IMAGE: A MAP OF THE STARS OF THE ORION CONSTELLATION

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Tracing the Origin Source of the Epidemics

Jian'an Wang

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ABSTRACT

This paper puts forward the hypothesis that the upper atmosphere is similar to a large liquid nitrogen tank storing various germs, and that water (drought) disasters and epidemic outbreaks are caused by abnormal large-scale motion of the atmosphere. Various possible causes of abnormal large-scale movement of the atmosphere are also given, including the solar system speeds up (or slows down) in the Milky Way, and the comet passes by the Earth.

Keywords: tracing of covid-19 origins, epidemic, epidemic situation, outbreak, pandemic, plague, flood, drought, comet.

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I. INTRODUCTION

From ancient times to the present, human beings have encountered countless epidemics, some of which are particularly serious and have a huge impact on human beings: COVID-19, SARS, plague, smallpox, influenza, cholera, malaria and so on [1]. How did the epidemics come about? It remains a mystery. To solve this mystery, we start by analyzing the abnormal weather that accompanies the epidemic outbreak.

COVID-19 broke out in Wuhan at the end of 2019, followed by a once-in-a-century flood in Southern China in June and July 2020. The winter of 2020-2021 is unusually cold in the Northern Hemisphere, especially since early to mid-February, a super cold wave swept almost the entire North America, Canada and the United States saw a once-in-a-century super cooling, and many places also saw rare snowstorms. In the spring and summer of 2021, a new wave of COVID-19 outbreak occurred, with the rapid spread of various new or variant version of COVID-19.

Historically, the outbreak of epidemics has always been accompanied by abnormal weather (such as drought, flood, and ice age). From the 15th to the 19th century, China experienced 15 droughts for 49 years, 14 of which were accompanied by epidemics [1]. The transformation between the Chinese Ming and Qing dynasties coincided with the long cold period of 1,500 s-1690s in the Little Ice Age. During this period, disasters such as drought and flood, locust plague, hail and epidemics frequently occurred [2].

Why is the abnormal climate, such as floods, droughts, and ice ages, often accompanied by the epidemic? This study aimed to solve this mystery and find out what caused the epidemic outbreak.

II. DISCUSSION

2.1 The surface of the atmosphere may have been home to all kinds of primitive germs

In the paper "On the dynamics of ocean currents and atmospheric circulation (Is it possible for germs to travel along latitudinal lines through atmospheric circulation? "[3], the author proposed the conjecture that germs could spread along the Earth's latitude through atmospheric circulation. The author further suspects that "primordium germs existed as protein molecules high up at the surface of the atmosphere." Because the temperature in the upper air is very low (the temperature drops 0.6 degrees centigrade for every 100 meters above sea level, for example, 20 degrees centigrade at sea level, and then -196 degrees centigrade at 36,000 meters, which is the temperature of liquid nitrogen). Such low temperatures allow these microbial proteins to be preserved for a long period, as in liquid nitrogen. And because germs have so little mass, they can stay suspended on the surface of the atmosphere for long periods and travel with the circulation of the atmosphere. The author suspects that these germ molecules began to exist on the surface of the Earth's atmosphere in its early days, because the atmosphere's surface was the first to cool to a temperature suitable for life during the Earth's evolution. These microbes may have come in part from space, and after entering the atmosphere, they stayed at the surface and moved with the circulation of the atmosphere. So, the atmosphere's surface is like a giant tank of liquid nitrogen for all kinds of primitive germs.

The Royal Astronomical Society announced at a press conference on September 14, 2020 that scientists have detected molecules of phosphine in the atmosphere of Venus. The finding, if confirmed, suggests that life could exist in Venus' atmosphere. [4], [5]. The discovery of phosphine is important because scientists believe the compound is vital evidence for the existence of life. On Earth, phosphine production is only associated with human industrial production or anaerobic microbial activity. If there is life on the surface of Venus' atmosphere, there is no reason to suspect that there is life on the surface of the Earth' atmosphere. Most of these original germs at the top of the Earth's atmosphere probably never came into contact with the ground, and never encountered by humans. As soon as these original germs came into contact with humans on a large scale, the outbreak began.

2.2 Why is the epidemic always accompanied by abnormal climate such as water (drought) disasters?

These original germs in the upper atmosphere, normally orbit the Earth with the circulation of the atmosphere, and do not come into contact with the ground, so they do not cause outbreaks. When the movement of the atmosphere is disrupted on a large scale, it can cause large amounts of cold air, which carries original germs, to fall to the ground, leading to outbreaks. Because the abnormal movement of the atmosphere can also lead to the occurrence of water (drought) disasters, it can be known that epidemics and water (drought) disasters often go hand in hand.

2.3 Why do big water (drought) disasters and epidemics occur once in a while (like decades or hundreds of years)?

We know that big floods (droughts) and epidemics on the Earth occur every once in a while (like decades or hundreds of years), so why not every year? If the epidemics and water (drought) disasters are indeed caused by abnormal movement of the atmosphere, then what causes abnormal atmospheric movement every once in a period? In article [3], it has been pointed out that the planetary atmospheric circulation is driven by the etheric wind on the planet, and the etheric wind on the planet is influenced not only by the solar ether but also by the galactic ether. Since the impact of the Earth's movement relative to the sun on the climate is periodic by year, the abnormal atmospheric movement (intervals of decades or centuries) causing the epidemics and big floods (droughts) should not be caused by the Earth's movement around the sun. Therefore, the movement of the Earth with the solar system through the Galaxy is most likely responsible for the periodic outbreaks of epidemics and big floods (droughts). Because the Earth moves at different speeds with the solar system in different regions of the Galaxy, when the motion is accelerated or decelerated, the galactic ether will affect the ether wind on the Earth, resulting in changes in the ether wind[6] on the Earth, and resulting in disturbances in the atmospheric movement, and resulting in big floods (droughts) and epidemic outbreaks.

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2.4 Why is the arrival of ice age always accompanied by epidemics?

Ice ages are caused by the accelerated motion of the solar system in the Galaxy [7],[8](because this will lead to the continuous decline of the solar luminous power, which leads to the generation of the ice age). The acceleration of the solar system in the Milky Way causes the change in the Earth's ether wind, which in turn causes disturbances in the atmosphere. Therefore, the arrival of the ice age is always accompanied by outbreaks of epidemics. Because the decelerated motion of the solar system in the Galaxy can also cause changes in the Earth's etheric winds, which in turn cause disturbances in the atmosphere, it could also cause epidemics as the Earth moves from an ice age to a warm one.

2.5 Will a comet passing by the Earth lead to an epidemic outbreak?

Comets are seen as a sign of misfortune and disaster, and people often link the emergence of comets to catastrophic events such as floods, droughts, famine, and epidemics. Because the comet passing by the Earth can also cause changes in the Earth's etheric winds, which in turn cause disturbances in the atmosphere, the emergence of the comet can also lead to the outbreak of epidemics and water (drought) disasters.

2.6 Are the mass deaths of humans and animals responsible for the outbreak of epidemics?

It is now widely accepted that outbreaks of epidemics after water (drought) disasters are caused by the large number of human and animal carcasses left by these disasters that were not properly disposed of. Because these carcasses are thought to breed a lot of germs. According to the theory of this paper, the outbreak of epidemics is not related to the death of people or animals, because it does not change the movement of the atmosphere, so it does not cause the original germs at the top of the atmosphere to fall to the ground, so it does not cause the outbreak of epidemics. This shows that war has nothing to do with the outbreak of epidemics.

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Pattern and Parity in Mathematics

Dr. Firoz Firozzaman

ABSTRACT

In this paper, we discuss how the simple concept of parity of numbers could be used to improve students' ability to understand a real-life problem in an efficient way and have a better retention rate. In college, first-year students enrolled in Algebra, Precalculus, or Calculus courses most likely have a lack of knowledge of operations in arithmetic in connection with algebra, geometry, and trigonometry. A certain group of students quite often face difficulty in recognizing mathematical patterns. One goal of this note is to recognize a mathematical pattern, connect it with other related areas of mathematics and science, and find a solution strategy as a general case based on the student's background knowledge. The overarching goal of this work is to identify the topics in first-year mathematics courses from algebra to calculus, where the students find it difficult because of a lack of understanding or lack of working knowledge and skills. The aim is to determine whether the difficulty involves conceptual or procedural deficiency and to develop resources that could be used to overcome the difficulties.

Keywords: parity, even numbers, odd numbers, singly and doubly even numbers, prime numbers, hinges, quartiles, deciles, and fractiles.

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We discuss some of the mathematical concepts and procedures students find most difficult and provide possible solution outlines. We argue that a possible understanding of the number system and mathematical pattern recognition may provide a strong foundation that enhances the ability, and confidence of a student for better performance, and good retention. Teaching and learning with mathematical parity prepare students for modeling real-life problems in STEM education.

Keywords: parity, even numbers, odd numbers, singly and doubly even numbers, prime numbers, hinges, quartiles, deciles, and fractiles.

I. INTRODUCTION AND DEFINITIONS

In mathematics, parity is a term to express if a given integer is even or odd. The parity of a number depends only on its remainder after dividing the number by 2. Suppose, $x \in Z^{*}$, $x \neq 0$, the set of all positive integers. It is known that if x is even than it is divisible by 2. Otherwise it is an odd integer.

Suppose $x^{*} \in Z^{*}$ is an even integer. If the quotient $\frac{x^{*}}{2}$ is again an even integer, it is doubly even or evenly even. Otherwise, it is known as singly even or oddly even.

The mathematical form of the evenly even and the oddly even numbers:

$S_{ee} = \{4x \mid x \in Z^{*}\}$ is the set of evenly even integers and
$S_{oe} = \{4x - 2 \mid x \in Z^{*}\}$ is the set of oddly even integers.

On the other hand, the odd numbers are not divisible by 2. The odd numbers set is $S_o = \{2x-1 \mid x \in Z^*\}$. It is very interesting to observe the unique behavior of odd numbers in many applications.

II. POWERS OF IMAGINARY NUMBER i=$\sqrt{-1}$

To evaluate i^n, $n \in Z^*$ and $m \in Z^*$

Table 1: Powers of i based on evenly even, oddly even, or odd integers.

Evenly even	$n = 4m,\ n \in S_{ee}$	$i^n = 1$
Oddly even	$n = 4m-2,\ n \in S_{oe}$	$i^n = -1$
Odd	$n = 4m-3,\ m \in Z^*$	$i^n = i$
	$n = 4m-1,\ m \in Z^*$	$i^n = -i$

The general rule: $n \equiv r \bmod 4$, where $r = 0, 1, 2, 3$.

III. PYTHAGOREAN THEOREM

In a right-triangle, the square of hypotenuse is equal to the sum of the squares of the other sides called the legs. This is known as the theorem of Pythagoras [1]. These three measurements are known as Pythagorean Triplets or Pythagorean Triples.

For the positive real numbers, $a, b,$ and c, which are the measures of three sides of a right triangle the Pythagorean Theorem is $a^2 + b^2 = c^2$, c is the measure of the hypotenuse.

We will discuss the known formulas in terms of evenly even, oddly even and odd numbers to determine the pattern.

Here we discuss some well-known results on Pythagorean triplets (a,b,c), $a^2 + b^2 = c^2$, $a,b,c \in Z^*$.

Proposition 1: Suppose a is an odd number greater than 1, then $b = \dfrac{a^2-1}{2}$ is evenly even of the form $b = 2m(m+1)$, $m \in Z^*$, and $c = \dfrac{a^2+1}{2}$ is odd, where $c - b = 1$.

Proposition 2. Suppose a is an evenly even number, then $b = \dfrac{a^2}{4} - 1$ is odd of the form $b = 4m^2 - 1,\ m \in Z^*$, and $c = \dfrac{a^2}{4} + 1$ is also odd, where $c - b = 2$.

Proposition 3. Suppose a is an oddly even number greater than 2, then $b = \dfrac{a^2}{4} - 1$ is even of the form $b = 4m(m+1),\ m \in Z^*$, and $c = \dfrac{a^2}{4} + 1$ is also even, where $c - b = 2$.

Table 2: The following results are Pythagorean Triples (a,b,c), a is an odd number, Proposition 1.

$(3,4,5)$	$(5,12,13)$	$(7,24,25)$	$(9,40,41)$
$(11,60,61)$	$(13,84,85)$	$(15,112,113)$	$(17,144,145)$
$(19,180,181)$	$(21,220,221)$	$(23,264,265)$	$(25,312,313)$
$(27,364,365)$	$(29,420,421)$	$(31,480,481)$	$(33,544,545)$

Table 3: The following results are Pythagorean Triples (a,b,c), $a > 2$ is an even number, Proposition 2 and 3.

$(2,0,2)$ Does not form a triangle	$(4,3,5)$	$(6,8,10)$	$(8,15,17)$
$(10,24,26)$	$(12,35,37)$	$(14,48,50)$	$(16,63,65)$
$(18,80,82)$	$(20,99,101)$	$(22,120,122)$	$(24,143,145)$
$(26,168,170)$	$(28,195,197)$	$(30,224,226)$	$(32,255,257)$

More Pythagorean Triples can be found by using the form $(ka)^2 + (kb)^2 = (kc)^2$, $k \in R^+$, set of positive real numbers.

Table 4: Pythagorean Triples are proportional with a scale factor of k, which forms the direct variations.

$k=1,\ (3,4,5)$	$k=2,\ (6,8,10)$	$k=3,\ (9,12,15)$	$k=4,\ (12,16,20)$
$k=0.1,\ (0.3,0.4,0.5)$	$k=0.2,\ (0.6,0.8,0.1)$	$k=0.3,\ (0.9,1.2,1.5)$	$k=0.4,\ (1.2,1.6,2)$

Another known approach: Two numbers can be selected and then find the third number of the Pythagorean Triples.

We will select two numbers p and q such that $a = 2pq$, then $b = p^2 - q^2$ and $c = p^2 + q^2$. The relation $(p^2 - q^2)^2 + (2pq)^2 = (p^2 + q^2)^2$ follows the Pythagorean Theorem.

Table 5: Pythagorean Triples when $p > q$.

