a quantitative research design that uses a structural equation model. The study targeted all the Senior High Schools in the Ashanti Region. However, five senior high schools were purposively selected. Out of this a total of 497 students using a simple random sampling technique were engaged.

They comprised form one, form two and form three students from the selected schools. The questionnaires were administered while the participants were in school. An official approval was sought from the school authorities before administering the questionnaires. Table 1 presents the demographic distributions of the study.

Table 1: Demographics Information

Demographics (response/option)	Frequencies (N)	Percentages (%)
Gender		
Female	244	49.1
Male	253	50.9
Total	497	100.0
Age		
Below 18 years	349	70.2
18 years and above	148	29.8
Total	497	100.0
Form		
Form One	134	27.0
Form Two	197	39.6
Form Three	166	33.4
Total	497	100.0

Out of the total of 497 students, 253 representing 50.9% were males and 244 constituting 49.1% were females. Also 349 students representing 70.2% were below 18 years and 148 students representing 29.8% were either 18 years or above. Out of the total participants 134 were in form one, 197 were in form two while 166 were in form three.

VIII. QUESTIONNAIRE AND MEASURES

There were five main variables in the study namely Teacher-Self Efficacy, Instructional Quality, Teacher-Student Relationship, Student Mathematics Interest and Student Performance. These variables were measured on a 5-point Likert scale of 1= Strongly disagree to 5= Strongly agree. The measurement items for teacher self-efficacy were adapted from (Adarkwah et al., 2022; Akman, 2021) those for instructional quality were adapted from (Arthur et al., 2022), those for teacher-student relationship were adapted from (Xu & Qi, 2019), those for mathematics interest were adapted from (Wong & Wong, 2019), while those for student performance were adapted from (Arthur et al., 2021, 2022).

IX. VALIDITY AND RELIABILITY ANALYSIS

Cronbach's alpha analysis was run in SPSS (v.26) to assessed the internal consistency of the measurement items. The expected value of alpha in order to achieve internal consistency is at least 0.7 (Pomegbe et al., 2020). As demonstrated in Table 2, Teacher Student Relationship had a CA of 0.930, Instructional Quality had a CA of 0.765, Teacher Self-Efficacy had a CA of 0.918, Mathematics Interest had a CA of 0.901 while Student Performance had a CA of 0.904.

For reliability and validity analysis of the model a confirmatory factor analysis (CFA) was run using Amos (v.23) software. Measurement items with poor factor loadings (score less than 0.5) were deleted from further analysis as suggested by Amoako et al. (2020). Two items were deleted from instructional quality whiles an item each was deleted from teacher self-efficacy, mathematics interest and student performance. The results are presented in Table 2 and Figure 2. Convergent validity was assessed using average variance extracted (AVE) and composite reliability (CR). Fornell & Larcker (1981) recommended an AVE score of at least 0.5 and CR score of at least 0.7 in order to achieve convergence. The least AVE as reported in Table 2 is 0.523 (Instructional Quality) and the least CR score was 0.723 (Instructional Quality).

As recommended by Hair et al. (2010) the model fit indices should be: CMIN/DF should be less than three, TLI and CFI should be greater than 0.9, GFI greater than 0.8 while RMR and RMSEA should be less than or equal to 0.08. From Table 2 it can be seen that all the model fit indices were achieved which suggest that the dataset fits the model appropriately.

Table 2: Confirmatory Factor Analysis

MFI: CMIN=269.009; df=176; CMIN/df=1.528; GFI=0.950; CFI=0.988; TLI=0.985; RMR=0.080; RMSEA=0.033	Std. Factor Loading
Teacher Student Relationship (TSR): CA=0.930; CR=0.933; AVE=0.702	
I get along well with my Mathematics teacher (TSR1)	0.843
My Mathematics teacher is very concerned about my physical and mental health (TSR2)	0.921
My Mathematics teacher is willing to provide extra help when I need it (TSR3)	0.877
My Mathematics teacher is very happy to listen to me (TSR4)	0.832
My Mathematics teacher is fair to me (TSR5)	0.859
My Mathematics teacher is willing to better explain concepts I don't understand to me outside classroom hours (TSR6)	0.672
Instructional Quality (IQ): CA=0.765; CR=0.767; AVE=0.523	
My Mathematics teacher provides good feedback for better understanding (IQ1)	0.684
My Mathematics teacher developing mathematical concepts systematically is a quality of a good teacher (IQ2)	0.730
My Mathematics teacher give enough classroom exercises to practice to test my understanding of the concept being taught (IQ3)	0.754
Teacher-Self Efficacy (TSE): CA=0.918; CR=0.910; AVE=0.717	
My Mathematics teacher encourages competition among students to motivate them to learn more (TSE1)	0.851
My Mathematics teacher is effective in monitoring mathematics experiments (TSE2)	0.923
My Mathematics teacher knows the steps necessary to teach mathematics concepts effectively (TSE3)	0.851
My Mathematics teacher encourages students to solve mathematics problems independently (TSE4)	0.755
Student Mathematics Interest (SMI): CA=0.901; CR=0.898; AVE=0.694	
I want to know all about how to do Mathematics problems (SMI1)	0.940
I am excited when a new Mathematics topic is announced in class (SMI2)	0.937
I want to know all about Mathematics (SMI3)	0.827
I want to talk about Mathematics with my friends (SMI4)	0.574
Student Mathematics Performance (SP): CA=0.904; CR=0.907; AVE=0.711	

I am confident in my understanding of Mathematics as a subject (SP1)	0.900
I score high marks in Mathematics Examination (SP2)	0.895
I teach my colleagues who need help in Mathematics (SP3)	0.881
Learning Mathematics enhances my thinking ability (SP4)	0.676

NOTE: CFI: Comparative Fit Index; CMIN/df: Chi-square/degree of freedom; TLI: Turkey-Lewis Index; RMR: Root Mean Square Residual; RMSEA: Root Mean Square Error of Approximation

The descriptive statistics and discriminant validity are shown in Table 3. From the analysis presented, instructional quality had the highest mean score of 3.614, whiles teacher self-efficacy had the lowest mean score of 3.026. The measurement items of the constructs were assessed on a 5-point Likert scale of 1=Strongly disagree to 5=Strongly agree, which has a highest possible mean of 5. A mean score of 3 and above indicates respondents agreed to the measurement items and this was the case for all the five constructs as presented in Table 3. According to Sarsah et al. (2020) discriminant validity is achieved when the least value of the square root of AVE is larger than the highest correlation coefficient. As demonstrated in Table 3 the least square root of AVE is 0.723 which is greater than 0.571 which is the highest correlation coefficient. This demonstrates that discriminant validity was achieved.