$2pq = 12,\ q = 1, p = 6$	$2pq = 12,\ q = 2, p = 3$	$2pq = 12,\ q = 0.5, p = 12$	$2pq = 12,\ q = 1.5, p = 4$
$(12, 35, 37)$	$(12, 5, 13)$	$(12, 143.75, 144.25)$	$(12, 13.75, 18.25)$

IV. PYTHAGOREAN PRIMES (FERMAT'S THEOREM ON SUMS OF TWO SQUARES)

In the number theory, Fermat's theorem on sums of two squares states that an odd prime P can be expressed as $P = x^2 + y^2$, with $x, y \in Z^*$, set of positive integers, iff $P \equiv 1 (\bmod 4)$, [3].

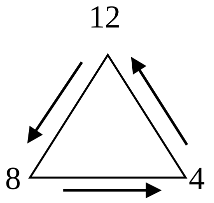
The known such prime numbers are

$5 = 1^2 + 2^2 \quad 13 = 2^2 + 3^2 \quad 17 = 1^2 + 4^2 \quad 29 = 2^2 + 5^2 \quad 37 = 1^2 + 6^2 \quad 41 = 4^2 + 5^2$

We propose a method to find the Pythagorean primes using the magic rule $8 - 4 - 12$.

Magic rule 8 – 4 – 12: Choose a Pythagorean prime, then add 8, 4, or 12 in order to collect the next Pythagorean prime. In this process one needs to observe the output. If the output is not a prime, filter it or mark it and continue with the process. Given below is an elastration step by step.

The first Pythagorean prime is $5 \equiv 1$ mod 4.



$5 + 8 = 13$ $\quad 13 + 4 = 17$ $\quad 17 + 12 = 29$
$29 + 8 = 37$ $\quad 37 + 4 = 41$ $\quad 41 + 12 = 53$
$53 + 8 = 61$ $\quad \boxed{61 + 4 = 65}$ $\quad 61 + 12 = 73$
$\boxed{73 + 8 = 81}$ $\quad \boxed{73 + 4 = 77}$ $\quad \boxed{73 + 12 = \enclose{circle}{85}}$, largest non-prime is 85 in this case.

Box the non-primes, if repeated three times in a row, use the largest non-prime as follows.

$\boxed{85 + 8 = 93}$ $\quad 85 + 4 = 89$ $\quad 85 + 12 = 97$

$\boxed{97 + 8 = 107}$ $\quad 97 + 4 = 101$ $\quad 97 + 12 = 109$

$\boxed{109 + 8 = 117}$ $\quad 109 + 4 = 113$ $\quad \boxed{109 + 12 = 121}$

$\boxed{113 + 8 = 121}$ $\quad \boxed{113 + 4 = 117}$ $\quad \boxed{113 + 12 = 125}$

$\boxed{125 + 8 = 133}$ $\quad \boxed{125 + 4 = 129}$ $\quad 125 + 12 = 137$

$\boxed{137 + 8 = 145}$ $\quad \boxed{137 + 4 = 141}$ $\quad 137 + 12 = 149$

$149 + 8 = 157$ $\quad \boxed{157 + 4 = 161}$ $\quad \boxed{157 + 12 = 169}$

$\boxed{169 + 8 = 177}$ $\quad 169 + 4 = 173$ $\quad \boxed{173 + 12 = 185}$

$173 + 8 = 181$ $\quad \boxed{181 + 4 = 185}$ $\quad 181 + 12 = 193$

Continuing in this process one may easily find infinitely many Pythagorean primes.

We further check the following:

$53 = 2^2 + 7^2$ $61 = 5^2 + 6^2$ $73 = 3^2 + 8^2$ $89 = 2^2 + 8^2$ $97 = 4^2 + 9^2$ $101 = 1^2 + 10^2$

$109 = 3^2 + 10^2$ $113 = 6^2 + 7^2$ $137 = 4^2 + 11^2$ $149 = 7^2 + 10^2$ $157 = 6^2 + 11^2$ $173 = 2^2 + 13^2$

It is interesting to note that each Pythagorean prime number is the sum of one even squares and one odd squares congruent to 1 mod 4 and this representation is unique.

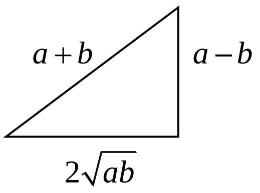
Following list shows the prime positions of Pythagorean primes. The Pythagorean primes are in row 1, their prime positions are in row 2.

5	13	17	29	37	41	53	61	73	89	97	101	109	113	137	149	157	173
3	6	7	10	12	13	16	18	21	24	25	26	29	30	33	35	37	40

V. PYTHAGOREAN TRIPLES IN FINDING AN ARC LENGTH

The parity among direct variations, Pythagorean triplets, and similar right triangles have applications in determining arc length of a certain types of functions.

The identity $4ab + (a-b)^2 = (a+b)^2$ is a Pythagorean.



Example 1. Find the arc length of the curve given by the function with the given restricted domain, $f(x) = \dfrac{x^4}{4} + \dfrac{1}{8x^2}$, $2 \le x \le 4$.

$f'(x) = x^3 - \dfrac{1}{4x^3}$, then to evaluate the definite integral $L = \displaystyle\int_2^4 \sqrt{1 + [f'(x)]^2}\,dx$.

It is expected that the readers know how to simplify the integrand.

It is not too difficult to check that $1 + \left(x^3 - \dfrac{1}{4x^3}\right)^2 = \left(x^3 + \dfrac{1}{4x^3}\right)^2$, which is a Pythagorean.

One has to verify that $2\sqrt{ab} = 1$ or $4ab = 1$.

VI. HINGES

Definition. Hinges are positions when we split an ordered data set into pieces. John Tukey's upper hinge and lower hinge are the measures of positions, known as third and first quartiles.

Let $p \neq 0$ be a positive integer. Two integers N and r are *congruent modulo p,* if there is an integer $k \geq 0, \quad 0 \leq r \leq p-1$ such that $N - r = kp$, and commonly known by the notation

$$N \equiv r \bmod p$$

Notice that the condition " $N - r = kp$" for some integer k" is equivalent to the condition "p divides $N - r$".

Suppose we have N discrete ordered data points and to find p segments keeping m data points in each segment. The number of hinges must be $p-1$.

To determine how many hinges are integer ranked and how many are non-integer ranked.

We observe that when $\frac{N}{p} = m + \frac{r}{p}$, the number of data points in each segment is $m = \frac{N-r}{p}$ and there are r integer ranked hinges.

For simplicity we discuss a special case for four equal divisions commonly known as quartiles [4], and the same idea is extended for deciles [2] and further on even order of divisions or segments.

Suppose N is an even number, the middle most hinge will be non-integer ranked.

Further if N is doubly even, the first and third hinge are non-integer ranked as well. The number of data points N is divisible by 4. This result is confirmed by the remainder rule

$\frac{N}{4} = m,\ r = 0;\ m,\ r \in Z^*$, there is no integer ranked hinges.

If N is singly even, then the remainder is 2 when N is divided by 4. The middle most hinge will be non-integer ranked and the other two must be integer ranked. The first hinge therefore is $(m+1)$th data point and the third one is $(N-r)$th data point, [4], [5].

Corollary 1: If the divisor p is an even number, then there exist midhinge (median) and data set shows symmetry about midhinge.

The midhinge H_2 is considered as the median of the ordered data set [4]. If p is odd, midhinge does not exist for the ordered data set and there is no symmetry.

Corollary 2: If the number of data points N is divisible p and $N = mp,\ r = 0$, then there is no integer ranked hinge. The positions of the hinges would be between each consecutive groups of m observations.

Note that if N is an odd number and $p=4$, then the remainder is either 1 or 3. On the other hand if N is an even number then remainder is either 0 or 2.

Remainder $r=1$ confirms the middle most hinge (median) as integer ranked and other two non-integer ranked keeping m data points in each segment.

Remainder $r=3$ or $p-1=3$ confirms all three hinges are integer ranked keeping m data points in each segment. Let us define d_m : m-th observation in the ordered data set.

Table 6: Thus we have the following table using average:

r	H_1: first quartile	H_2: median	H_3: third quartile
0	$(d_m + d_{m+1})/2$	$(d_{N/2} + d_{N/2+1})/2$	$(d_{N-m} + d_{N-(m+1)})/2$
1	$(d_m + d_{m+1})/2$	$d_{(N+1)/2}$	$(d_{N-m} + d_{N-(m+1)})/2$
2	d_{m+1}	$(d_{N/2} + d_{N/2+1})/2$	d_{N-m}
3	d_{m+1}	$d_{(N+1)/2}$	d_{N-m}

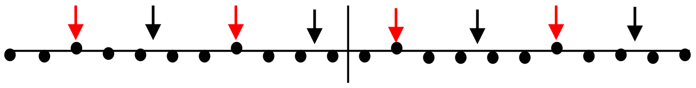
The remainder rule we propose works for hinges when the divisor p is an even number. But the remainder rule still works when p is an odd number. In this case, the number of hinges is even, which shows an interesting behavior. Finding integer ranked hinges we keep as an open question.

For example, we have 22 ordered data points and to find 8 hinges for nine segments.

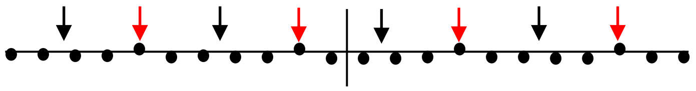
We have $N=22,\ p=9$, therefore $\dfrac{22}{9} \equiv 4 \bmod 9$, where $r=4,\ m=2$. It is not difficult to verify that there are 4 integer-ranked hinges and remaining 4 hinges are non-integer ranked. The number of data points in each segment is $m=2$.

Following are the possible selections.

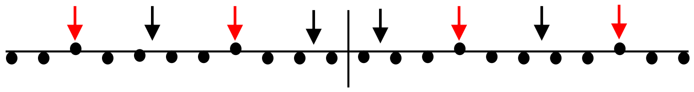
Observation 1: Integer ranked to non-integer ranked respectively.



Observation 2: Non-integer ranked to integer ranked



Observation 3: With symmetry around the middle line.



VII. GENERAL MODEL FOR FRACTILES

Suppose there are N elements in an ordered data set, we are interested to find $f-1$ fractiles. In this model f is an even number, [2], [4].

Let us consider $N \equiv r \bmod f$, where $r = 0, 1, 2, \cdots f-1; \quad f = 2, 3, 4, \cdots, N$

The number of observations in each segment is known by $m = \frac{N-r}{f}$.

The α-th fractile is calculated as follows

$$F_\alpha = \frac{N+1}{f}\alpha = \left(i + \frac{d}{f}\right), \text{ where } i \text{ and } d < f \text{ are positive integers.}$$

The following model produces fractiles by the following rounding notion:

Condition 1. If $r < \frac{f}{2}$, round F_α to the nearest integer when $d \le r$.

Condition 2. If $r \ge \frac{f}{2}$, round F_α to the nearest integer when $d < \frac{f}{2}$ or $\frac{3}{2}f - r - 1 \le d \le f-1$.

Condition 3. Otherwise take average of two consecutive terms in the groups with m data points in each.

Example 1. Suppose $N = 64, \; f = 8, \; 64 \equiv 0 \bmod 8, \; r = 0$.
In this example all the fractiles are non-integer ranked, with 8 ordered data points in each segment.

Table 7: The average position of the fractiles based on $F_\alpha = \frac{N+1}{f}\alpha$

Hinge	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Position	8th - 9th	16th – 17th	24th – 25th	32nd – 33rd	40th – 41st	48th – 49th	56th – 57th

Example 2. Suppose $N = 65, \; f = 8, \; 65 \equiv 1 \bmod 8, \; r = 1$.

In this example all the fractiles are non-integer ranked except the median

$F_4 = \frac{65+1}{8} \cdot 4^{th} = 33^{rd}$, with 8 ordered data points in each segment.

Table 8: The average position of the fractiles based on $F_\alpha = \frac{N+1}{f}\alpha$

Hinge	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Position	8th - 9th	16th – 17th	24th – 25th	33rd	40th – 41st	48th – 49th	56th – 57th

Example 3. Suppose $N = 66$, $f = 8$, $66 \equiv 2 \bmod 8$, $r = 2$

In this example we have two integer ranked hinges. There are 8 ordered data points in each segment.
One needs to identify integer ranked hinges using the proposed conditions.

Table 9: The average position of hinges based on $F_\alpha = \dfrac{N+1}{f}\alpha$

Hinge	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Position	8th - 9th	16th – 17th	25th	33rd – 34th	41th – 42nd	50th	56th – 57th

It is very easy to check that F_3 and F_6 are integer ranked.

The third position is $F_3 = 25 + \dfrac{1}{8} \approx 25$ and the sixth position is $F_6 = 50 + \dfrac{2}{8} \approx 50$

VIII. CONCLUSION

In this paper, we proposed several methodologies to solve complex mathematical problems, portraying pattern recognition and parity which could make complex math problems easier. We proposed strategies to determine powers of imaginary roots, calculating Pythagorean triplets and Pythagorean primes using our “Magic Rule 8-4-12”, and finding arc lengths and fractiles. Applying these methods could enhance students’ background knowledge and skills to face the challenges in STEM education. These methods would be further studied to determine if students are able to implement these tactics to solve mathematical problems more efficiently.

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On the Geometrical Structure of Natural Numbers

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I. INTRODUCTION

In the current literature, a large volume of work describes the structure of vector spaces; see, for example, [1]. Lattices, a close concept defined over the set of natural numbers, appear as a helpful framework in several physical applications [2]. Also, a similar setup has been developed in a mathematical application context and named natural vector (semi-)[1]spaces [3-9]. Previous work on natural numbers varied problems has also been developed in this laboratory [11-22].

This paper will try to find out how starting from natural numbers, more concretely from natural prime numbers, an infinite dimensional vector space can be built up so that a conveniently defined vector norm can generate the set of natural composite numbers. Also, one can consider another kind of vector space structured from the natural powers of prime numbers, which one can connect with Mersenne numbers and the recursive generation of natural numbers. Finally, the previous framework will generate information not only on Fermat's theorem and his extension but also on the Euler-Riemann zeta function.

Essentially the present paper will possess four parts to develop the above-proposed scheme.

First, the prime numbers' powers and the accompanied structure will be minutely described. The second part will describe the infinite-dimensional natural vector space, whose vector elements one can use to generate all the natural numbers. Third, the extension of Mersenne numbers will be analyzed, along with their twins, discussing their prime and composite nature and their relationship to the recursive generation of natural numbers. Finally, with some simple operations performed over the previously constructed natural vectors, one will display information about the connection of the

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[1] From now on, these constructs will be called *natural* vector spaces. They, besides the name semispaces, can also be called *orthants*.

described geometrical structure of natural numbers and the extension of Fermat's theorem, the Euler-Riemann function.

II. PRIME POWER SETS: NATURAL POWERS OF PRIME NUMBERS

Before describing a geometrical framework generating the natural number set, this section discusses some aspects of prime numbers and their natural powers.

2.1 The Set of Prime Numbers

Suppose that one defines the entire set of natural prime numbers as:

$$\mathbb{P}=\{2\}\cup\mathbb{T}, \tag{1}$$

at that point, the set $\mathbb{T}$ holds all *odd* prime numbers.

Without loss of generality, one can also suppose the set $\mathbb{P}$ ordered.

2.2 Powers of Prime Numbers

2.2.1 Prime Power Sets

One can also construct the natural power set associated with every prime number $p\in\mathbb{P}$

That is, after choosing any prime number *p*, one can structure a set of its natural powers $\mathbf{p}^{\mathbb{N}}$, using:

$$\forall p\in\mathbb{P}\wedge\forall n\in\mathbb{N}:\mathbf{p}^{\mathbb{N}}=\left\{p,p^{2},p^{3},\ldots p^{n},\ldots\right\}; \tag{2}$$

one can call such a set a *prime power set*. The prime number *p* used to construct the set $\mathbf{p}^{\mathbb{N}}$ can be named the *prime source (or origin)* of the power set.

If one studies the prime power set of any pair of prime numbers, the corresponding prime power sets are disjoint. Then one can write:

$$\forall\{p,q\}\in\mathbb{P}\Rightarrow\mathbf{p}^{\mathbb{N}}\cap\mathbf{q}^{\mathbb{N}}=\varnothing. \tag{3}$$

2.2.2 The Union of Prime Power Sets

It is evident that the cardinality of every prime power set is equal to the cardinality of the whole set of natural numbers, that is:

$$\forall p\in\mathbb{P}:Card\left(\mathbf{p}^{\mathbb{N}}\right)=Card\left(\mathbb{N}\right). \tag{4}$$

While $\mathbb{U}$ the union of all the prime power sets is defined as:

$$\mathbb{U}=\bigcup_{\forall p\in\mathbb{P}}\mathbf{p}^{\mathbb{N}}, \tag{5}$$

when observed as a collection of natural numbers, produces what could be named a prime power set *paradox*.

Because not all natural numbers are contained in $\mathbb{U}$, therefore:

$$\mathbb{U} \subset \mathbb{N}, \tag{6}$$

then one might admit that:

$$Card(\mathbb{U}) \leq Card(\mathbb{N}), \tag{7}$$

but at the same time, by construction, every prime power subset contained in $\mathbb{U}$ has the cardinality of the natural number set.

Moreover, one can also write:

$$\mathbb{P} \subset \mathbb{U}. \tag{8}$$

2.2.3 Natural Powers of the Prime Set

The nature of the set of prime powers can be made more explicit by considering the set $\mathbb{U}$ constructed differently. By the definitions of the previous sections, one can also think of defining the *natural powers of the prime set*: $\mathbb{P}^r$, as follows:

$$\forall r \in \mathbb{N} : \mathbb{P}^r = \{2^r, 3^r, 5^r, \ldots p^r, \ldots\}. \tag{9}$$

The sequence of such powers starts at:

$$r = 1 : \mathbb{P}^1 \equiv \mathbb{P}, \tag{10}$$

then the definition above is the same as supposing that there are several sets $\mathbb{P}^r$, as one can compute any natural power of the elements of the prime set $\mathbb{P}$. As many as the cardinality of the natural number set $\mathbb{N}$. Every set $\mathbb{P}^r$ associated with the power of any prime number has the cardinality of the prime set itself:

$$\forall r \in \mathbb{N} : Card(\mathbb{P}^r) = Card(\mathbb{P}). \tag{11}$$

Then, one can construct the union of all the powers of the primes sets $\mathbb{P}^r$, which shall be coincident with the union of all prime power sets:

$$\mathbb{U} = \bigcup_{\forall r \in \mathbb{N}} \mathbb{P}^r. \tag{12}$$

However, the cardinality of the union of powers of the prime set is at least the one corresponding to the natural number set:

$$Card(\mathbb{U}) \geq Card(\mathbb{N}), \tag{13}$$

a result that paradoxically contradicts the previous cardinality considerations on the union set $\mathbb{U}$.

This paradoxical situation appears again from this alternative point of view.

2.3 The Matrix of Powers of Prime Numbers

One can reorder the two ways of constructing the union of the powers of prime numbers into an array $\mathbf{U}$ of dimensions: $\left(Card\left(\mathbb{P}\right)\times Card\left(\mathbb{N}\right)\right)$, with elements defined in the following way:

$$\mathbf{U}=\left\{u_{IJ}=\left(p_I\right)^J\middle|p_I\in\mathbb{P}\middle|I=1,Card\left(\mathbb{P}\right);J=1,Card\left(\mathbb{N}\right)\right\},\tag{14}$$

where the rows of the matrix $\mathbf{U}$ are made by the ordinal index of the *I*-th prime number, and the columns of the matrix $\mathbf{U}$ follow the natural powers to which the prime number in question is raised.

One can write a partial matrix $\mathbf{U}$ example using this finite sample:

$$\begin{pmatrix} \ldots & 2^3 & 2^2 & 2 \\ \ldots & 3^3 & 3^2 & 3 \\ \ldots & 5^3 & 5^2 & 5 \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix},\tag{15}$$

noting that one has reversed the ordering of columns from the customary way: using left to right order, while the usual up-to-down order for rows is maintained. The reason for reversing the column order will be evident when the related natural vector spaces are constructed below in the next section.

2.4 Ordering the Prime Power Sets and the Powers of the Prime Set

Considering the set $\mathbb{P}$ ordered, then the set $\mathbb{U}$ can be supposedly (supra)ordered, so one can write:

$$\mathbb{U}=\left\{\mathbf{2}^{\mathbb{N}},\mathbf{3}^{\mathbb{N}},\mathbf{5}^{\mathbb{N}},\ldots,\mathbf{p}^{\mathbb{N}},\ldots\right\}\Rightarrow\mathbf{2}^{\mathbb{N}}\prec\mathbf{3}^{\mathbb{N}}_{,}\prec\mathbf{5}^{\mathbb{N}}\ldots\prec\mathbf{p}^{\mathbb{N}}\ldots\quad,\tag{16}$$

with the symbol $\prec$ indicating a prime power set's (supra)order.

Of course, one can also (supra)order the set $\mathbb{U}$ according to its alternative construction as the union of the powers of the prime set, using the order of the natural powers of the prime set:

$$\mathbb{U}=\left\{\mathbb{P}^1,\mathbb{P}^2,\mathbb{P}^3,\ldots\mathbb{P}^r,\ldots\right\}\Rightarrow\mathbb{P}^1\prec\mathbb{P}^2\prec\mathbb{P}^3\prec\ldots\mathbb{P}^r\prec\ldots\tag{17}$$

2.5 The Complement of the Union of Prime Power Sets

The whole set of prime powers sets: $\mathbb{U}$, when compared with the set of natural numbers, permits writing the construction of a new set $\mathbb{K}$, say, which one can identify as the complement of the set $\mathbb{U}$, and defined like:

$$\mathbb{K}=\mathrm{C}\left[\mathbb{U}\right]\Rightarrow\forall x\in\mathbb{K}:x\notin\mathbb{U}.\tag{18}$$

Therefore, this is the same as considering the set $\mathbb{K}$ formed by **all** natural composite numbers, which are **not** expressible as a **unique** prime number power.

One must take into account that with this definition of the set $\mathbb{K}$, one can write:

$$\mathbb{N} = \mathbb{U} \bigcup \mathbb{K} \wedge \mathbb{U} \bigcap \mathbb{K} = \varnothing . \tag{19}$$

2.6 Some Thoughts about Prime Number Powers and Composite Natural Numbers

The nature of the elements of the prime power sets is such that one can separate them from the composite natural numbers.

Powers of prime numbers are composite numbers whose divisors are restricted to be the prime source or some of its adequate powers.

Thus, the prime powers have some *intermediate* character between the prime numbers and the composite numbers, which one can suppose constructed at least with the product of ***two*** *distinct* prime number powers.

Therefore, it seems adequate to partition natural numbers into three different classes: primes, prime powers, (without the first power) and composites.

III. NATURAL PRIME POWER SPACES

These preliminary considerations about composite natural numbers, primes, and their power sets, permit the definition of a vector space structure defined over the set $\mathbb{U}$, the union of prime power sets, as defined in equations (16) and (17).

3.1 Introduction

First, one can outline a *natural prime power space* as a set of infinite-dimensional vectors possessing a structure in the form of a row[2] (or column), with their components chosen over the elements belonging to the union of the prime power sets $\mathbb{U}$.

One can note an infinite-dimensional natural prime power space with the symbol: $\mathrm{V}_{\infty}(\mathbb{U})$. Also, to shorten this verbose naming, one can refer along this paper to a (natural) (prime) power (vector) space. That is: a *power space*, the words in parenthesis elided for literary convenience.

3.2 Canonical Basis Set

As it is well-known, one defines the canonical basis set of such a power space as an infinite set of infinite-dimensional vectors forming the row vector set **E**:

$$\forall I \in \mathbb{N} : \left\langle \mathbf{e}_I \right| = \left(0,0,\ldots,0,1,0\ldots,0\right) \Rightarrow \left\langle \mathbf{e}_I \right| = \left\{ e_{JI} = \delta_{JI} \middle| \forall J \in \mathbb{N} \right\}. \tag{20}$$

---

[2] Here, Dirac's bra symbol: $\langle \mathbf{v} |$ is used to denote row vectors. Column vectors, if needed, are written with the ket symbols: $| \mathbf{v} \rangle$. That means the relationship: $\langle \mathbf{v} | = | \mathbf{v} \rangle^{T}$ and $| \mathbf{v} \rangle = \langle \mathbf{v} |^{T}$, holds; with the supraindex T meaning transposition. The present work will represent prime number collections as row vectors.

All the components in one vector belonging to the infinite canonical basis set are null $\{0\}$, except for a unique unit $\{1\}$ element at some fixed position associated with a natural order.

One can order the canonical basis vectors set: $\mathbf{E}=\{\langle\mathbf{e}_I\| \forall I \in \mathbb{N}\}$ according to the order of natural numbers. This ordering corresponds to a construction where the *I*-th canonical vector has the unit $\{1\}$ coincident with the *I*-th vector component.

From the usual definition of the Euclidean scalar product point of view, the canonical basis set is orthonormalized, and one can write:

$$\forall\{I,J\}\in\mathbb{N}:\langle\mathbf{e}_I|\mathbf{e}_J\rangle=\delta_{IJ}\,, \tag{21}$$

where $\forall\{I,J\}\in\mathbb{N}:\delta_{IJ}$ is a Kronecker's delta.

Suppose a square infinite-dimensional unit matrix, whose elements are Kronecker's delta symbols: $\mathbf{I}_\infty=\{I_{\infty;IJ}=\delta_{IJ}|\forall\{I,J\}\in\mathbb{N}\}$, the rows (or columns) of such a matrix corresponding to the canonical basis set.

3.3 Canonical Prime Vectors

One can associate every vector of the canonical basis set $\mathbf{E}$ with an ordered prime number just using the simple rule, where the unit component position is substituted by a prime number, like in the ordered sequence:

$$\begin{gathered}\langle\mathbf{2}_1|=(0,\ldots,0,0,0,2);\langle\mathbf{3}_2|=(0,\ldots,0,0,3,0);\langle\mathbf{5}_3|=(0,\ldots,0,5,0,0);\ldots\\ \ldots\langle\mathbf{p}_I|=(0,\ldots,0,p_I,0,\ldots0);\ldots\end{gathered} \tag{22}$$

where the vector subindices correspond to the natural order of the prime numbers, that is: $p_I\in\mathbb{P}$ corresponds to the *I*-th prime number. The row vector set: $\mathbf{P}=\{\langle\mathbf{p}_I\|\forall I\in\mathbb{N}\}$ can be called the *canonical prime* vector set. The canonical prime vector set forms an orthogonal set:

$$\forall\{I,J\}\in\mathbb{N}:\langle\mathbf{p}_I|\mathbf{p}_J\rangle=\delta_{IJ}\,p_I^2\,, \tag{23}$$

whose Euclidean norms coincide with the squared prime number associated with the involved canonical prime vector.

The canonical prime vectors can be alternatively defined as products of a canonical base vector by the corresponding ordered natural prime number, that is:

$$\forall I\in\mathbb{N}:\langle\mathbf{p}_I|=(0,\ldots,0,p_I,0,\ldots0)=p_I\langle\mathbf{e}_I|\,. \tag{24}$$

3.4 Dimension and Inner Dimension

The above-described construction of the power space original or the prime canonical vectors supposes every vector being of *infinite* dimension, that is, for instance:

$$\forall\langle \mathbf{e}_I| \in \mathbf{E} \wedge \forall\langle \mathbf{p}_I| \in \mathbf{P} \Rightarrow \langle \mathbf{e}_I| \in \mathrm{V}_\infty(\mathbb{U}) \wedge \langle \mathbf{p}_I| \in \mathrm{V}_\infty(\mathbb{U}). \tag{25}$$

Then one can assume the existence associated with some power space elements of a vector's *inner* dimension, made by the number of components different from zero within a given power space vector.

Considering this, the inner dimension of each original or prime canonical vector is one.