Table 3: Descriptive and Discriminant Validity

Variables	Mean	Std Dev.	Teac-Std Rel.	Instruc. Qual	Efficacy	Interest	Performance
Teac-Stud Rel.	3.134	1.107	0.838				
Instruc. Qual	3.614	0.932	0.245***	0.723			
Efficacy	3.026	1.281	0.280***	0.121*	0.847		
Interest	3.084	1.214	0.096*	0.084	0.571***	0.833	
Performance	3.112	1.249	0.111*	0.227***	0.469***	0.392***	0.843

*√AVE are bold and underlined; ***~p-value significant at 1%; *~p-value significant at 5%*

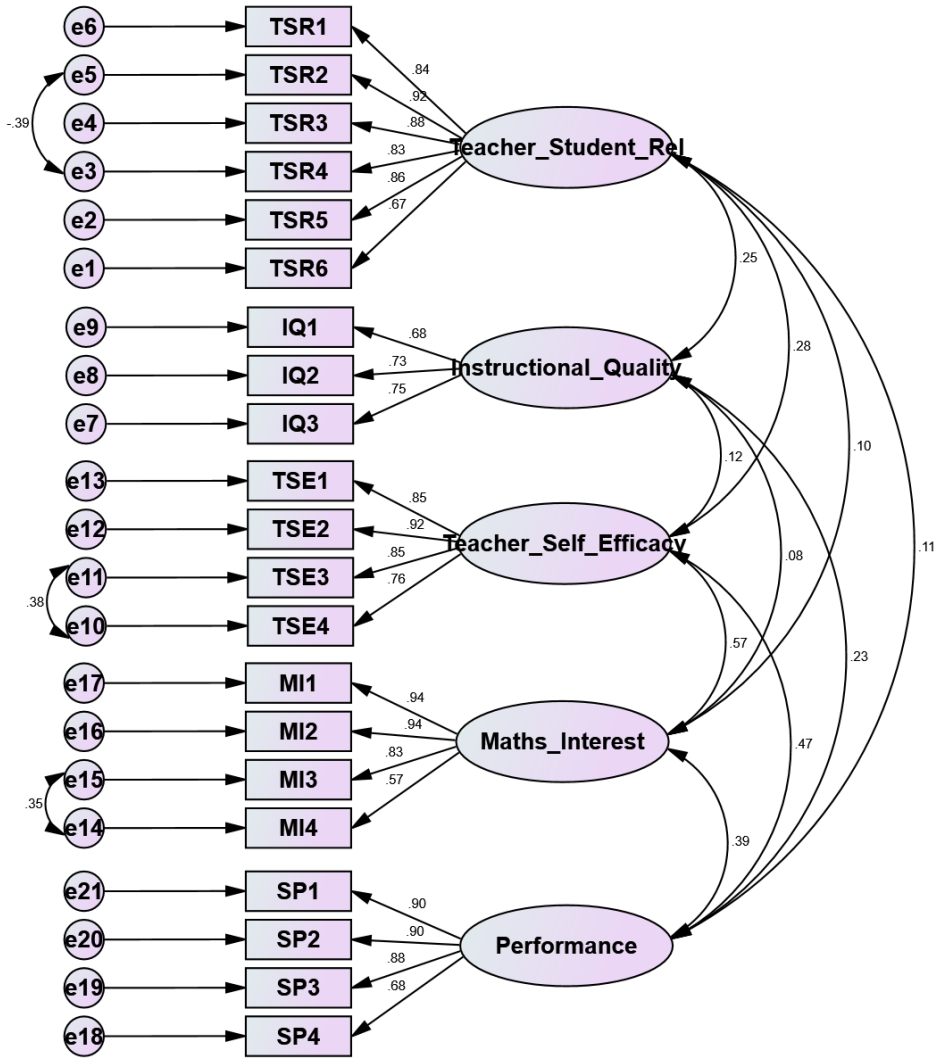

Figure 2: Confirmatory Factor Analysis

X. RESULTS

The path analysis was estimated using the Structural Equation Modelling (SEM) run in Amos (v.23). the result is presented in Table 4 and Figure 3. The estimation was based on 5000 Bootstrap samples, with Bias-Corrected Confidence Interval of 95%.

For the hypothesized paths, it was identified that Teacher Student relationship had a negative but statistically insignificant effect on Student Performance ($\beta = -0.031; p > 0.05$).
Hypothesis H1 *"Teacher Student Relationship has a direct positive effect on Student Performance"*, was therefore rejected.

Instructional Quality on the other hand had a significant positive effect on Student Performance ($\beta = 0.194; p < 0.01$). This suggests that as the quality of instruction in the classroom becomes better, students' performance is likely to be enhanced by about 19.4%, and vice versa.

Hypothesis H2 *"Instructional Quality has a direct positive effect on Student Performance"* was therefore accepted. Again, from the results, Teacher Self-Efficacy had a significant positive effect on Student Performance ($\beta = 0.307; p < 0.01$). This shows that a good teacher's self-efficacy enhanced students' performance by about 30.7% and vice versa.

Hypothesis H3 *"Teacher Self-Efficacy has a direct positive effect on Student Performance"* was therefore accepted.

To assess the mediating effect of student mathematics interest, in the relationship between teacher student relationship and student performance, the effect of teacher student relationship on mathematics interest was first estimated. The result indicated that teacher student relationship had a negative but statistically insignificant effect on mathematics interest ($\beta = -0.048; p > 0.05$). Teacher student relationship also had an insignificant effect on student performance. The coefficient of the indirect effect was -0.010 which was statistically insignificant (since zero (0) can be found in-between the lower and upper bounds).

Hypothesis H4 *"Student Mathematics Interest mediates the relationship between teacher student relationship and student performance"* was therefore rejected.

The mediating effect of student mathematics interest, in the relationship between instructional quality and student performance was ascertained. The effect of instructional quality on mathematics interest was first tested. The results indicated that instructional quality had a positive but statistically insignificant effect on student mathematics interest ($\beta = 0.030; p > 0.05$). The coefficient of the indirect path was 0.006 which was statistically insignificant (since zero (0) can be found in-between the lower and upper bounds).

Hypothesis H5 *"Student Mathematics Interest mediates the relationship between instructional quality and student performance"* was therefore rejected.

Finally, in determining the mediating effect of mathematics interest in the relationship between teacher self-efficacy and student performance, the effect of teacher self-efficacy on mathematics interest was first tested. The results showed a significant positive effect of teacher self-efficacy on mathematics interest ($\beta = 0.421; p < 0.01$). This suggests that a good teacher self-efficacy could enhance students' interest in mathematics by about 42.1% and vice versa. Student mathematics interest also had a positive significant effect on student performance ($\beta = 0.210; p < 0.01$). An increase in student mathematics interest could enhance student performance by about 21.0% and vice versa. The coefficient of the indirect effect was 0.003 which was statistically significant (because both lower bound and upper bound were positive and zero (0) cannot be found in-between them). This represents a partial mediating effect since teacher self-efficacy had a direct positive effect on student performance.

Hypothesis H6 *"Student Mathematics Interest mediates the relationship between teacher self-efficacy and student performance"* was therefore accepted.

Table 4: Direct and Indirect Paths

Direct paths	Un-standard estimate	Standard error	Composite reliability
Teach_Stud_Rel → Maths_Interest	-0.048	0.033	-1.469
Instruct_Quality → Maths Interest	0.030	0.040	0.736
Teach_Self_Efficacy → Maths Interest	0.421	0.043	9.684**
Teach_Stud_Rel → Performance	-0.031	0.040	-0.766

Instruct_Quality → Performance	0.194	0.052	3.727**
Teach_Self_Efficacy → Performance	0.307	0.050	6.108**
Maths Interest → Performance	0.210	0.065	3.214**
Indirect paths	Un-standard estimate	Lower BC	Upper BC
Teach_Stud_Rel → Interest → Performance	-0.010	-0.030	0.003
Instruct_Quality → Interest → Performance	0.006	-0.011	0.029
Self_Efficacy → Interest → Performance	0.089	0.035	0.148

*Bias-Corrected (BC) Percentile Method; 5000 Bootstrap sample; 95% Confidence level ** ~ P-value significant at 1% (0.01)*