3.5 Homotheties of the Canonical Prime Vectors

Also, one can study the homotheties of the prime canonical basis set vectors by the powers of the prime source number involved in each case. One can express these homothetic vectors as:

$$\forall p_I^r \in \mathbb{U}: \langle \mathbf{p}_I^r| = (p_I)^{r-1}\langle \mathbf{p}_I| = \left(0,\dots,0,(p_I)^r,0,\dots,0\right) = (p_I)^r\langle \mathbf{e}_I|. \tag{26}$$

According to the previous point of view, the inner dimension of the homothetic prime vectors is also the unit. They are monodimensional, say. Also, these vectors correspond to an orthogonal set whose Euclidian norms are the squared prime power:

$$\forall\{I \neq J\} \in \mathbb{N}: \langle \mathbf{p}_I^r | \mathbf{p}_J^s \rangle = 0 \wedge \forall\{I,r\} \in \mathbb{N}: \langle \mathbf{p}_I^r | \mathbf{p}_I^r \rangle = p_I^{2r}. \tag{27}$$

3.6 The Sum of Vectors in Power Spaces

One can define the *pseudosum* between two prime canonical basis set vectors, or their homotheties, as the commutative operation expressed by:

$$\forall I \neq J: \langle \mathbf{p}_I^r| \oplus \langle \mathbf{q}_J^s| = (1-\delta_{IJ})\left(0,\dots,0,(p_I)^r,0,\dots,(q_J)^s,0,\dots,0\right), \tag{28}$$

which requires a different symbol from the usual vector addition.

Because, in the natural power space definition case, the sum of two (or more) vectors possessing in the same position a non-zero component is forbidden, which is the same as saying it doesn't exist by definition.

Considering this, the pseudosum of two canonical prime vectors, or their homotheties, results in a vector with an inner dimension of two.

Of course, one can extend the pseudosum to any number of vectors belonging to the homothetic prime canonical basis set up to the chosen inner dimension of the resultant vector.

For example, one can express the construction of a vector of inner dimension N as:

$$\{I_K | K=1,N\} \subset \mathbb{N} \wedge \{r_K | K=1,N\} \subset \mathbb{N}: \langle \mathbf{x}_N| = \bigoplus_{K=I_1}^{I_N} \langle \mathbf{p}_K^{r_K}|, \tag{29}$$

provided that the set of subindices corresponds to *different* prime order numbers. In this way, one can generate any power space vector within a chosen inner dimension.

3.7 Projection of a Pseudospace Vector into the Internal Space

One can project any natural power space row vector with inner space dimension N into an N-dimensional power space by using a projector matrix of dimension $\left(\infty \times N\right)$ constructed as:

$$\mathbf{J}=\left(\left|\mathbf{e}_{I_N}\right\rangle,\left|\mathbf{e}_{I_{N-1}}\right\rangle,\ldots,\left|\mathbf{e}_{I_2}\right\rangle,\left|\mathbf{e}_{I_1}\right\rangle\right). \tag{30}$$

The columns of the projection matrix $\mathbf{J}$ correspond to the canonical basis set vectors, coincident with the non-null components positions of the power space vector of inner dimension N.

Then for any power space vector, there exists a projector matrix $\mathbf{J}$ that transforms the infinite-dimensional vector into a finite-dimensional one:

$$\forall\left\langle\mathbf{v}_\infty\right|\in \mathrm{V}_\infty\left(\mathbb{U}\right)\Rightarrow\exists\mathbf{J}_{\left(\infty\times N\right)}:\left\langle\mathbf{v}_\infty\right|\mathbf{J}_{\left(\infty\times N\right)}=\left\langle\mathbf{v}_N\right|\in \mathrm{V}_N\left(\mathbb{U}\right). \tag{31}$$

3.8 Linear Combinations of the Power Space Canonical Basis Set

One can restate the paragraphs above with more straightforward vector space algebra.

Taking into account that one might express any power space vectors as a linear combination of vectors of the canonical basis set with coefficients, the coordinates with respect to such a basis set, taken from the union of prime powers set $\mathbb{U}$:

$$\Pi=\left\{p_{I_K}^{r_K}\middle|K=1,N\right\}\subset\mathbb{U}\Rightarrow\sum_{K=1}^{N}p_{I_K}^{r_K}\left\langle\mathbf{e}_{I_K}\right|=\left\langle\mathbf{v}\right|\in \mathrm{V}_\infty\left(\mathbb{U}\right). \tag{32}$$

It is simple to consider the set $\prod$, made by arbitrarily chosen elements of the set $\mathbb{U}$, as the coordinates of the vector $\left\langle\mathbf{v}\right|$ concerning the canonical basis set $\mathbf{E}$.

The projection of the vector $\left\langle\mathbf{v}\right|$ results in an N-dimensional vector, with inner dimension N, coincident with the cardinality of $\prod$.

That is one can write:

$$\left\langle\mathbf{v}\right|\mathbf{J}=\left\langle\mathbf{u}\right|\in \mathrm{V}_N\left(\mathbb{U}\right)\to\left\langle\mathbf{u}\right|=\left(p_{I_N}^{r_N},\ldots,p_{I_2}^{r_2},p_{I_1}^{r_1}\right). \tag{33}$$

3.9 Geometrical Norm of a Projected Power Space Vector

To every projected power space N-dimensional vector, one can associate a *geometrical norm* by the easily defined algorithm:

$$\forall \langle \mathbf{v}_N | = \left(v_N, v_{N-1}, \dots, v_K, \dots, v_2, v_1\right) \in \mathbf{V}_N(\mathbb{U}) \Rightarrow$$
$$\mathrm{G}\left[\langle \mathbf{v}_N |\right] = \prod_{K=1}^{N} v_K \to \mathrm{G}\left[\langle \mathbf{v}_N |\right] \in \mathbb{K} . \tag{34}$$

It is easy to see that the natural values of the geometrical norm of a projected vector of a power space generate the *composite natural numbers* contained in the set $\mathbb{K}$.

To these elements, one can add the prime powers contained in the set $\mathbb{U}$ when also considering the geometrical norm of the homotheties of the canonical basis set.

3.10 The Odd and Even Natural Power Spaces

The constructed vectors of a natural power space can have or do not have a non-null component at the position of the first prime number $\{2\}$, associated in turn with the $\langle \mathbf{e}_1 |$ canonical or prime canonical vector $\langle \mathbf{2}_1 |$. All the vectors lacking the components in the first position produce, via the corresponding projections and their geometric norms, prime power, or composite **odd** natural numbers. The projected odd vectors have the form:

$$\langle \mathbf{o} | = \left(\langle \mathbf{v} |, 0\right).$$

Therefore, all vectors with this characteristic create an odd natural power subspace. Calculus of their vector projections' geometric norms to the inner space yields odd composite numbers.

To every odd natural power subspace, there exists an **even** power subspace. One can obtain the vectors belonging to such an even structure from the subspace odd vectors by adding an element of the power set $\mathbf{2}^{\mathbb{N}}$ in the first zero component. That is, for example:

$$\langle \mathbf{e} | = \left(\langle \mathbf{v} |, 2^r\right) \tag{35}$$

Therefore, to every odd power subspace, one can associate an infinite sequence of even power subspaces, where every vector, instead of a zero element within the first component, has an element of the power set $\mathbf{2}^{\mathbb{N}}$.

Then, while the geometric norm of the odd vectors generates odd composite numbers, the same operation on even vectors generates even composite numbers.

3.11. The In and Out Natural Power Spaces

One can present the same odd-even scheme symbolically:

$$\left\{ Odd\left(\sim \exists \mathbf{2}^{\mathbb{N}}\right) \leftrightarrow Even\left(\exists \mathbf{2}^{\mathbb{N}}\right) \right\}, \tag{36}$$

that resumes the construction of odd and even spaces to any power space vector.

However, this possibility will also indicate that, from the point of view of the natural numbers structure, the power set $\mathbf{2}^{\mathbb{N}}$ might not necessarily be considered prevalent in front of any other odd prime power set $\mathbf{p}^{N}$.

In decimal representation, even numbers have been made significant when compared to odd numbers, because humans and other livings are two-sided. However, other creatures might be three, five, seven, or multiple-sided ... So, if they could be aware of it, the power sets: $\mathbf{3}^{\mathbb{N}}, \mathbf{5}^{\mathbb{N}}, \mathbf{7}^{\mathbb{N}}$, or $\mathbf{p}^{\mathbb{N}}$ ... can be as crucial as the set $\mathbf{2}^{\mathbb{N}}$.

In the present construction of the power spaces, one can contemplate power subspaces with the position, the *I*-th, say, corresponding to the *I*-th prime number p_I power set $\mathbf{p}_I^{\mathbb{N}}$, holding a zero $\{0\}$ instead of the corresponding prime powers.

Therefore, infinite power subspaces with the corresponding prime number zeroed from their ordered position exist. So, one can see the situation corresponding to the even numbers set is just one of many infinite possibilities.

One can thus propose a general binary classification of the kind:

$$\left\{Out\left(\sim \exists \mathbf{p}_I^{\mathbb{N}}\right) \leftrightarrow In\left(\exists \mathbf{p}_I^{\mathbb{N}}\right)\right\}, \tag{37}$$

when the prime number chosen in the above classification is $\{2\}$, the number sorting coincides with the customary *Odd-Even* one in the equation (36).

Such possibilities come from the fact that every prime number, with its power set, corresponds to one direction independent of the other in the power space. If the defined prime power space is isotropic, no direction of space has to be prevalent from the rest. One can choose any direction, and thus the same partition as customarily done with the even-odd classification of natural numbers can create two power space classes.

Of course, using large prime numbers to create alternative power subspaces to the even-odd structure, although on the same footing, might not be as enjoyable as those power subspaces created with the first prime numbers less than 20, say: $\{2,3,5,7,11,13,17,19\}$. And from this possibility, a less reduced one involving less than 10 four primes looks handy.

3.11.1 General Classification of Composite Numbers

Resuming what one discussed in the previous section: if the power set elements position $\mathbf{2}^{\mathbb{N}}$ produces composite numbers of even or odd structures, the elements of any power set $\mathbf{p}^{N}$ can do the same. They partition the power spaces into two classes, one divisible by some elements of the power set and others not. Precisely the same behavior as the even – odd numbers.

One can conclude that the even – odd classification is just a conventional particular way to study composite numbers. There are equivalent independent infinite ways to do the same partition with any prime number.

However, one can imagine a general partition of composite numbers corresponding to constructing them into classes that use the in – out binary classification discussed so far, but not referring to one prime number only, but two, three, ... and so forth.

A simple example will be the class of composite numbers not divisible by the elements of two prime power sets: $\{\mathbf{p}^{\mathbb{N}};\mathbf{q}^{\mathbb{N}}\}$, formed by the power space vectors, wherein the proper component positions of the primes $\{p,q\}$, one encounters zeros instead. Therefore, one can imagine a ternary, quaternary, ... composite number classification.

3.12 Structure of the Matrix of Prime Numbers Powers

In the equation , the already defined matrix **U**, holding all the powers of prime numbers, as defined in section *2.3*, can be revisited using the row and column structure of the prime powers described so far.

Therefore, one can write the prime powers matrix in two manners. Using the column set and the supraindex *T* as the transposition symbol to spare typing space:

$$\forall J\in\mathbb{N}:\left|\mathbf{p}^{[J]}\right\rangle=\left(2^J,3^J,5^J,\ldots p^J,\ldots\right)^T, \tag{38}$$

one can write the matrix **U** in the form of a row made of columns:

$$\mathbf{U}=\left(\ldots\left|\mathbf{p}^{[J]}\right\rangle,\ldots\left|\mathbf{p}^{[3]}\right\rangle,\left|\mathbf{p}^{[2]}\right\rangle,\left|\mathbf{p}^{[1]}\right\rangle\right). \tag{39}$$

Also, defining the matrix **U** row vector set as:

$$\forall p_I\in\mathbb{P}:\left\langle\mathbf{p}_I^{\mathbb{N}}\right|=\left(\ldots,p_I^J,\ldots,p_I^3,p_I^2,p_I^1\right), \tag{40}$$

one can also write the matrix **U** like a column made of rows:

$$\mathbf{U}=\left(\left\langle\mathbf{2}^{\mathbb{N}}\right|,\left\langle\mathbf{3}^{\mathbb{N}}\right|,\left\langle\mathbf{5}^{\mathbb{N}}\right|,\ldots,\left\langle\mathbf{p}_I^{\mathbb{N}}\right|,\ldots\right)^T. \tag{41}$$

3.12.1 Alternative Basis Sets of $\mathrm{V}_\infty(\mathbb{U})$.

The vectors defined in equations (38) and (40) correspond to infinite sets of infinite vectors belonging to $\mathrm{V}_\infty(\mathbb{U})$. However, vectors in the equation (38) are in column form for the convenience of constructing the matrix $\mathbf{U}$ columns. One can easily transpose them, forming rows, as the vectors used in previous sections. In each of the (38) and (40) vector series, every vector is linearly independent of the rest. Thus, one can alternatively use both sequences as basis sets of the space $\mathrm{V}_\infty(\mathbb{U})$.

As an example, take two vectors of the sequence : $\{\langle\mathbf{p}_I|,\langle\mathbf{p}_K|\}$. To show the linear independence of the vectors is sufficient to see that any (2×2) determinant with one row from the first vector and another from the second is not null:

$$\forall I \neq K \wedge \forall R \neq S : Det\begin{vmatrix} p_I^R & p_I^S \\ p_K^R & p_K^S \end{vmatrix} = p_I^R p_K^S - p_I^S p_K^R \neq 0 . \tag{42}$$

One can say the same about the sequence vectors.

Consequently, the columns and rows of matrix **U** are two sets of linearly independent vectors, and one can say that because of this, the determinant of this matrix is non-null: $Det|\mathbf{U}| \neq 0$.

IV. NATURAL POWER OF PRIMES SPACES

4.1 Mersenne Numbers: Definition and Connection with Binary Representation.

Mersenne numbers are natural numbers that, in a decimal base, can be calculated as the set of powers of 2 minus one unit:

$$\forall r \in \mathbb{N} : M_{(-)}(r) = 2^r - 1 . \tag{43}$$

They have another interesting property. When represented in binary form can be written as the binary unity vector of dimension *r*: $\langle \mathbf{1}_r |$, see, for example [10,14]. So, the unity vector $\langle \mathbf{1}_r |$ representing the vertex of an *r*-dimensional Boolean hypercube, maximally far away from the corresponding origin or binary zero vector of the same dimension: $\langle \mathbf{0}_r |$. The many applications and properties of zero and unity vectors [18-22] and the Mersenne numbers have been studied in our laboratory as two elements forming part of Boolean hypercube vertices.