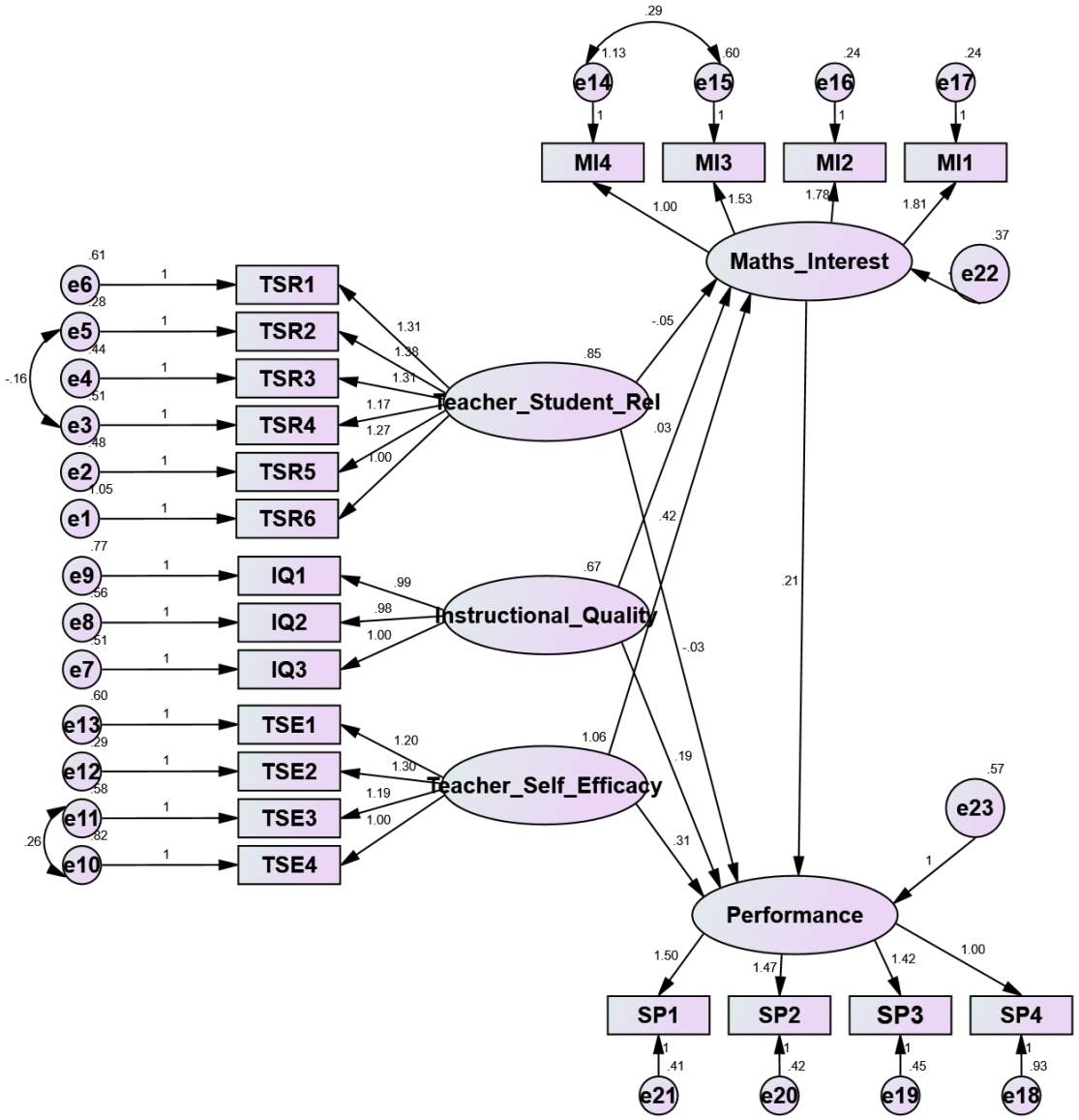

Figure 3: Structural Paths

XI. DISCUSSION

After the Structural equation modelling (SEM) was used to estimate the model, it was found that teacher student relationship had no significant effect on students' mathematics performance. Instructional quality and teacher self-efficacy on the other hand had significant positive effects on students' mathematics performance. This finding is in line with studies like (Dupuis et al., 2020; Hachfeld & Lazarides, 2021; Affuso et al., 2022) which have confirmed that quality instruction can improve students' performance.

Also, it was found that student mathematics interest did not mediate the relationship between teacher student relationship and mathematics performance. Similarly, mathematics interest played no mediating role in the relationship between instructional quality and mathematics performance. Finally, it was concluded that student mathematics interest partially mediated the relationship between teacher self-efficacy and students' mathematics interest. This also in tandem with the study of (Arthur et al., 2022; Khodarahmi et al., 2022; Wong & Wong, 2019b).

XII. CONCLUSION

The findings of the study have revealed that instructor quality is very key determinant of student mathematics interest which intend affects student performance.

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ABSTRACT

The main purpose of this study was to examine the role of counselling on the mental health of survivors of extrajudicial executions in the Mathare informal settlement of Nairobi. For the realization of this research aim, the study was guided by four specific objectives. These objectives comprised to examine the prevalence of mental illnesses and disturbances among the survivors of extrajudicial executions in Mathare Informal Settlement; to identify the key problems leading to mental health complications in Mathare Informal Settlement; to determine the contribution of counselling on the mental health of survivors of extrajudicial execution in Mathare Informal Settlement; to identify possible obstacles that prevent survivors of extrajudicial execution in Mathare Informal Settlement from accessing and utilizing counselling services. The research design and methodology used were ex post facto and mixed methodology, respectively. Data collection was done using questionnaires, whereas for descriptive analysis, SPSS (statistical package for social sciences) was used. The study also utilized measures of central tendency such as mean, mode, tables, and frequencies. After the data analysis process, the study found that counselling positively impacted the extrajudicial execution survivors' mental and general wellbeing, thus concluding that a significant relationship existed between counselling services and the extrajudicial execution survivors' mental health.

Keywords: counselling, mental health, survivors, extrajudicial executions

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I. INTRODUCTION

One of the globally acclaimed barometers for assessing how civilized and politically developed a society is, includes its capacity to protect and safeguard people's inalienable and fundamental rights. However, in several countries worldwide, extrajudicial executions (EJE) have been at the forefront of adversely affecting civilization by impacting human rights, especially violating the right to life (Aceves, 2018). Extrajudicial executions are defined as killings, which can rationally be presumed to be generated by a policy at any government level for the eradication of specific individuals as a substitution for arresting them and bringing them to justice (Amnesty International, 2020).

In Kenya, the evidence of extrajudicial executions is somewhat overwhelming. It has been documented by numerous entities, including the media, the Kenya National Commission on Human Rights (KNCHR), the Human Rights Watch (HRW), Community Justice Centres under the umbrella of the Police Reform Working Group (PRWG) and Missing Voices Kenya. However, despite the wave of extrajudicial killings that has been witnessed throughout the country, Mathare remains one of the most impacted by the killings. In Mathare, young males are constantly executed in the name of ethnic politics and violence. Besides, in these killings, Jones et al. (2016) stress that 'bonokos,' a common term for weapons placed next to the victim's body after the shooting by the police, are often used as justifications for the executions. The reality in Mathare Informal Settlement, therefore, entails the

incessant violence enacted upon the young male population, but the government and the society neglect the existence of such an issue.

According to a study carried out between 2013 and 2015, Mathare had over 800 extrajudicial cases within this period (MSJC, 2015). An evaluation of these cases shows that the Mathare extrajudicial executions victims' average age is 20 years, with most victims being male. The extrajudicial executions adversely impact the survivors, family members, witnesses, and the community. As a result, the number of extrajudicial executions in Mathare continues to escalate. Any survivors of these inhumane acts are constantly subjected to a myriad of issues, including feeling stigmatized, living in fear, depressed, traumatic scars, overwhelmed with psychological disorders and other mental issues. The proliferation brings about a need for a study focusing on solutions, particularly counselling, to the mental health complications that extrajudicial execution survivors experience.

1.1 Statement of the Problem

In the contemporary world, extrajudicial executions have become more rampant, and the issue has taken up an astronomic increase in numerous nations across the globe. In most nations, particularly Indonesia, the Philippines, and Nigeria, the issue of extrajudicial killings remains unsolved, for not only does it involve government apparatus but also the ordinary citizens' support of these killings (Jones et al., 2016). In Kenya, similar to other countries with cases of extrajudicial executions, despite the existence of an avalanche of extrajudicial killings by the police, the Kenyan authorities hardly investigate the security agencies for their conduct but instead regularly defend the security agencies and deny that this issue exists in the country (Human Rights Watch, 2016).

A glance through existing studies on the subject matter also indicated that despite the issue of extrajudicial execution being deeply embedded in the contemporary culture, especially in informal settlement areas, the topic remains understudied. In the Mathare context, existing studies on extrajudicial killings within the region primarily focus on police killings (Stapele, 2019) and the correlation between extrajudicial executions of young ghetto men and conceptions of citizenship (Stapele, 2016). As such, a research gap exists in that no studies are focusing on the role of counselling on the mental health of survivors of extrajudicial executions within the Mathare Informal Settlement region.

1.2 Objectives of the Study

The following objectives guided the study:

i. To examine the prevalence of mental illnesses and disturbances among the survivors of extrajudicial executions in Mathare Informal Settlement.
ii. To identify the key problems leading to mental health complications in Mathare Informal Settlement.
iii. To determine the contribution of counselling on the mental health of survivors of extrajudicial execution in Mathare Informal Settlement.
iv. To identify possible obstacles that prevent survivors of extrajudicial execution in Mathare Informal Settlement from accessing and utilizing counselling services.

II. LITERATURE REVIEW

This section is divided into two segments, the empirical review and the theoretical review. The empirical review segment provides a detailed evaluation of existing studies on the role of counselling on the mental health of extrajudicial execution survivors and discusses the identified literature gaps. The theoretical review discusses the theories that guided the study.

2.1 Empirical Review

The Prevalence of Mental Illnesses among Survivors of Extrajudicial Executions

According to the World Health Organization (WHO), approximately 10-20 percent of adolescents and children in the global context suffer from mental disorders (WHO, 2014). On the other hand, KNHCR (2011) projects the global estimates of individuals that will experience a mental health illness at 25 percent, with about 20 percent and 10 percent of patients and the general population respectively seeking primary care presenting with mental health disorder symptoms at any specific time. These mental health disorders comprise primary insomnia, panic disorder, obsessive and compulsive disorder, post-traumatic stress disorder, Alzheimer's and other dementias, alcohol and selected drug user disorders, epilepsy, schizophrenia, bipolar affective disorder, and unipolar depressive disorders. Regarding psychosis, Kiima and Jenkins (2012) asserted that there is a likely prevalence of the disorder in the country at an average of one percent of the Kenyan population. Nonetheless, the most often diagnosis of mental health disorders made in the Kenyan general hospital settings are anxiety and stress disorders, substance abuse, and depression (Kenya Mental Health Policy, 2015). As a result of the diversity of mental health disorders, at any specific time, there is a high likelihood of a member of one in every four families be suffering from a mental illness (KNHCR, 2011).

Besides, the prevalent onset of the disorders during the early years of development and its continued perseverance through adulthood remains a crucially significant concern. Of the mental disorders, however, attention deficit hyperactivity disorder (ADHD) is among the most prevalent mental illness among adolescents and children. Particularly, ADHD is reported to affect about five to seven percent of individuals under the age of eighteen years of age, in the international context, with its pervasiveness being higher in males than in females. However, despite that ADHD has a high prevalence rate, depression is reported to have the highest prevalence rate, affecting about 5.6 percent of the world's male population and 11.7 percent of the female population, making it the most common form of mental illness (WHO, 2017). Besides, this mental illness can adopt numerous forms such as seasonal affective, postpartum, psychotic, and major depression based on risk factors, symptoms, and causes. The third most prevalent mental disorder among adolescents and children, autism spectrum disorder (ASD), is said to affect about one percent of the global population, with males being five more times to develop the condition compared to the female population.

The prevalence of mental illnesses is higher among individuals exposed to any form of a traumatic event in comparison to those not exposed to such situations. Particularly, Knipscheer et al. (2020) assert that exposure to a potentially traumatic event results in the development of post-traumatic stress disorder (PTSD). The study further examined the lifetime prevalence of PTSD across numerous nations and discovered that the rates ranged from below one percent in Switzerland and Nigeria to about five to nine percent in Norway, the Netherlands, and the United States and as high as 37 percent in post-conflict nations, such as Cambodia, Algeria, and Liberia. These fluctuations in prevalence were primarily associated with the vulnerability conveyed in numerous socioeconomic country characteristics, type of event, the disparities in the levels of wealth, and the deviating risks of exposure to possibly traumatic incidents. Alternatively, in the study of PTSD and trauma exposure in young people, Lewis et al. (2019) posited that youths at greater risk of developing PTSD following exposure to a traumatic incident include those who were subjected to interpersonal types of index trauma, those living in disrupted or disadvantaged families, those with a history of psychopathology, those who have undergone previous victimisation, and girls. Nonetheless, despite the numerous factors that have a significant influence on the onset of PTSD among individuals exposed to possible traumatic incidents, Kessler et al. (2019) posit that the trauma type has the most substantial impact on the prevalence of PTSD among these populations.

The rate of mental illnesses is particularly high among individuals subjected to police harassment and killings and other forms of extrajudicial executions. Luitel et al. (2013) found that the prevalence of mental illnesses was higher for individuals who had witnessed the harassment of others, those who were exposed to others being killed, as well as those who got hurt in the process. For these people, the study ascertained that they had a higher likelihood of developing PTSD, depression, and anxiety symptoms compared to the rest of the population. Regarding police violence and its association with the prevalence of mental illnesses, DeVylder et al. (2018) observed that police violence posed a substantial risk to public mental health. The study established that exposure to physical violence at the hands of the police force generated great odds of subclinical psychotic experiences, suicide attempts, depression, and psychological distress.

In the Kenyan context, psychiatrists have established that approximately 25 percent of out-patients visiting hospitals in search of care suffer from a certain form of mental health condition, such as obsessive compulsive disorder, alcohol dependence, generalised anxiety disorder, panic disorder, post-traumatic stress disorder, or depression. Furthermore, in the in-patient context, the number of individuals with any form of mental health condition increases to approximately 40 percent of in-patient clients. Concerning depression, Kenya was ranked at position four in Africa, with 1.9 million people suffering from depression, followed by Major Depressive Disorders (MDD), this is according to (WHO 2014). Besides, the prevalence of mental health disorders is attributed to be higher for vulnerable groups such as elderly persons, survivors of violence, the unemployed, individuals living in difficult conditions or poverty, adolescents and children with disrupted upbringing, or those with a chronic or serious physical illness. From this perspective, the young extrajudicial survivors from Mathare Informal Settlement can be denoted as having a very high probability of suffering from mental illnesses, for they are among the vulnerable groups. Specifically, these young people are not only victims of an extreme form of violence (extrajudicial killings) but are also brought up in difficult conditions, thus making the prevalence of mental health disorders.

The Key Problems Leading to Mental Health Complications in Mathare Informal Settlement

People develop mental health complications due to a broad array of reasons. First, according to the World Health Organization, mental health and a diverse collection of mental disorders are influenced to a substantial extent by the physical, economic, and social environments in which individuals reside (WHO, 2014). Secondly, Walsh (2011) asserted that the significance of lifestyle factors is an underestimated factor in the prevalence of mental health complications. The study attributes lifestyle factors to contribute to various psychopathologies, to the optimization and preservation of cognitive functions, and to fostering social and individual wellbeing. In this context, an unhealthy lifestyle, which is commonplace in Mathare Informal Settlement, functions as a key problem resulting in the development and advancement of mental health complications.