4.2. Mersenne Numbers in the Prime Power Space

Due to the definition of Mersenne numbers, one can quickly write the whole set $\mathbf{M}_{(-)}^{\mathbb{N}}$ of Mersenne numbers with a simple algorithm using the vectors of the prime power spaces. For example:

$$\langle \mathbf{M}_{(-)}^{\mathbb{N}} | = \langle \mathbf{2}^{\mathbb{N}} | - \langle \mathbf{1}_{\mathbb{N}} | . \tag{44}$$

Where one uses row vectors to contain the implied sets. That is: $\langle \mathbf{1}_{\mathbb{N}} |$ contains a vector of the dimension of the natural number cardinality, made entirely by ones $\{1\}$. The already described vector $\langle \mathbf{2}^{\mathbb{N}} |$ contains the ordered set of powers of 2, acting as coordinates associated with the canonical basis set:

$$\langle \mathbf{2}^{\mathbb{N}} | = \left(\dots, 2^K, \dots, 2^3, 2^2, 2 \right) = \bigoplus_{K=1}^{\infty} 2^K \langle \mathbf{e}_K | . \tag{45}$$

Finally, the vector $\langle \mathbf{M}_{(-)} |$ holds all the elements forming the set of Mersenne numbers, which also can be written as:

$$\begin{aligned}\left\langle \mathbf{M}_{(-)}^{\mathbb{N}} \right| &= \underset{K=1}{\overset{\infty}{\oplus}} M(K)\langle \mathbf{e}_K| = \underset{K=1}{\overset{\infty}{\oplus}} \left(2^K - 1\right)\langle \mathbf{e}_K| \\ &= \underset{K=1}{\overset{\infty}{\oplus}} 2^K \langle \mathbf{e}_K| - \underset{K=1}{\overset{\infty}{\oplus}} \langle \mathbf{e}_K| = \left\langle \mathbf{2}^{\mathbb{N}} \right| - \left\langle \mathbf{1}_{\mathbb{N}} \right|\end{aligned} \quad (46)$$

4.3. Mersenne Twins

In the same way that Mersenne numbers are defined, one can construct a set of Mersenne twins, see reference [14], which one can describe as:

$$\forall r \in \mathbb{N} : M_{(+)}(r) = 2^r + 1, \quad (47)$$

and the prime power space form is immediately written like this:

$$\left\langle \mathbf{M}_{(+)}^{\mathbb{N}} \right| = \left\langle \mathbf{2}^{\mathbb{N}} \right| + \left\langle \mathbf{1}_{\mathbb{N}} \right|. \quad (48)$$

Mersenne twins also have a characteristic binary representation, which one can write as the $(r+1)$ -dimensional bit string: $\left(1, \left\langle \mathbf{0}_{(r-1)} \right|, 1\right)$. Note that the canonical basis set **E**, now with every vector taken in binary form as a bit string, is the basis that permits the expression of natural numbers as sums of powers of two.

And this one can make it as in the previous equations used to express the Mersenne numbers in terms of the binary canonical basis set.

4.4. The Linear Independence of Mersenne Vectors ad other Questions

The vector pair defined in equations (44) and (48) constitute two linearly independent vectors as any (2×2) determinant with two elements of the Mersenne vector and two taken from the Mersenne twin vector is non-zero:

$$\forall p \neq q : Det \begin{vmatrix} 2^p - 1 & 2^q - 1 \\ 2^p + 1 & 2^q + 1 \end{vmatrix} = 2\left(2^p - 2^q\right) \neq 0. \quad (49)$$

Therefore, the vector pair: $\left\{ \left\langle \mathbf{M}_{(-)}^{\mathbb{N}} \right|, \left\langle \mathbf{M}_{(+)}^{\mathbb{N}} \right| \right\}$ is linearly independent. The elements of such vectors play different roles in natural number generation [14,19,20] and related subjects [21,22].

Moreover, several elements of $\left\langle \mathbf{M}_{(-)}^{\mathbb{N}} \right|$ are prime and well-studied; see, for example [23]. The same occurs with the twin counterpart vector $\left\langle \mathbf{M}_{(+)}^{\mathbb{N}} \right|$ but has not been so exhaustively studied, possibly because of its binary representation and scarcity of primes in the sequence compared with the Mersenne counterpart.

As evident upon observing both vectors' structure, the basic vector in natural number generation corresponds to the elements of the power vector $\left\langle \mathbf{2}^{\mathbb{N}} \right|$, as one has previously analyzed [14,19].

The use of vector $\left\langle \mathbf{2}^{\mathbb{N}} \right|$ elements to recursively generate natural numbers has been discussed in deep [19] and will not be repeated.

4.5. Extended Mersenne Numbers

Although the representation of numbers constructed with a similar algorithm to Mersenne's does not present a simplistic binary representation, they can also be a source of large prime numbers. Some previous study has been performed [14,18]. Still, it is interesting to rewrite in the present notation and framework the possibility of writing extended Mersenne numbers involving the rest of the primes.

One can observe that, for instance, one can construct a set of odd numbers using the prime number 3 as a source, like:

$$\forall r \in \mathbb{N} : L\left(r\right) = 3^{r} \pm 2 , \tag{50}$$

which can be associated with a vector structure similar to the one used in the Mersenne twin case, as one can write instead of the equation (50), the expression:

$$\left\langle \mathbf{L}_{3}^{\mathbb{N}} \right| = \left\langle \mathbf{3}^{\mathbb{N}} \right| \pm 2 \left\langle \mathbf{1}_{\mathbb{N}} \right| . \tag{51}$$

Some of these extended Mersenne twins can be prime, like the large one obtained in the following way:

$$3^{123} + 2 = \\ 48519278097689642681155855396759336072749841943521979872829 ,$$

which is composed of 59 digits. The corresponding twin with a negative form is a composite number.

This situation permits constructing a superfamily of extended Mersenne numbers using the notation that one has used.

Then the sequence:

$$\forall p \in \mathbb{P} : \left\langle \mathbf{L}_{p\ell}^{\mathbb{N}} \right| = \left\langle \mathbf{p}^{\mathbb{N}} \right| \pm \left(\ell = 2, p-1; 2 \right) \left\langle \mathbf{1}_{\mathbb{N}} \right| , \tag{52}$$

provides a set of extended Mersenne vectors depending on the number of possible vectors homothetic to the unity vector $\left\langle \mathbf{1}_{\mathbb{N}} \right|$.

V. EXTENDED APPLICATIONS OF NATURAL NUMBERS POWER SETS

In the present section, one will study two applications of natural number powers to show the far-reaching potential of what has been commented on until now.

5.1 Fermat's Theorem and its Extension.

This section will discuss the extension of the Fermat theorem to higher dimensions than the usual one having the origin of the so-called Pythagorean triples. One can find previous work in our laboratory in the references [13,15,16].

5.1.1 Sums of Powers of the Natural Numbers Set

Let's recall the natural number set $\mathbb{N}$. The symbol $\mathbb{N}^{[P]}$ means the *P*-th power of *all* the elements of the natural set $\mathbb{N}$.

That is, if $\mathbb{N}=\{1,2,3\ldots,n,\ldots\}$ then: $\mathbb{N}^{[P]}=\{1,2^P,3^P\ldots,n^P,\ldots\}$.

Therefore: $\mathbb{N}^{[P]}\subset\mathbb{N}$. In the same way, as one has discussed in the first paragraph of this work referred to the prime number set $\mathbb{P}$.

The exciting thing about this definition is the possibility to sum the elements of every natural power set an indefinite number of times. The present sum formalism now will differ from the earlier definition of the pseudosum connected with powers of prime numbers and acting on homothetic canonical basis vectors.

One can construct sets like: $\mathbb{S}^{[P]}_{[Q]}$, that is, sets of *Q* natural numbers, all *P* powered and ordered. One can write to express this operation of summing equal powers of *Q* natural numbers as:

$$\forall\{P,Q\}\in\mathbb{N}:\mathbb{S}^{[P]}_{[Q]}=\left\{s^P_K \,\middle|\, K=1,Q\right\}\subset\mathbb{N}^{[P]}\Rightarrow S^{[P]}_{[Q]}=\left\langle\mathbb{S}^{[P]}_{[Q]}\right\rangle=\sum_{K=1}^{Q}s^P_K. \tag{54}$$

However, this does not imply that all the elements of the resultant sums $S^{[P]}_{[Q]}$ belong necessarily to $\mathbb{N}^{[P]}$.

For example, one can write the two-dimensional Fermat's theorem as:

$$\forall P\in\mathbb{N}\wedge P>2:S^{[P]}_{[2]}=\left\langle\mathbb{S}^{[P]}_{[2]}\right\rangle\notin\mathbb{N}^{[P]}. \tag{55}$$

But it is empirically true, see for instance [13], that one can also write the *Q*-dimensional version of the usual Fermat's theorem:

$$\forall Q\in\mathbb{N}\wedge Q\geq 2:\exists S^{[2]}_{[Q]}=\left\langle\mathbb{S}^{[2]}_{[Q]}\right\rangle\Rightarrow S^{[2]}_{[Q]}\in\mathbb{N}^{[2]}. \tag{56}$$

The same is empirically admissible [15] for higher powers and sums, like:

$$\forall P\in\mathbb{N}\wedge P>3:S^{[P]}_{[3]}=\left\langle\mathbb{S}^{[P]}_{[3]}\right\rangle\notin\mathbb{N}^{[P]}, \tag{57}$$

and also, it can be written that:

$$\forall Q \in \mathbb{N} \wedge Q \geq 3 :\Rightarrow \exists S_{[Q]}^{[3]} = \left\langle \mathbb{S}_{[Q]}^{[3]} \right\rangle \in \mathbb{N}^{[3]}, \tag{58}$$

it corresponds to behavior similar to Fermat's theorem but involves sums of $\mathbb{N}^{[3]}$ elements.

Perhaps this comportment can be generalized in the terms:

$$\forall P \in \mathbb{N} \wedge P > N : S_{[N]}^{[P]} = \left\langle \mathbb{S}_{[N]}^{[P]} \right\rangle \notin \mathbb{N}^{[P]} \tag{59}$$

and

$$\forall Q \in \mathbb{N} \wedge Q \geq N :\Rightarrow \exists S_{[Q]}^{[N]} = \left\langle \mathbb{S}_{[Q]}^{[N]} \right\rangle \in \mathbb{N}^{[N]}. \tag{60}$$

Although such two statements have not been fully proved, even empirically, one shall understand them as conjectures.

There is an exception when $N = 4$ where it seems, after a large sample of tested cases, that:

$$\exists S_{[4]}^{[5]} = \left\langle \mathbb{S}_{[4]}^{[5]} \right\rangle \in \mathbb{N}^{[5]}, \tag{61}$$

from vast computational experience, it seems that one can also write:

$$\forall P > 5 : S_{[4]}^{[P]} = \left\langle \mathbb{S}_{[4]}^{[P]} \right\rangle \notin \mathbb{N}^{[P]}. \tag{62}$$

Provisionally, in the light of the gathered empirical results, one can write a generalized Fermat theorem provided that the condition $\forall N \geq 5$ holds:

$$\forall P \in \mathbb{N} \wedge P > N > 5 : S_{[N]}^{[P]} = \left\langle \mathbb{S}_{[N]}^{[P]} \right\rangle \notin \mathbb{N}^{[P]} \tag{63}$$

and

$$\forall Q \in \mathbb{N} \wedge Q \geq N :\Rightarrow \exists S_{[Q]}^{[N]} = \left\langle \mathbb{S}_{[Q]}^{[N]} \right\rangle \in \mathbb{N}^{[N]}. \tag{64}$$

Due to the combinatorial explosion, expression (64) appears very hard to test for large natural numbers.

5.1.2. Minkowski Spaces and Extended Fermat Theorem

Natural vectors might possess a zero norm in Minkowski spaces; see, for example, [12,16,25]. Therefore, one might define the Fermat theorem in a $[2+1]$ space, that is, a three-dimensional space, not a two-dimensional one...

The notation $[N+M]$ in the context of Minkowski spaces corresponds to one of such structures possessing *N*-dimensional Euclidian and *M*-dimensional Minkowskian parts.

Concerning understanding the idea of the Minkowskian part, one supposes to write the metric vector in one of these spaces as the row vector: $\left(\langle \mathbf{1}_N | ; - \langle \mathbf{1}_M |\right)$.

In the following extended Fermat theorem discussion, one will use Minkowski spaces of the kind: $\left[N+1\right]$.

Let us suppose that one can describe the possible Fermat vectors by the symbol: $\left\{\left[N+1\right],P\right\}$ involving the dimension of the space and the power of the natural number set.

Then a set of vectors can be defined, like: $\mathbb{X}\left\{\left[N+1\right],P\right\}$. One can construct vectors with elements of the natural power set $\mathbb{N}^{[P]}$ such that: (1) do not contain 0 nor 1, (2) are all different, and (3) possess an ascending order.

Such vectors might be called perfect whole vectors [9,24]. One might define them as:

$$\begin{gathered}\left\langle\mathbf{x}\right|\in\mathbb{X}\left\{\left[N+1\right],P\right\}\Rightarrow\left\langle\mathbf{x}\right|=\left(x_1^P;x_2^P;x_3^P;\ldots x_N^P;x_{N+1}^P\right)\\ \wedge\left\{x_I^P\left|I=1,N+1\right.\right\}\subset\mathbb{N}^{[P]}\wedge x_1^P<x_2^P<x_3^P<\ldots x_N^P<x_{N+1}^P\end{gathered}. \tag{65}$$

One can suppose these vectors to belong to a vector space $\mathrm{V}_{(N+1)}\left(\mathbb{N}^{[P]}\right)$ where scalar products are subject to a metric vector which one can write like $\left(\left\langle\mathbf{1}_N\right|;-1\right)$.