In addition, the lack of mental healthcare literacy and the absence of awareness regarding the existence and prevalence of mental health disorders in informal settlements result in the development of mental health complications. Particularly, in typical African and Kenyan contexts, mental illnesses and mental health disorders are perceived as sorcery or witchcraft (Gikonyo, 2009; Hugo, 2011). Another study by Reid et al. (2014) discovered that most communities and families considered the onset of mental health disorders as a punishment from the 'gods' for immoral conduct by a family member. As a result of the perception of mental illnesses as a punishment for immoral behaviour, for the individual suffering from mental illness, a strong sense of self-esteem is essential for effective coping with the mental illness. For those with low self esteem, watching their families' breakdown and the blame for their mental disorder being shifted to the mothers further result in the development of mental health complications (Munika et al., 2018).

In other instances, the social environment within the Mathare Informal Settlement plays a vital role in the development of mental health complications among the youths. Specifically, Fisher et al. (2013) argue that the pre-natal experience and the mother's maternal health influence the mental wellbeing of young children. For areas such as the Mathare Informal Settlements, the mothers are exposed to poor health and nutrition, poor environmental conditions, highly demanding physical labour, stress, alcohol and drug misuse, and smoking, which increases the children's probability of developing mental health complications during their early and adolescent years. In addition, in a meta-analysis and systematic review on 17 studies, early childhood growth and maternal depression or the development of depressive symptoms uncovered that children born and brought up by depressed mothers had a significant risk of being stunted and underweight, which amplified their risk of being diagnosed with a mental illness in later life (Surkan et al., 2011). In the case of Mathare Informal Settlement, the likelihood of maternal depression or the development of depressive symptoms is quite high due to insufficient emotional and practical support, the subjection to intimate partner violence, having hostile in-laws, the lack of intimate partner support and empathy, being unmarried, being young, unintended pregnancies, and socio-economic disadvantage. As a result, the children born and brought up by such mothers have a high probability of manifesting mental health complications in their early and later lives.

Finally, the exposure of the Mathare Informal Settlement youths to environments that induce stress response contributes to an increased prevalence of mental health complications. A WHO (2014) report establishes that exposure to stress-inducing settings over a young person's life course leads to the accumulation of stress-related behavioural responses such as drug and alcohol abuse, which transform to drug or related dependency, which are categorized as mental disorders. Besides, the accumulation of these stress responses over time significantly affects the individuals' behavioural, physiological, psychosocial, and epigenetic attributes, which leads to the development of factors that most immediately affect a person's mental health (WHO, 2014). Therefore, the exposure to stressors at the Mathare Informal Settlement serves as an amplifier to the young persons' risk of developing mental health complications, for these stressors leave them vulnerable to acquiring these complications. However, the provision of counselling services can play a crucial role in the restoration or improvement of this population's mental health outcomes.

Counselling in Promoting the Mental Health of Extrajudicial Execution Survivors

Counselling is defined as a way of assisting individuals in solving their own interpersonal social, or emotional problems (Matliwala, 2017). From this perspective, counselling does not entail giving advice or solving the clients' problems on their behalf; however, it entails helping clients to gain more insight into the underlined issues and objectively solving the issues. The counsellor's responsibility, in this context, comprises of showing the client a different dimension of understanding a particular situation as well as enabling the client to know their weaknesses and strengths without any judgment. From the client's perspective, counselling provides them with an objective and supportive environment for the exploration of their problems as well as an avenue for the determination of alternative courses of action that might lead to problem-solving.

Besides, individuals seek counselling for diverse reasons, including an inability to change, where despite possessing a desire to change, they lack the personal insight or the self-awareness to drive them to alter their behaviours or actions. Other individuals seek out counselling when they are concerned by physical symptoms that fail to react to medical remedies or investigations. These psychosomatic issues may comprise stomach problems, tiredness, sleep disorders, tension headaches, skin problems, among other similarly debilitating symptoms. On the other hand, for other people, lack of direction or motivation, low self-esteem, lack of assertiveness, difficulties at work, or academic underachievement may be the driving force towards engaging in counselling sessions. Finally, other

people's search for counselling help is grounded on feelings of worthlessness, anxiety troubles, addictions and phobia, and frequently the belief that the failure may further worsen their conditions. Nonetheless, anxiety and depression are identified as the main issues that instigate the search for counselling help (Sharf, 2012).

Counselling plays a critical role in the enhancement of people's mental health and general wellbeing. There is a general understanding that problem-related to mental health could be handled in a counselling context. According to Sharma (2019), the purpose of counselling is to enhance the individual's positive personality development and growth, assist in conflict management, improve their relationships, help them in coping with situational crises such as prolonged medical illnesses, pain, or bereavement, assist in the reversal or modification of problem behaviours, eradicate negative symptoms such as depression or anxiety, and help in the treatment of mental, behavioural, or emotional dysfunctions. According to Wango (2015), the effectiveness of counselling really depends on the ability of the client to bring out the desired changes, harness their own potential, and address their issues with certainty.

However, different counselling approaches serve specific purposes in the improvement of a person's mental and general wellbeing. For instance, cognitive behavioural therapy (CBT) is employed in the identification and correction of irrational, irregular, and negative thoughts that may have become automatic due to repetition. Particularly, the CBT approach works by challenging an individual's ways of thinking and enabling them to generate more realistic and helpful thought patterns of treatment, thus making it effective in the treatment of post-traumatic stress disorder, body dysmorphic disorder, panic, and generalised anxiety disorder, and depression. Relaxation therapy is employed in dealing with stress as well as in the decrease of autonomic hyperactivity and anxiety. For the modification of unhelpful and harmful behaviours that a person may have, behavioural therapy is utilized. However, despite its effectiveness, a majority of persons with mental health complications rarely gain access to counselling services due to numerous barriers.

Possible Obstacles to the Extrajudicial Survivors' Access and Utilization of Counselling Services

There exist numerous probable obstacles that impede the access and utilization of counselling services by individuals with mental disorders. To begin with, the stigma and discrimination connected to mental illnesses are commonly suggested impediments to the search for and access to counselling services (Zartaloudi & Madianos, 2010). In psychiatry, stigma is denoted as the attitude of disapproval towards individuals diagnosed with mental illnesses (Zhang et al., 2020). Meyer and Ndetei (2015) state that there are three types of stigmas related to mentally ill individuals, which include label avoidance, self-stigma, and public stigma. The public stigma is the most impactful form of stigma for mentally ill persons, for it may result in other members of the society acting against the stereotyped individuals. As such, these pejorative attitudes induce society to distance themselves, reject, and fear individuals with mental complications. Consequently, for individuals with mental illnesses, social stigma results in decreased help seeking behaviour, shame, concealment of symptoms, lowered self-esteem, and diminished opportunities (Seacat, 2014). Moreover, these stigmatizing attitudes towards individuals with mental illnesses have been found to be prevalent and resulting in influenced subsequent treatment behaviours on a global scale (Henderson et al., 2013). In the Kenyan context, the stigma associated with mental illnesses influences not only people's help-seeking behaviours but also the health professionals' willingness to work in mental health settings. Marangu et al. (2014) assert that the discrimination and stigma linked to mental health disorders is the predominant factor that explains why only a few health professionals decide to work in mental health services. The outcome is a decrease in the number of counsellors, which limits the number of individuals that can gain access to counselling services.

Secondly, financial barriers also serve as a substantial impediment to the access and utilization of counselling services. Corburn and Kaanja (2016) argue that the cost of care has been among the most cited hindrances to mental health treatment, especially in informal settlements. Besides, the possession of health insurance, through the public or private sector, is a primary determinant of access and use of health services. As a result, individuals without health coverage have greater unmet needs, delay seeking care, and experience greater barriers to care. Similarly, for mental health illnesses, individuals without health insurance face impediments to accessing counselling services. In the Kenyan context, individuals with mental illnesses lack not only health insurance but also the funds to cater to their medical care. A study by Musyimi et al. (2017) found that most Kenyans lacked the resources to get to the hospitals as well as pay for their treatment. During the study, some of the subjects reported a lack of food at the time of the research and, as such, argued that they were required to earn money to feed themselves and their families instead of seeking treatment. Therefore, financial inadequacy can be cited as among the critical hindrances to the access and utilization of proper mental health care by Kenyans with mental illnesses.

Thirdly, the disintegrated organization of mental health services has been identified as a significant obstacle to the access and utilization of counselling services. In the Kenyan context, not only does the country lack a formal, sanctioned mental health policy but also sufficient facilities for the provision of high-quality mental health services. The lack of a formally endorsed mental health policy has significantly limited the mental health reform agenda within the nation (Marangu et al., 2014). As a result, the country only has a single well-known psychiatric hospital, the Mathari Hospital in Nairobi, which focuses on the provision of in-patient services for all mental health clients across the nation (Ndetei et al., 2008). On the other hand, the country lacks mental health care services at the community level as well as in primary care facilities. In addition, the healthcare facilities that offer mental health care services, comprising of the district and sub district hospitals, lack the capacity to deliver out-patient mental health care services at the community level (KNHCR, 2011). As a result, for people residing in rural areas or informal establishments such as the

Mathare Informal Settlement, gaining access to mental health services is almost impossible due to a lack of essential resources and knowledge of this facility.

Fourthly, the lack of prioritization of mental healthcare in Kenya also plays a vital role in individuals' access and utilization of counselling services. Prioritization of resource apportionment for mental healthcare in a country like Kenya is constantly a challenge due to competing for health priorities, including the increment of chronic ailments such as renal failure, cardiac diseases, and diabetes, and the prevalence of infectious diseases such as malaria and HIV (Ndetei et al., 2007). Particularly, the country's reforms have primarily focused on addressing and eradicating communicable diseases as well as making improvements with regard to maternal and child mortality. The resulting outcome has been a low budget priority and the receipt of little attention by the mental healthcare domain, which adversely affects the growth of this field and the availability of low resources in the treatment of mental disorders, thus affecting the access to mental health services within Kenya. Furthermore, deficiencies have surfaced in the delivery of mental healthcare within the Kenyan context, where the Kenyans' mental health needs exceed the available mental health services at all the healthcare services delivery system levels, creating a "treatment gap" where significant individuals with mental illnesses lack the proper treatment (KNHCR, 2011). Besides, on a global level, this "treatment gap," particularly in lower-middle- and low-income countries, is projected to be at 75 to 85 percent, which equates to about 8.5 million in Kenya that lack the access and utilization of the care they need. In addition, for persons with mental illnesses, access to proper care is impeded because their families and carers lack sufficient information and knowledge of how to cope with a relative's mental health disorder.

The existing "treatment gap" is further widened by the availability of a limited mental health workforce in Kenya. Marangu et al. (2014) state that the key challenge for mental healthcare within the country is the critical shortage, or in some regions, the total inadequacy of a professional mental health workforce. Specifically, the country only has 54 psychiatrists, ten medical social workers, 418 trained psychiatric nurses, and a limited number of psychologists obligated with catering to the mental healthcare needs of a population of approximately 43 million people, with about four percent of this population having a major mental disorder. Furthermore, the Kenyan health workers at all levels are associated with low degrees of mental health literacy, which further compounds the issue of lack of access to mental healthcare surfacing from shortages of the mental health workforce, thus serving as an obstacle to the access and utilization of mental healthcare by the individuals suffering from mental illnesses.

2.2 Theoretical Framework

The Psychodynamic Theory

The psychodynamic theory has its roots in the psychoanalytic practice and theories of Sigmund Freud (Freud, 1923). This theory concentrates on the certainty that a significant portion of our mental functioning is unconscious (Dryden & Mytton, 1999). Besides, this unconscious section of our mental functioning is attributed to contain feelings and thoughts as well as memories that we have repressed for the avoidance of the conflict and pain they might generate. This theory further asserts that despite this material being somewhat repressed, it continues to have an intense impact on individuals' feelings, thoughts, and behaviours. In addition to the unconscious concept, this theory is also founded on an understanding of the structure of human personality.

The psychodynamic theory is primarily rooted in the understanding that the threat of punishment from the Superego, when combined with the demands of the Id, creates a substantial pressure that generates anxiety (Hough, 2014). Freud (1923) considered the Superego as encompassing internalized moral precepts, ideals, and values gained from parental and other authority figures, especially around the age of three. The Id is identified as the most primitive component of the system, which remains present from birth, and as such, is recognized as the repository for everything that is inherited, instinctual, and fixed in an individual's makeup (Freud, 1923). Besides, the theory establishes that the conflict linked to an individual's wishes and external reality results in the stemming of defence mechanisms that protect them against tension and extreme discomfort. These defence mechanisms comprise displacement, regression, introjection, reaction formation, denial, rationalisation, projection, humour, and repression. However, the persistent and prolonged use of these mechanisms tends to be counter-productive in the long run, for they not only require substantial amounts of vigilance and energy but also result in the falsification of experiences and reality distortion (Hough, 2014).

This theory, however, is among the critiqued models based on its application to clinical practice. Fonagy and Target (2010) critique the theory as not logically connected to the psychoanalytic technique clinical theory, as inductive instead of deductive reasoning, as well as its Reconstructionist clinical stance. However, Fulmer (2018) posits that the contemporary psychodynamic theory has evolved from its psychoanalytic roots, making it the most versatile and inclusive school of thought accessible to therapists.

The modern psychodynamic theory is ideally conceptualized as a system. From this perspective, contrary to the conventional approach that perceives the theory as either limited to personality or therapeutic model, the contemporary approach views the theory as a versatile system that both explains and embraces irrationality. As a result, Shedler (2010) asserts that the theory has proven its efficacy for a diverse range of populations and conditions, including personality disorders,

substance-related disorders, eating disorders, somatoform disorders, panic, anxiety, and depression. Its broad application, therefore, makes the theory significant for this study in the determination of the contribution of counselling in promoting the mental health of survivors of extrajudicial execution in Mathare Informal Settlement. Besides, the theory's capacity to propose origins and explanations of problems makes it efficient in the identification of the key problems leading to mental health complications in Mathare Informal Settlement.

III. METHODS

The study adopted an ex post facto research design. The selection of the ex post facto research design was because of its capacity to allow a researcher to obtain conclusions that are specific and relevant without interfering with the study population (Salkind, 2010). Besides, this design allowed for the fusion of both quantitative and qualitative data, hence providing a means of exhausting the extrajudicial executions subject matter.

The study was conducted in Mathare informal settlement of Nairobi in Nairobi County. The researcher purposefully selected Mathare informal settlement for the study because a majority of the area's residents were living under harsh living conditions; some got involved in violence; lacked employment, and hence profiling and killing of crime suspect by the police officers. Of the settlement residents, those identified as survivors of extrajudicial execution, those who had reported cases of extrajudicial executions at Mathare Social Justice Centre formed the target population.