Then a Minkowski norm over the set $\mathbb{X}\left\{\left[N+1\right],P\right\}$ is simply well-defined corresponding to the algorithm:

$$M\left[\left\langle\mathbf{x}\right|\right]=\left\langle\left\langle\mathbf{m}\right|*\left\langle\mathbf{x}\right|\right\rangle=\left(\sum_{I=1}^{N}x_I^P\right)-x_{N+1}^P\Leftarrow\left\langle\mathbf{m}\right|=\left(\left\langle\mathbf{1}_N\right|;-1\right), \tag{66}$$

such that one can define a Fermat vector of order P as:

$$\left\langle\mathbf{f}\right|\in\mathbb{X}\left\{\left[N+1\right],P\right\}\wedge M\left[\left\langle\mathbf{f}\right|\right]=0. \tag{67}$$

If one finds a Fermat vector, all its homotheties are Fermat vectors. That is, the following property holds:

$$\forall\left\langle\mathbf{f}\right|:M\left(\left\langle\mathbf{f}\right|\right)=0\Rightarrow\forall\lambda\in\mathbb{N}:M\left(\lambda\left\langle\mathbf{f}\right|\right)=0. \tag{68}$$

There must be a *simple* and general reason providing the clue about the growing scarcity or lack of Fermat vectors when the parameters N and P become large. If found, it could also be a crucial mathematical result that can illuminate the nature of the extension of the Fermat theorem.

5.1.3. Copies of the Whole Perfect Vectors

One can define copies of all the perfect whole [9,24] vectors in all the vector spaces defined over the powers of the natural number set $\mathbb{N}^{[P]}$.

One can consider the vector sets $\mathbb{X}\{[N+1],P\}$, starting from $N=2$ and with larger values, as copies of themselves. Meaning that, provided the arrangement of increasing powers:

$$\mathbb{X}\{[N+1],1\} \to \mathbb{X}\{[N+1],2\} \to \ldots\ \mathbb{X}\{[N+1],P\} \to \ldots, \tag{69}$$

a one-to-one correspondence connects every element of all the sets in the sequence, involving all dimensions and powers. But such a correspondence does not imply all the vectors within the sequence fulfill the property of being Fermat vectors.

5.2 Inward Inverse of a Prime Power Vector and the Euler-Riemann's Zeta Function

This section will present the connection between the vectors made of natural number powers with the Euler-Riemann function [1,26,27].

5.2.1. Inward Powers of the Prime Number Vector

Among the vector structures, which one can construct within the vector space $\mathrm{V}_\infty(\mathbb{U})$, appear the vectors whose elements are sequences of the same power, *r* say, associated with the prime number set $\mathbb{P}$. One can write such vectors as the columns of the matrix **U**, as defined in the equation (38),or use the transposed vectors directly in the form of the rows instead:

$$\forall r \in \mathbb{N} : \left\langle \mathbf{p}^{[r]} \right| = \left(\ldots, p^r, \ldots, 5^r, 3^r, 2^r\right) \in \mathrm{V}_\infty(\mathbb{U})\ . \tag{70}$$

One can obtain such kind of vectors from the initial prime number vector holding the ordered prime number set $\mathbb{P}$ as its elements:

$$\langle \mathbf{p}| = (\ldots, p, \ldots, 5, 3, 2) \in \mathrm{V}_\infty(\mathbb{U}) \tag{71}$$

through the inward power operation:

$$\left\langle \mathbf{p}^{[2]} \right| = \langle \mathbf{p}| * \langle \mathbf{p}| = (\ldots, p\cdot p, \ldots, 5\cdot 5, 3\cdot 3, 2\cdot 2) = \left(\ldots, p^2, \ldots, 5^2, 3^2, 2^2\right), \tag{72}$$

where one multiplies the vector elements in the same position. Thus, the repetition of the inward power produces the vectors of the equation :

$$\left\langle \mathbf{p}^{[r]} \right| = \overset{r}{\underset{K=1}{*}} \langle \mathbf{p}|. \tag{73}$$

5.2.2. Inverse Powers of the Prime Number Vector

The vector $\langle \mathbf{p}|$ was described in a previous paper as whole and perfect [9,24], meaning a vector with components ordered and non-zero. One has used a similar arrangement to study the vectors in extending the Fermat theorem in paragraph 5.1.

In this case, one can define a more general possibility by constructing the inward inverse of a given vector. In the present case, the inward inverses of the perfect vectors $\left\langle \mathbf{p}^{[r]} \right|$ are easy to define using the following algorithm:

$$\langle \mathbf{p}^{[-r]} | = \left(\ldots, p^{-r}, \ldots, 5^{-r}, 3^{-r}, 2^{-r} \right) \;, \tag{74}$$

where one can use as a source the inward inverse of the vector $\langle \mathbf{p} |$ as defined in the equation :

$$\langle \mathbf{p}^{[-1]} | = \left(\ldots, p^{-1}, \ldots, 5^{-1}, 3^{-1}, 2^{-1} \right), \tag{75}$$

upon the procedure of the inward power construction, which in this case is written like this:

$$\langle \mathbf{p}^{[-r]} | = \overset{r}{\underset{K=1}{*}} \langle \mathbf{p}^{[-1]} | . \tag{76}$$

Of course, the infinite vector set: $\left\{ \langle \mathbf{p}^{[-r]} | \,\middle|\, \forall r \in \mathbb{N} \right\}$ no longer belongs to the vector space $\mathrm{V}_\infty\left(\mathbb{U}\right)$ but to an extended one. To define this new vector space over all the powers described by the integer set, one can define the set $\mathbb{U}^{[-1]}$ simply using the following construct:

$$\forall p^r \in \mathbb{U} \Rightarrow p^{-r} \in \mathbb{U}^{[-1]} \tag{77}$$

in such a manner that:

$$\mathbb{W} = \mathbb{U} \bigcup \mathbb{U}^{[-1]} \subset \mathbb{Q} \,, \tag{78}$$

therefore, one can suppose that:

$$\forall p \in \mathbb{P} \wedge \forall s \in \mathbb{Z} : p^s \in \mathbb{W} \,. \tag{79}$$

Consequently, one can redefine the vector space of the prime powers as the vector space of the positive and negative prime powers $\mathrm{V}_\infty\left(\mathbb{W}\right)$.

Of course, the vector pair: $\left\{ \langle \mathbf{p}^{[r]} | ; \langle \mathbf{p}^{[-r]} | \right\}$ can be considered the inverse one from the other, and one can see that their inward product produces the *unity* vector of the appropriate dimension:

$$\langle \mathbf{p}^{[r]} | * \langle \mathbf{p}^{[-r]} | = \langle \mathbf{p}^{[-r]} | * \langle \mathbf{p}^{[r]} | = \langle \mathbf{1}_{Card(\mathbb{P})} | \,. \tag{80}$$

Furthermore, the scalar product of any whole perfect [9,24] vector and its inward inverse is the space dimension, as one can write:

$$\left\langle \langle \mathbf{p}^{[r]} | * \langle \mathbf{p}^{[-r]} | \right\rangle = \left\langle \langle \mathbf{p}^{[-r]} | * \langle \mathbf{p}^{[r]} | \right\rangle = \left\langle \langle \mathbf{1}_{Card(\mathbb{P})} | \right\rangle = Card\left(\mathbb{P}\right). \tag{81}$$

5.2.3. Splitting the Riemann Zeta Function

One can define the well-known Riemann zeta function $\zeta(z)$ as a complex variable function possessing a simple form:

$$\zeta(z)=\sum_{n=1}^{\infty} n^{-z}=\sum_{n\in\mathbb{N}} n^{-z}\,. \tag{82}$$

However, it can be split in two contributions, whenever one splits the natural number set into the prime set $\mathbb{P}$, which we have been dealing with in the present study, and the composite natural numbers, $\mathbb{K}$ say, in such a way that one can write:

$$\mathbb{N}=\mathbb{P}\cup\mathbb{K}\,, \tag{83}$$

therefore, one can rewrite the equation (82) like this:

$$\begin{gathered}\forall z\in\mathbb{C}:\ \zeta(z)=1+\pi(z)+\kappa(z)=\\ 1+\sum_{p\in\mathbb{P}} p^{-z}+\sum_{k\in\mathbb{K}} k^{-z}=1+\left(2^{-z}+3^{-z}+5^{-z}+\ldots\right)+\left(4^{-z}+6^{-z}+8^{-z}\ldots\right)\cdot\end{gathered} \tag{84}$$

Of course, the variable in the functions of the equation (84) is a complex number.

Still, one can also obtain some particular values of the Riemann function for variable values lying within the natural number set, as simply: $\mathbb{N}\subset\mathbb{C}$.

The Riemann function part possessing inverses of prime numbers has been described by Euler, constituting a previous construction leading Riemann to the function $\zeta(z)$ over the complex field variables.

One can find more information in references [2,27]. Interestingly enough, the Euler function $\pi(z)$, with the variable defined as $z\in\mathbb{N}$, diverges for $z=1$, but converges for $\forall z>1$.

5.2.4. The Euler Prime Function and Generalized Inward Powers of the Prime Vector.

Then, for natural numbers, the Euler prime function values: $\pi(s)$ are related to the set of prime vector powers: $\left\{\langle\mathbf{p}^{[s]}|\;|\forall s\in\mathbb{Z}\right\}$.

However, one can easily extend the inward powers of a vector within the real and complex fields, just extending the definitions of equations (72), (75), and (76) with a direct inward power provided with the complex field elements:

$$\forall z\in\mathbb{C}:\langle\mathbf{p}^{[\pm z]}|=\left(\ldots,p^{\pm z},\ldots,5^{\pm z},3^{\pm z},2^{\pm z}\right). \tag{85}$$

One can connect this last expression with Hadamard's [1,2,2828] extended idea of a function over a diagonal matrix, acting over an isomorphic vector form.

Once the extended diagonal powers of the equation defined, one can quickly obtain the Euler prime function $\pi(z)$, employing the complete sum of a vector, defined and applied in many cases, see references [12,16,25] for more details, along our research path, and in this paper.

5.2.5. Complete sum of a vector, norms, and multiple scalar products.

Indeed, the complete sum of any vector like:

$$\forall z\in\mathbb{C}:\left\langle \mathbf{p}^{[-z]}\right| = \left(\ldots,p^{-z},\ldots,5^{-z},3^{-z},2^{-z}\right), \tag{86}$$

say, can be particularly described by the sum:

$$\forall z\in\mathbb{C}:\left\langle\left\langle \mathbf{p}^{[-z]}\right|\right\rangle = 2^{-z}+3^{-z}+5^{-z}+\ldots+p^{-z}+\ldots=\sum_{\forall p\in\mathbb{P}} p^{-z} \tag{87}$$

which is coincident with the Euler prime function so that one can write:

$$\forall z\in\mathbb{C}:\pi(z)=\left\langle\left\langle \mathbf{p}^{[-z]}\right|\right\rangle\in\mathbb{C}. \tag{88}$$

One must remark now that the resultant vector constructed as in the equation shall be considered as belonging to a broader vector space: $\left\langle \mathbf{p}^{[-z]}\right|\in \mathrm{V}_{\infty}(\mathbb{C})$.

At the same time, the Euler prime function generally possesses values within the complex field, as the equation (88) indicates. One can extend such properties to the Riemann composite function $\kappa(z)$.

The complete sum of a vector, like in the equations (87) and (88), corresponds within the vector sets one has been working up to here to the sum leading to the Euler or Riemann functions. At the same time, one can associate a vector's complete sum with a first-order vector norm, a Minkowski norm, as described in the previous paragraph about the extension of Fermat's theorem.

More than this, the inward product of two or more vectors, being a vector, the result, if put under a complete sum operation, corresponds to a norm of the order of the number of vectors involved:

$$\left\langle \overset{r}{\underset{K=1}{*}}\left\langle \mathbf{p}\right|\right\rangle = \left\langle\left\langle \mathbf{p}^{[r]}\right|\right\rangle = \sum_{K=1}^{\infty} p_K^{r}\,. \tag{89}$$

The same can be said when the involved vectors are different. Suppose one has a vector set like: $\left\{\left\langle \mathbf{p}^{\mathbb{N}}_{I_K}\right| \,\middle\|\, K=1,N\right\}$, one can construct the scalar product involving N vectors using the simple algorithm:

$$\overset{N}{\underset{K=1}{*}}\left\langle \mathbf{p}^{\mathbb{N}}_{I_K}\right| = \left\langle \mathbf{q}^{\mathbb{N}}\right| \Rightarrow \left(\mathbf{p}^{\mathbb{N}}_{I_1};\mathbf{p}^{\mathbb{N}}_{I_2};\mathbf{p}^{\mathbb{N}}_{I_3};\ldots\mathbf{p}^{\mathbb{N}}_{I_N}\right) = \left\langle\left\langle \mathbf{q}^{\mathbb{N}}\right|\right\rangle\in\mathbb{N}\,. \tag{90}$$

The elements of the inward products vector $\left\langle \mathbf{q}^{\mathbb{N}} \right|$ also present the products of the chosen N primes, power to every natural number.

VI. DISCUSSION AND RESULTS

Along this work, one has studied the powers of prime numbers as a source to construct vectors whose elements conveniently manipulated yield composite natural numbers.

Initially, one has described the sets of powers of prime numbers as a first step to building up a Euclidian vector space.

The basis set made of the canonical basis homothecies of the prime numbers, and their natural powers can be presented, with the ancillary use of a geometric norm, as the origin of the composite numbers.

Several characteristics of some natural numbers have been discussed, among others, the possibility of describing the twins of Mersenne numbers and extending such initial Mersenne pairs to the whole set of prime numbers.

Furthermore, one has studied two applications of the powers of natural numbers.

First, in Fermat's theorem, transforming the original problem into a zero norm of a tri-dimensional vector.

Then, one has studied the extension to higher dimensions of the Fermat theorem, employing the whole natural number set. One achieves this by defining a natural Minkowski metric space, where vectors with null norms correspond to Fermat vectors, whose components fulfill the extended version of the original theorem.

Second, one has discussed the Euler and Riemann functions. One has described from the point of view of this paper the Euler function involving primes, using the whole natural set, and the Riemann function.

While one can initially keep the vectors belonging to natural spaces, the nature of the Euler and Riemann functions needs, at least, the use of natural numbers powers chosen as integers, constructed similarly as in the previous sections of the present work.

Starting from previously defined perfect whole [9,24] natural vectors containing natural numbers powers, one can construct inward inverse natural perfect whole vectors. However, the inward natural vector inverses induce the need to transport the initial algebraic tools from vector spaces defined over the natural numbers into at least vector spaces defined over the field of rational numbers.