The researcher utilized simple random sampling in the selection of the study participants. Only 40 individuals from the Mathare Social Justice Centre database and who were residents of Mathare Informal Settlement formed the sample size. Rashid and Azeem (2011) specified that for an experimental study, it is desirable to have a minimum of 15 study participants in each group. Thus, a total of 40 study participants from the Mathare Social Justice Centre database were considered a sufficient sample size for the determination of the role of counselling on the mental health among Mathare extrajudicial survivors. From the 40 study participants, 20 were assigned to the experimental group and 20 to the control group.

Additionally, the study used questionnaires in collecting data from the 40 respondents. However, before undertaking the research, the researcher partook in a pilot test where ten questionnaires were administered to the targeted population. After the pilot test, any areas that were identified as requiring amendment were rectified before the main study. Finally, the researcher utilized SPSS (statistical package for social sciences) for descriptive analysis (using measures of central tendency such as mean, mode, and median, as well as tables and frequencies).

IV. RESULTS AND DISCUSSION

The researcher administered 40 questionnaires to Mathare Informal Settlement residents. The 40 questionnaires were duly filled and returned to the questionnaire, hence achieving a 100 percent response rate, which was sufficient for analysis.

4.1 Demographic Information

Of the 40 respondents, 52.5% (21) were female, whereas 47.5% (19) were male. Of these respondents, 22 were aged between 18 and 28, 11 were between 28 and 39, and seven were 39 years and above. As such, a majority of the respondents (55%) were between the years of 18 and 28. Concerning residency, two had been residents of Mathare Informal Settlement for less than a year, three had stayed in Mathare for between one to four years, two between four and seven years, four between seven and ten

years, and 29 had stayed in Mathare for ten years or more. The findings that most of the respondents (73%) had been Mathare residents for ten or more years increased the study reliability for these participants had a better knowledge of the occurrence of extrajudicial executions events within the area.

Moreover, when asked whether they had witnessed any extrajudicial execution occurrences, 29 of the 40 respondents answered yes, whereas 11 answered no. These figures show that the probability of Mathare residents witnessing extrajudicial executions was high, for 72% of the respondents had already witnessed such incidences. On the other hand, when asked whether they had been victims of any extrajudicial execution occurrences, 17 of the 40 respondents answered yes, whereas 23 answered no.

4.2 Prevalence of Mental Illnesses Among Extrajudicial Execution Survivors and Witnesses

In the evaluation of the prevalence of mental illnesses among extrajudicial execution survivors and witnesses, the study focused on anxiety, insomnia or sleep disorder, depression, panic disorder, alcohol and drug abuse, post-traumatic stress disorder (PTSD), and other forms of mental illnesses. Table 1 below shows the obtained results on the identified mental illnesses' prevalence among extrajudicial execution survivors and witnesses.

Table 1: Prevalence of Mental Illnesses Among Extrajudicial Execution Survivors

Mental Illness	Cases	Percentage
Anxiety	22	76%
Insomnia/Sleep disorder	21	72%
Panic Disorder	21	72%
PTSD	16	55%
Alcohol and drug abuse disorder	10	34%
Depression	8	28%
Other forms of mental illness	3	10%

These findings inferred that exposure to extrajudicial executions resulted in the development of mental illnesses among the witnesses and survivors. Of these illnesses, anxiety, insomnia and other sleep disorders, and panic disorder were the most prevalent among this population, with approximately three-quarters of the extrajudicial execution survivors developing anxiety disorders. Alternatively, the least prevalent mental illness among extrajudicial execution survivors was depression.

4.3 Key Problems Leading to Mental Health Complications Among Survivors of Extrajudicial Executions in Mathare Informal Settlement

A WHO (2014) report posited that a diverse collection of mental disorders is influenced to a substantial extent by the physical, economic, and social environments in which individuals reside. In the Mathare Informal Settlement context, the current study found the physical environment as a substantial contributor to the development of mental health complications among extrajudicial execution survivors. The survivor' subjection to poor environmental conditions was recognised as the primary problem resulting in their development of mental health conditions. The study also attributed the survivors' being born to and brought up in a family exposed to a mental breakdown and in a hostile

home to an increased probability of developing mental health complications. Other contributors to mental health complications among extrajudicial execution survivors comprised experiencing unhealthy lifestyles and poor health and nutrition at the settlement.

Additionally, the study found that the absence of awareness regarding the existence and prevalence of mental health disorders in Mathare Informal Settlement and lack of mental healthcare literacy among Mathare Informal Settlement residents resulted in an increased likelihood of mental health complications. Similarly, Hugo (2011) established that the lack of mental healthcare literacy and the absence of awareness regarding the existence and prevalence of mental health disorders in informal settlements resulted in the development of mental health complications. Other environmental and social problems increasing the survivors' probability of developing mental health complications encompassed found smoking, alcohol, drug misuse, and exposure to police brutality and killings in Mathare Informal Settlement. Therefore, in addition to the trauma associated with being exposed to police brutality and extrajudicial executions, physical, environmental, and social problems also led to the extrajudicial execution survivors' development of mental health complications.

4.4 The Impact of Counselling on the Mental Health of Survivors of Extrajudicial Execution

The study compared the data collected from the experimental and the control groups to determine the impact of counselling on the survivors' mental health. The experimental group comprised individuals who sought counselling services to improve their mental well-being. The control group encompassed respondents who had never received counselling services after witnessing and surviving an extrajudicial execution. After asking the experimental group the number of times they sought counselling to improve their mental health, the figure below was obtained.

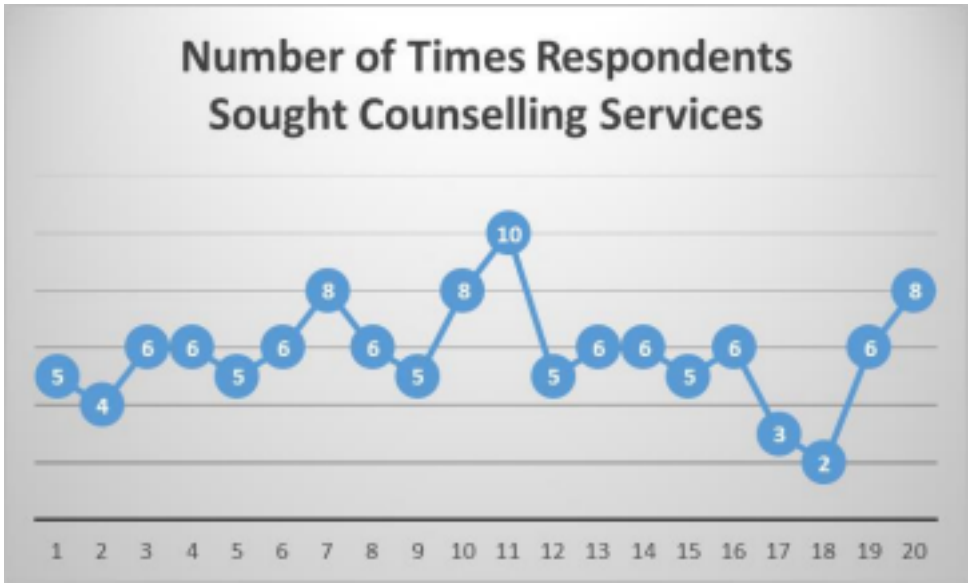

Figure 1: The Number of Times Respondents Sought Counselling Services

Figure 1 above indicates that the least number of times that respondents within the experimental group sought counselling services was twice, and the highest was ten, whereas a majority of them sought counselling six times for the improvement of their mental state.

Table 2: below shows a comparison between the experimental and control groups' results on the impact of counselling on the survivors' mental wellbeing.