With the variable defined over the complex field, the Riemann function furthers the original natural space simplicity into even more extended vector spaces. Therefore, one has not studied this issue in depth, as the present work essentially studies the humble set of natural numbers.

To perform the mentioned goals, one has constructed a suggestive theoretical framework where the natural numbers can be studied from many points of view. Generally speaking, one has used a set of well-known simple operations over natural whole perfect vectors.

Such operations are (1) the inward product and power of a vector, (2) a Minkowski norm related to the total sum of the components of a vector, (3) a geometric norm related to the product of the total components of a vector, and (4) the inward inverse of a vector.

Finally, it can be said, as a resumé, that a simple analysis of the natural powers of prime numbers allows for presenting a picture where vector spaces, Euclidian and Minkowskian, and natural numbers can be easily connected.

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Compliance with ethical standards

Conflict of interest: The author state that this work has no conflict of interest.

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ABSTRACT

Organic produce have of recent been among the positive outlook for health safety in bids to avoid the bad impact of synthetic insecticides to human and the environment. The use of non-chemical control measures has been practiced on sweetpotato weevils with uncertainty. Very often, farmers either do nothing or rely heavily on synthetic insecticides. Field experiments were conducted from 2015 to 2016 to assess the effectiveness of grass mulch in controlling sweetpotato weevils and millipede on sweetpotato tubers at TARI, Kibaha in Tanzania. Comparison was made with Dimethoate 40EC insecticide and a negative control treatment where no weevil management technique was applied. The treatments were tested on four popular and farmer preferred sweetpotato varieties; Ukerewe, Simama, Mataya and Kiegea established on naturally infested fields. Obtained results suggested that the use of mulch significantly ($P<0.05$) increased sweetpotato yield (weight) but did not reduce Cylas puncticollis infestation on roots. The influence of test varieties on C. puncticollis infestation were highly significant ($P<0.005$). The effect of dipping sweetpotato vines into Dimethoate 40EC against C. puncticollis and millipedes prior to planting was insignificant ($P>0.05$). Conclusively, neither grass mulch nor Dimethoate 40EC insecticides may offer effective control of sweetpotato weevils in pest endemic areas.

Keywords: cylas spp; pest management, sweetpotato, organic mulch, insecticides.

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Keywords: cylas spp; pest management, sweetpotato, organic mulch, insecticides.

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I. INTRODUCTION

Sweetpotato (*Ipomoea batatas* L. Lam) is a dicotyledonous plant of the family Convolvulaceae, perennial herb but it behaves as an annual herb when is cultivated (Bartolini 1985). It is cultivated in the tropical and subtropical zones and in warm temperate regions (Saranya et al. 2006). It is an important staple food and cash tuber crop in Africa and in the economy of poor households in Tanzania (Ndunguru and Kapinga 2007). The crop is a source of dietary fiber, complex carbohydrates, proteins, vitamins A, C and B, iron, calcium as well as making of industrial starch (Korada et al. 2010). The immature leaves are consumed as vegetable (Masumba et al. 2007), and also for animal feed (Etela et al. 2008). Furthermore, the crop is easy to produce at rural level with minimum input, due to its tolerance to a wide range of edaphic and climatic condition, low demands on soil nutrients, low requirements for external inputs such as fertilizers, flexibility in planting and harvesting periods (Lebot 2009). Sweetpotato is currently grown more in developing countries than any other root crop following active sweetpotato promotion for food security and its high nutritious substances toward alleviating vitamin A deficiency (VAD) by many governments (CIP 2009).

In Tanzania, sweetpotato ranks third among the most important root and tuber crops after cassava and Irish potato (Masumba et al. 2007). The major sweetpotato producing areas in the country are the regions surrounding Lake Victoria, Southern Highlands, Eastern, Western zone and Zanzibar (Ndunguru & Kapinga, 2007). Farmers preferences on selection of the varieties are based on parameters related to dry matter content, sweetness, marketability, early maturity, high yielding, root shape and size, vigour, fiber content and little pests infestation especially in the Central and Eastern zone farming (Masumba et al., 2007). According to FAOSTAT (2014), sweetpotato production in Tanzania increased from 1 322 000 metric tons in 2007 to 1 381 120 metric tons per annum in 2009. Although the production of sweetpotato has been shown to increase in Tanzania, the potential and sustainability of the crop has been significantly threatened by insect pests, diseases and limited availability of improved varieties (Ebregt et al. 2007). These threats have of recent resulted into a significant reduction in yield. Insect pests cause major losses to sweetpotato production in developing countries, Tanzania inclusive. Among the important insect pests of sweetpotato are weevils namely sweetpotato weevils (SPW) the *Cylas puncticollis* (Boheman) and rough weevils (*Blosyrus sp.*) and other various pests and diseases (Ebregt et al. 2007). Among those, *C. puncticollis* is the most serious pest in drier agro-ecological zones (Rees et al. 2012). This weevil causes yield losses ranging from 20% to 50% on many farms and can reach 100% depending on the season, variety and time during which the crop remains in the field (Oswald et al. 2009).

Larvae development takes about 10 and 35 days at 30 °C and 40 °C, respectively, before pupating. The increase or decrease in temperature affects larval development (Oswald et al. 2009). The infested roots could be recognized by holes or tunnels, spongy in appearance and dark in colour when pulled up from the soil. Furthermore, the larvae also suck and mine the vine causing malformation and cracking of the tissues (Korada et al. 2010). Rees et al (2012) reported that the damaged vines are the main reason for yield loss in sweetpotato. The damage caused by adult is less severe than the one caused by the larvae because the former has limited access to the roots.

Most of the sweetpotato varieties grown in Tanzania are susceptible to weevil infestation but some of them have high levels of tolerance revealed by low levels of root damage. Stathers et al (2003) reported on lower levels of damages inflicted on varieties characterized by high foliage weight. Yield and quality of sweetpotato storage roots can be improved by planting clean or less infested varieties associated with proper management practices (Ebregt et al. 2007). However, in Tanzania the integration of less infested varieties and sweetpotato weevils management methods such as use of mulches and dipping of the vine cuttings into insecticides (often dimethioate) before planting under natural infestation had never been comprehensively explored. The findings reported here emanates from a study conducted to assess the effectiveness of grass mulch and pre-planting disinfection of vine cuttings with Dimethoate 40EC insecticide when applied to sweetpotato varieties with varied susceptibility to weevils namely; Kiegea, Mataya, Simama and Ukerewe.

II. MATERIALS AND METHODS

2.1 Description of the study area

The study was conducted at the Tanzania Agricultural Research Institution (TARI), Kibaha, Coast Region located at 06°46´S, at a longitude 38°55´E and altitude of 302 meters above sea levels (masl). The field is predominantly sandy clayey loam soils (Masumba et al. 2007). The area recorded an average annual rainfall of 728.5 mm with unreliable distribution and intensity characterized by a weak bi-modal pattern which was below the rainfall average usually received per year (1000 mm).

2.2 Experimental design, layout and experimentation

The experiment was laid down in a factorial (4×3) design with two factors, factor A being the test varieties: (i) Kiegea; (ii) Mataya; (iii) Simama (SPNO); (iv) Ukerewe while Factor B were the weevils management practices namely; (i) Chemical control; (ii) Mulch; (iii) negative control (not treatment applied). All treatments were replicated three times. None infested planting materials were obtained from a germplasm at TARI Kibaha. The experiment was set in March 2015 and repeated in March, 2016 during the onset of the main rainy season. Each experimental plot measured 6.0 m × 6.0 m in size and it consisted of six ridges each of 1.0 m × 6.0 m in size. The vine cuttings (each) measuring about 30 cm long, with 4-6 nodes were planted singly along each ridge, at a spacing of 0.6 m × 0.3 m which gave a population of 20 plants per ridge. Considering the three replications, a total of 360 plants were established. Accounting for the inter-plot space of 1 m apart, the total experimental area was [(6.0 m x 6.0 m) x 4 + (1.5 m x 3 m)] x 3. Vine cuttings for the insecticide (chemical) treatment, were dipped in a Dimethoate 40 EC at a rate of 5 mls in 20 L of water (1ml/ltr) for 10 minutes then allowed to dry under the tree shade for three hours prior to planting. For the mulch treatment 50 kg of dry grasses was applied per ridge 20 days after planting (Fig. 1A).

Fig. 1: Mulched (A) and chemically treated and untreated (B) experimental plots

2.2 Routine management of the experiments

Earthing up of soil around the base of plants and re-mounding of the ridges was done at 3 and 6 weeks after planting. Weeding was done at 3, 6 and 9 weeks after planting, and whenever the necessity arose until harvest (Fig. 1B). Weeding was done timely particularly before the onset of flowers and roots. There were no fertilizer and irrigation applications done in the experimental plots. This aimed at imitating practices similar to those undertaken by the farmers in Coast region. Harvesting of the roots was done at 130 days after planting.

2.3 Data collection

The data collected were the percentage plant establishment which was done 2 weeks after planting, plant vigour, vine crown damage, number of harvested roots, number of weevil infested roots, percentage root infestation (incidence), severity of root damage at harvesting which was done by using a five-point score approach where 1= 0%; 2=1-25%; 3= 26-50%; 4= 51-75%; 5=76-100% as described by Stather et al (2003). The damage severity index (SI) was calculated using the formula:

$$\text{Severity Index} = \frac{(a \times \text{score } 0) + (a \times \text{score } 1) + (a \times \text{score } 2) \ldots\ldots\ldots + (a \times \text{score } 5)}{\text{Total number of roots}}$$

Where, ‘a’ is the number of roots with a particular score

In addition, the data related to percentage infestation was computed using the formula below:

$$\text{Percentage infestation} = \frac{\text{Total number of infested roots}}{\text{Total number of harvested roots}} \times 100$$

Other collected data were the number and weight of storage roots per plot (g) damaged by *C. puncticollis, C. brunneus* and millipedes, total number and weight of storage roots harvested, weight of vine , number and weight of marketable and unmarketable storage roots, number and weight of storage roots damaged.

2.4 Statistical analyses

All data were subjected to analysis of Variance using GenStat release 16.3DE statistical software. The Least Significance Difference (LSD) at 5% error limit and Duncan’s New Multiple Range Test (DNMRT) were used for means separation tests.

III. RESULTS

3.1 Crop establishment and vigour

The plant establishment and vigour results are presented (Table 1). Generally all varieties had a good establishment rate with an average of 99.5%. The plant vigour score was intermediate at class 3.

Table 1: Plant establishment and vigour of the tested sweetpotato varieties

Variety	Percentage plant establishment	Crop/plant vigour (average)
Ukerewe	99.4	3.4
Simama	99.6	3.6
Mataya	99.4	3.1
Kiegea	99.7	3.5

3.2 Effects of variety and management practices on vines and root yields

The recorded number of storage roots were significantly ($p < 0.05$) different among the four varieties (Table 2; Fig. 2 & 3). The highest number of harvested roots was recorded from Kiegea (589) and the lowest was recorded from Simama (305). The differences observed in weights of the total harvested storage roots among the four varieties at different management practices were insignificant ($p > 0.05$). However, a significantly ($p < 0.05$) higher vine weight was recorded in variety Simama (13938 g) and the lowest in variety Mataya (6304 g). Furthermore, there was a significantly ($p < 0.05$) higher number of marketable roots in the variety Mataya than the rest of sweetpotato varieties. The number and weight of non-marketable roots was significantly ($p < 0.05$) higher in the variety Kiegea than other varieties. The effect of weevils management practices on the number of harvested storage roots was not statistically significant ($p > 0.05$). However, there was a very significantly ($p < 0.05$) higher root weight recorded from the mulched plots than other treatments. In addition, a very highly significant ($p < 0.05$) difference in vine weight was recorded among the management practices.

Table 2: Effect of variety and management practices on weight of vines, number and weight of harvested roots

Variety	Management	Harvested roots		Marketable roots		Unmarketable roots		Vine
		No.	Wt (g)	No.	Wt (g)	No.	Wt (g)	Wt (g)
UKERE WE	Control	106^{a}	4017^{ab}	36^{a}	2683^{a}	70^{ab}	1333^{a}	5267^{bcd}
	Chemical	130.7^{ab}	6417^{abc}	56^{ab}	5000^{abc}	74.67^{abc}	1417^{a}	5467^{bcd}
	Mulch	98.7^{a}	6100^{abc}	52^{a}	5033^{abc}	47.33^{a}	1067^{a}	1837^{abc}
SIMAMA	Control	83.3^{a}	3333^{a}	32.33^{a}	2300^{a}	51^{a}	1033^{a}	5667^{cd}
	Chemical	111.3^{a}	6300^{abc}	53.67^{ab}	4933^{abc}	57.67^{a}	1367^{a}	6633^{d}
	Mulch	110^{a}	8300^{bc}	61.33^{ab}	6800^{bc}	48.67^{a}	1500^{ab}	1638^{ab}
MATAYA	Control	132.3^{ab}	4967^{ab}	55.67^{ab}	3367^{ab}	77^{abcd}	1600^{ab}	2133^{abc}
	Chemical	193.3^{c}	6750^{abc}	72^{ab}	4167^{abc}	122^{cde}	2583^{cd}	2833^{a-d}
	Mulch	199.3^{c}	10133^{c}	93.67^{b}	7450^{c}	106.33^{b-e}	2683^{cd}	1335^{a}
KIEGEA	Control	186^{bc}	5467^{ab}	62^{ab}	3567^{abc}	$124d^{e}$	1900^{abc}	3767^{a-d}
	Chemical	199^{c}	5667^{abc}	60.67^{ab}	3267^{ab}	138.33^{e}	2400^{bcd}	3467^{a-d}
	Mulch	204.3^{c}	7850^{abc}	67^{ab}	4567^{abc}	137.33^{e}	3283^{d}	3342^{a}
SE±		34.02	2344.9	21.01	2006.7	25.95	498.9	1986.6
CV (%)		23.3	37.4	35.9	45.3	29.5	27.0	59.5
LSD(p=0.05)	V	33.26	2292.5	20.54	1961.8	25.36	487.8	1942.1
	M	28.81	1985.4	17.79	1699.0	21.97	422.4	1681.9
	V × M	57.61	3970.7	35.58	3398.0	43.93	844.8	3363.9
F stat.	V	***	ns	*	ns	***	***	*
	M	ns	**	ns	**	ns	*	***
	V × M	ns	ns	ns	ns	ns	ns	ns