Table 2: The Impact of Counselling on Mental Health

Statements	Experimental Group		Controlled Group	
	Mean	Standard Deviation	Mean	Standard Deviation
Counselling helps Mathare extrajudicial execution survivors and witnesses in addressing their traumatic exposures.	4.45	0.94	4.10	1.33
Counselling provides a supportive environment for Mathare extrajudicial execution survivors and witnesses to objectively evaluate their mental state.	4.50	0.83	4.30	0.98
Counselling helps Mathare extrajudicial execution survivors and witnesses to adopt positive and constructive behaviours.	4.45	1.10	3.85	1.23
Counselling promotes psychological wellbeing and restoration among Mathare extrajudicial execution survivors and witnesses.	4.50	0.69	3.90	1.17
Counselling helps survivors and witnesses of Mathare extrajudicial executions to be empowered and or create survivor's social support network.	3.25	0.85	3.90	1.12
Counselling helps Mathare extrajudicial execution survivors and witnesses in addressing alcohol and drug-related addictions.	3.75	1.02	3.85	0.88
Counselling helps Mathare extrajudicial execution survivors and witnesses in improving relationships and interaction with others.	3.45	1.05	3.85	1.35
Counselling helps Mathare extrajudicial execution survivors and witnesses achieve personality development and growth.	3.60	0.94	3.55	1.28
Counselling helps Mathare extrajudicial execution survivors in reporting and following up with cases.	3.90	0.91	3.80	1.15
Counselling helps Mathare extrajudicial execution survivors and witness in addressing psychological and mental disorders i.e., anxiety, depression PTSD	4.65	0.75	4.00	1.08

The experimental group reported higher degrees of and more positive impacts of counselling on their mental health in comparison to the controlled group, as indicated in the table above. The standard

deviation for the experimental group was relatively small, inferring to a small convergence in the study participants' assessments. Contrarily, the standard deviation for the controlled group was somewhat large, especially in comparison to the experimental group's, indicating a significant variation in the respondents' assessments concerning the impact of counselling on mental health for Mathare Informal Settlement extrajudicial survivors and witnesses.

For the survivors who sought counselling, the services positively impacted their mental and general wellbeing in multiple ways. Firstly, counselling helped them address their traumatic exposures. Secondly, counselling provided them a supportive environment to objectively evaluate their mental state. Thirdly, counselling services contributed to the adoption of positive and constructive behaviours and enabled them to create a social support network. Finally, counselling empowered and restored them, promoted their psychological wellbeing, and enabled them to address alcohol and drug-related addictions, improve relationships and interactions with others, and achieve personality development and growth. Therefore, seeking counselling can be significantly beneficial for extrajudicial execution survivors. However, several obstacles may impede these individuals' access and using counselling survivors.

4.5 Possible Obstacles That Prevent Survivors of Extrajudicial Executions in Mathare Informal Settlement from Accessing and Utilizing Counselling Services

The study found four primary obstacles that impeded the extrajudicial execution survivors' access and utilization of counselling services. The stigma connected to psychological disorders was the first obstacle preventing extrajudicial execution survivors' access and use of counselling services. Meyer and Ndetei (2015) found three types of stigmas related to mentally ill persons: label avoidance, self-stigma, and public stigma. Of the three forms of stigma, the public stigma might be the most impactful for the extrajudicial execution survivors with mental illnesses. Public stigma may result in other members of the society acting against the stereotyped individuals by distancing themselves, rejecting, and fearing the persons with psychological disorders. The second obstacle comprised the discrimination of persons with mental illnesses at the Mathare Informal Settlement. Thus, in fear of being discriminated against, survivors of extrajudicial executions in Mathare avoided seeking counselling services to enhance their mental wellbeing after exposure to traumatic experiences.

The third obstacle consisted of financial barriers. Notably, a majority of Mathare Informal Settlement's extrajudicial execution survivors lacked the financial resources required to obtain quality counselling services. Corburn and Kaanja (2016) backed these findings by citing financial barriers as among the primary barriers preventing individuals from accessing counselling services in informal settlements.

The final obstacle to obtaining counselling services for Mathare Informal Settlement's extrajudicial execution survivors consisted of lack of mainstreaming of counselling services in Mathare, the lack of high quality mental health services at Mathare Informal Settlement, and the inadequacy of professional counsellors and mental health workforce. KNHCR (2011) posited that the aforementioned obstacles arose because Kenya, as a country, lacked mental health care services at the community level as well as in primary care facilities. Besides, the healthcare facilities that offer mental health care services, comprising of the district and sub-district hospitals, lack the capacity to deliver out-patient mental health care services at the community level. As a result, for people residing in Mathare Informal Settlement, gaining access to counselling was almost impossible. Therefore, there is a need to minimize stigmatization and discrimination of mentally ill persons and to eradicate financial barriers and lack of quality counselling services for improved access and use of counselling services by extrajudicial execution survivors at Mathare Informal Settlement.

V. CONCLUSION AND RECOMMENDATIONS

The study results established that all Mathare Informal Settlement residents who had either witnessed or been a victim of extrajudicial executions suffered from a broad array of mental health complications. The identified mental disorders comprised anxiety, depression, alcohol and drug abuse disorders, panic disorders, post-traumatic disorders, and other forms of mental illnesses. Of these illnesses, anxiety was identified as the most prevalent among Mathare Informal Settlement extrajudicial survivors. Besides exposure to extrajudicial executions, the study identified physical, economic, and social environments as either exacerbating or leading to the development of mental health complications among extrajudicial execution survivors and witnesses. Of the persons that developed mental health complications, the study found that those who sought counselling services obtained positive mental health and overall wellbeing changes. Particularly, counselling services helped extrajudicial execution survivors address the traumatic exposures, offered them a supportive environment to objectively evaluate their mental states, and assisted them to adopt positive and constructive behaviours. Consequently, counselling empowered them to address alcohol and drug addictions, form a survivor's social support network, improve their relationships and interactions, and achieve personal growth and development. However, despite the numerous benefits of counselling, a substantial number of the extrajudicial execution survivors at Mathare Informal Settlement lacked access to counselling services. The obstacles impeding their access to and utilization of counselling services consisted of financial barriers, the stigmatization and discrimination of mentally ill individuals, and the lack of quality counsellors and counselling services at the Mathare Informal Settlement as well as the rest of the country.

The study recommended the implementation of education, community sensitization and awareness programs, and facilities offering quality mental health care as capable of minimizing or eradicating the extrajudicial execution survivors' access to counselling services. Specifically, the Mathare Informal Settlement residents should be educated on the adverse impacts of exposure to extrajudicial executions on their mental health, how to use counselling to minimize some of the negative psychological effects, and the available mental health care facilities. The county and local government should consider providing these residents with community sensitization and awareness on the importance of counselling to break the existing stigma associated with mental illnesses and the ensuing discrimination, as well as encourage extrajudicial execution survivors to seek help. Besides, the government should consider increasing healthcare facilities that offer mental health care services, comprising of the district and sub-district hospitals, especially in informal settlements like Mathare, where there is a prevalence of extrajudicial executions cases. The government should also ensure the employment of a highly-skilled workforce with the ideal counselling competencies, skills, and knowledge for offering quality services to those in need and using counselling in improving the mental and general wellbeing of extrajudicial execution survivors and other individuals who undergo traumatic experiences.

VI. SUGGESTIONS FOR FUTURE STUDIES

This study employed ex post facto research, which may have produced a certain degree of bias because of the lack of statistical tests. Besides, the research method cannot verify or test the research problem statistically, thus bringing about accuracy and integrity concerns. As such, future studies on the role of counselling on the mental health of survivors of extrajudicial execution in Mathare Informal Settlement should consider using quantitative data and quantitative data analysis methods. Moreover, this study's sample was confined to Mathare Informal Settlement, which decreases its generalizability. For improved generalizability of the research findings, future studies on the subject matter should attempt to cover a larger target population, such as the entire Nairobi County.

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