*Key: F stat: *** = p <0.001 very highly significant; **= p ≤0.01 very significant; *= p ≤0.05 significant; n.s= p >0.05 Not Significant;*
M= management, V= Variety, V X M= interaction between Variety and Management.
**The means along the same column bearing similar letter(s) do not differ significantly at 5% level of probability based on Duncan's Multiple Range Test (DMRT)*

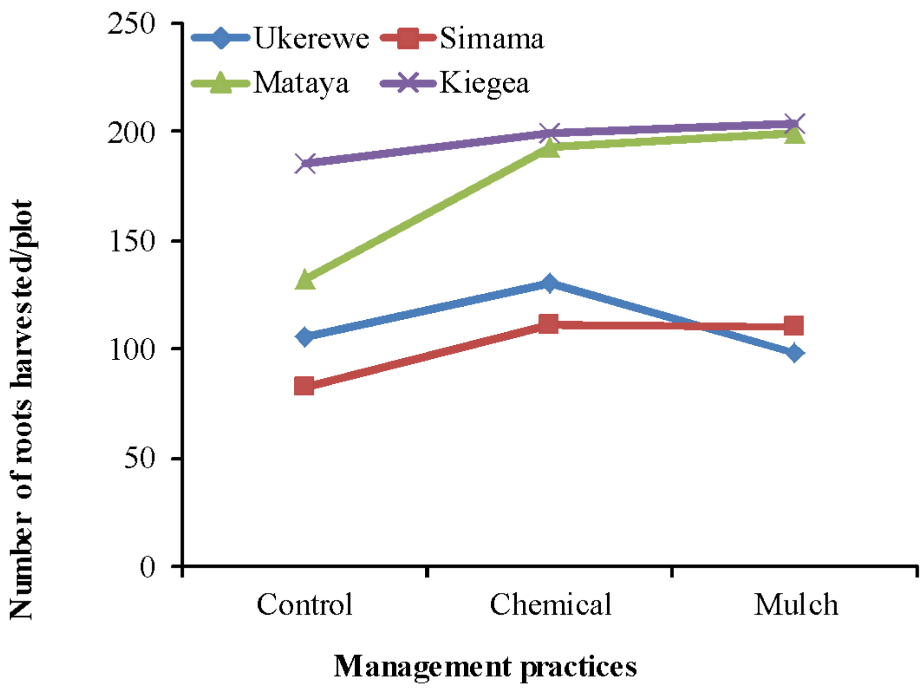

Fig. 2: The trends on the effect of varieties and management practices on number of harvested roots

Table 3: Mean sum of squares for the total number of roots and the vines of the tested sweetpotato varieties in different management practices

Source of variation	DF	M.S. for Roots			M.S. for Vines
		Total roots	Marketable roots	Unmarketable roots	
Replication	2	5961	1481.5	1714.6	6588433
Variety (V)	3	19605***	1355.7*	12213.8***	14475170*
Management (M)	2	3478ns	1494ns	1032.9ns	41444212***
V × M	6	852ns	212.8ns	486.7ns	2305446ns
Residual/error	22	1161	443.7	671.1	3946417
Total	35				
F prob.	V	<.001	0.050	<.001	0.028
	M	0.071	0.053	0.237	<.001
	V × M	0.628	0.816	0.634	0.739

*Key: M.S = Meam square; F stat: *** = p <0.001 very highly significant; **= p ≤0.01 very significant; *= p ≤0.05 significant; n.s= p >0.05 Not Significant*

Generally, the highest vine weight was obtained on chemical treated plots and the lowest weight was obtained on mulched plots. In addition, the number of harvested roots was found to be significantly ($p < 0.05$) affected by the varieties characteristics and management practices (Table 3). These variables affected the number of roots and vines at all levels of measurements. The influence of management practices on the number of vines was highly significant ($p < 0.001$).

3.3 Effect of varieties and management practices on the number and weight of damaged roots

Numbers of damaged roots among the studied varieties were not statistically different ($p > 0.05$). However, the management practices had a significant ($p \leq 0.05$) effect on the weight of the damaged storage roots (Table 4; Fig. 3).

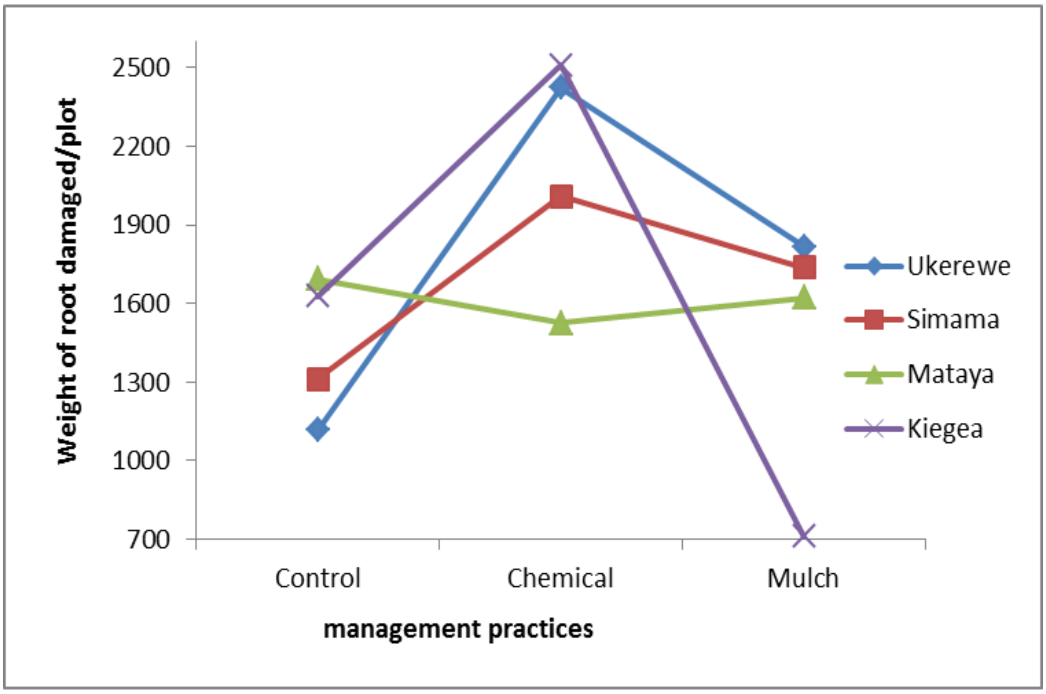

Fig. 3: The trends of the effect of variety and management practices on weight of damaged roots.

Table 4: The number and weight of damaged storage roots, percentage infestation of the pests and the number of damaged vines

Variety	Management	Damaged roots		Percentage infestation			Damaged vine
		Number	Wt (g)	*C .punctic*	*C. brunneus*	Millipedes	
UKEREWE	Control	33.67^{ab}	1117^{ab}	25.74^{abc}	4.698^{a}	4.231^{a}	10.8^{a}
	Chemical	52.33^{ab}	2423^{c}	30.06^{abc}	2.122^{a}	5.745^{a}	25.0^{a}
	Mulch	62.00^{ab}	1817^{abc}	42.70^{c}	12.917^{b}	6.369^{a}	11.04^{a}
SIMAMA	Control	44.67^{ab}	1311^{abc}	43.53^{c}	6.247^{ab}	3.544^{a}	24.63^{a}
	Chemical	34.00^{ab}	2008^{bc}	26.73^{abc}	7.018^{ab}	2.993^{a}	29.78^{a}
	Mulch	53.00^{ab}	1738^{abc}	38.15^{bc}	6.36^{ab}	6.556^{a}	24.17^{a}
MATAYA	Control	43.67^{ab}	1692^{abc}	22.69^{abc}	5.279^{a}	3.445^{a}	14.34^{a}
	Chemical	41.00^{ab}	1525^{abc}	14.50^{a}	2.681^{a}	1.854^{a}	15.77^{a}
	Mulch	45.33^{ab}	1621^{abc}	17.62^{ab}	2.779^{a}	3.445^{a}	26.25^{a}
KIEGEA	Control	30.33^{a}	1628^{abc}	11.46^{a}	2.485^{a}	2.251^{a}	24.45^{a}
	Chemical	69.33^{b}	2508^{c}	26.71^{abc}	7.874^{ab}	3.075^{a}	23.00^{a}
	Mulch	28.33^{a}	711^{a}	9.50^{a}	1.721^{a}	2.739^{a}	27.60^{a}
SE±		19.23	623.2	12.03	3.580	2.681	15.72

CV (%)		42.90	37.2	46.6	69.1	71.6	73.4
LSD (0.05)	V	18.80	609.2	11.76	3.500	2.621	1942.1
	M	16.28	527.6	10.18	3.031	2.270	15.37
	V × M	32.56	1055.2	46.6	6.062	4.539	26.62
F stat.	V	ns	ns	**	ns	ns	ns
	M	ns	*	ns	ns	ns	ns
	V × M	ns	ns	ns	*	ns	ns

*Key: F stat: *** = p <0.001 very highly significant; **= p ≤0.01 very significant; *= p ≤0.05 significant; n.s= p >0.05 Not Significant*

Inferring to management practices, the highest weights of damaged roots were recorded in the chemically management plots for Kiegea (2508 g), Ukerewe (2423 g) and Simama (2008 g) in the order of decreasing magnitudes (Table 5). Conversely, variety Mataya recorded the highest average weight (1692 g) of damaged roots under the control treatment. Interestingly, the weights of the damaged roots in each management practice did not follow the same trend as of the number of obtained damaged roots. Suggestively, the number of damaged roots does not necessarily reflect on the weights of the roots, which entails that the number of roots could be small but may record a substantial amount of weight. The reverse to these observations were the positive implication of other management practices used in this study. These findings suggest that the damage of roots by weevils could not be controlled by using mulching materials. As such, neither mulching nor disinfecting vines using insecticides offered a viable protection of sweetpotato storage roots against weevils' infestation in the pest endemic soils.

Table 5: The mean sum of squares for the percentage infestation by *C. brunneus, C.* puncticollis, Millipede.

Source of variation	DF	*C. brunneus*	*C. puncticollis*	Millipede
Replication	2	51.58	2324.2	0.573
Variety	3	23.11 ns	933.6**	17.966ns
Management	2	5.42 ns	182.0ns	4.524ns
Interaction (V × M)	6	43.57*	251.0ns	4.264ns
Residual/error	22	12.82	144.7	7.186
Total	35			
F prob.	V	0.176	0.003	0.086
	M	0.661	0.879	0.542
	V × M	0.016	0.160	0.732

*Key: F stat: **= p ≤0.01; *= p ≤0.05; N.S= Not Significant; * = significant; ** = very significant; *** = very highly significant*

The detailed analysis of the damage causes on storage roots suggested that *C. puncticollis*, *C. brunneus*, and millipedes, invariably (p > 0.05; Tabe 4; Table 5) inflicted damages to sweetpotato roots. The number of storage roots which were damaged by millipedes did not differ significantly (p > 0.05) among the studied varieties. However, a very significant (p ≤ 0.05) difference was established on storage roots which were damaged by *C. puncticollis* and *C. brunneus* (Table 5). Further to that, the number and weight of storage roots damaged by *C. puncticollis* was lower on variety Kiegea under mulched and negative control plots while in Mataya variety the lowest damage was recorded under chemical management treatment.

IV. DISCUSSION

The weight of vines and yield of marketable and non-marketable storage roots were highly varied among different varieties under similar management practices. An application of mulching materials on the sweetpotato fields contributed to the increase in number and weight of sweetpotato storage roots. The varieties, Kiegea and Mataya yielded more roots than other varieties. Highest root weight was recorded on Mataya variety only under mulched treatment. This signifies the ability of variety Mataya to potentially utilize the microclimate created by the mulching materials to yield more. Improved nutrients availability and soil moisture content have been reported to be resultants of grass mulch which subsequently improves growth conditions for the plants. In other words, the varieties Ukerewe and Simama produced fewer numbers of roots under all levels of management. Although the largest weight of marketable roots in all four varieties was recorded under mulched treatment, the order of decrease in magnitude of the marketable root weight was: Mataya > Simama > Ukerewe > Kiegea. The findings of this study indicate that among the studied varieties, the variety Mataya is potentially promising in terms of the weight of marketable roots based on the low-costing management practices employed. These findings are consistent with those of Nedunchezhiyan et al (2012) who reported that management practices recorded significantly high root yield in sweetpotato. Similar authors stressed, however, that the yield differed between management practices used for crop establishment, which is also observed in the existing study.

The effect of variety and management practices on the number and weight of damaged roots differed inconsistently among varieties and management practices. The highest numbers and weights of damaged roots recorded for Ukerewe, Simama and Mataya varieties when mulching materials were used could be attributed to the host-weevil relationship between these varieties which was clearly manifested. However, the detail of this speculation is beyond the scope of this study and could need further investigation using the same varieties. It could, however also, be deduced that the use of chemicals in controlling weevils in sweetpotato fields does not offer effective control probably because of the ability of the chemicals to escape by volatilization upon exposure to the external environment. Similarly, this observation suggests that the activity of the chemical throughout the shelf-life of sweetpotato in the field is questionable despite the residual effects reported by Addor (1995). This could be the contribution of the large number and weight of damaged roots obtained under chemical control practice in all plots. On the other hand, the efficiency of the mulch could be attributed to the ability of these materials to remain in the soil for a long time. These materials also decompose slowly and at the sometime release organic acids and cast unpleasant odour which might have hindered infestation of the sweetpotato by weevils. Similar findings were reported by Jayaraj et al (2012) that the physical soil cover made by mulching materials reduced access of roots to the weevil even if the soil cracks. Similar authors found that mulching with *Eupatorium odoratum* leaves at a rate of 3 tons per hectare at 30 days after planting was effective in suppressing weevil attack in India. These findings also indicate that the number of vines of the studied sweetpotato varieties is highly dependent on the management practices. In addition, the number of reliable or marketable roots is very dependent on the type of sweetpotato variety used. These findings also indicate that the number of vines of the studied sweetpotato varieties is highly dependent on the management practices. In addition, the number of reliable or marketable roots is very dependent on the type of sweetpotato variety used.

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Discloser

The author declared that he has no conflict of interests.

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