IMAGE: A MAP OF THE STARS OF THE ORION CONSTELLATION

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A.E. Avramenko

ABSTRACT

In the article, from the unified positions of the theory of relativity, physical processes are generalized that occur synchronously in the space of inertial coordinate systems in the entire range of possible movements - from the observed movements of celestial bodies to intra-atomic interactions of electromagnetic fields and quantum particles. Atomic objects manifest and fixed by direct detection of elastic wave interactions on a pulsar time scale in their natural states, excluding particle collisions. In the inertial frame, the body moves according to the Kepler-Newton laws (inertiality in the usual mechanical sense).

During the passage of the wave front, the electromagnetic field of the pulsar, interacting with the electromagnetic field of microparticles, fixes their discrete mechanical states on its scale with an accuracy of up to a quantum of time allowed by the pulsar scale. The discrete mechanical states of microparticles measured on the pulsar scale are multiples of the constant value ∂T = 5.5511151231257800E-17s, they are repeated in any inertial coordinate system. Their number is finite, regardless of the duration of the measurements.

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Pulsar: Metric Generalization of Time-Space of Celestial and Quantum Mechanics

A.E. Avramenko

In the article, from the unified positions of the theory of relativity, physical processes are generalized that occur synchronously in the space of inertial coordinate systems in the entire range of possible movements - from the observed movements of celestial bodies to intra-atomic interactions of electromagnetic fields and quantum particles. Atomic objects manifest and fixed by direct detection of elastic wave interactions on a pulsar time scale in their natural states, excluding particle collisions. In the inertial frame, the body moves according to the Kepler-Newton laws (inertiality in the usual mechanical sense).

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It is proved that the solution of the missing inverse problem of celestial mechanics in the time domain, based on the observed propagation of the pulsar electromagnetic radiation front, also determines the inertiality with respect to the field in measurements of discrete states of microparticles, generalized in 4-dimensional space on the pulsar time scale.

Axiomatic transformations of the space-time states of the material world as a whole strictly correspond to the fundamental physical principles - the integral laws of conservation of energy momentum and angular momentum. Thus, the physical processes of celestial and quantum mechanics proceed in the space of inertial coordinate systems synchronously in the entire range of possible movements - from the observed movements of celestial bodies to the interactions of electromagnetic fields and quantum particles. Atomic objects are manifested and fixed by direct detection of elastic wave interactions on a pulsar time scale in their natural states, excluding particle collisions, similar to collisions in a collider.

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I. INTRODUCTION

Space and time are primary concepts, the essence of the forms of existence of matter. These concepts are obtained by appropriate abstractions from the concept of space-time connections between material processes. The simplest concept related to space and time is a material point of space, considered at a certain point in time. To mark a point in space, you need to place a material body of sufficiently small dimensions there. When a frame of reference is chosen, this position can be given by three coordinates referring to a certain point in time, counted by the base clock.

It should be especially noted that both the coordinates themselves and the mutual distances, angles and other quantities calculated with the help of them, characterizing the relative positions of bodies, take on a certain meaning only on the assumption of a certain basis. In the same way, moments of time, which include coordinates and distances, as well as time intervals, become certain only on the

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assumption of a certain basis and a certain time on the basis, that is, on the assumption of a certain frame of reference. It is enough to measure a certain basis in the sense of ordinary triangulation and detect directions to an object from different points of this basis using an electromagnetic wave in order to calculate, according to the laws of Euclidean geometry, the distance to the object and all other data characterizing its position “with a huge degree of accuracy” [1].

All these requirements are fully satisfied by Poincare's theory of relativity, in which the laws of Euclidean geometry for real physical space are valid with absolute accuracy. Einstein's theory of universal gravitation, which he called the general theory of relativity (GRT), on the contrary, allows for the inhomogeneity of space and, as a consequence, deviations from the physical laws of Euclidean geometry. This manifested its metrical contradiction with Poincaré's theory of relativity, based on axiomatic transformations of coordinates and time in inertial frames.

A.A. Logunov, referring to the work [2], drew attention to the fact that in it Poincaré, having formulated his fundamental idea, attributed it completely to Lorentz. Poincaré always highly appreciated and noted everyone who gave impetus to his creative thought. However, in no article did he write about Einstein's role in the creation of the theory of relativity. It is inconceivable that Poincaré, having read Einstein's 1905 papers, would not understand them. On the contrary, Einstein's work simply contained nothing new for him. Based on his previous works, as well as on the studies of Lorentz, Poincaré formulated everything that is the content of the theory of relativity, discovered the laws of relativistic mechanics, and extended Lorentz's transformations to all forces of nature.

After the appearance of Einstein's works on the theory of relativity (1905), Poincaré stopped publishing on this topic. Their personal meeting took place only once - in 1911. About this, Einstein himself wrote to his Zurich friend Dr. Zangger:

"Poincare in relation to the relativistic theory] rejected everything completely and showed, for all his subtlety of thought, a poor understanding of the situation."

In his characterization of Einstein, which Poincaré gave in November 1911 at the request of the Zurich ETH in connection with Einstein's invitation to a professorship, he wrote:“Einstein does not hold on to classical principles and, when faced with a physical problem, is ready to consider any possibilities. Thanks to this, his mind anticipates new phenomena, which in time can be experimentally verified. I do not mean to say that all these predictions will stand the test of experience when possible; on the contrary, since he seeks in all directions, it is to be expected that most of the paths he enters will turn out to be dead ends; but at the same time, one must hope that one of the directions indicated by him will turn out to be correct, and this is enough. That is exactly what should be done. The role of mathematical physics is to ask questions correctly; only experience can solve them”.

What happened next only confirmed this assessment of Einstein's theory and Poincaré's prediction. Its metrical contradiction with Poincaré's theory of relativity, which has survived experimental testing, and based on the postulate of the independence of phenomena from the unaccelerated motion of a closed system within which they occur, appeared. At the same time, in the practice of physical research and their expert assessments, preference is often given to the mathematical theory of Einstein, without taking into account its features and fundamental limitations in physical applications. So, for example, the Nobel Prize in Physics in 2020 was awarded "*for the discovery that the formation of a black hole is a reliable prediction of the general theory of relativity*", and in the next year, 2021 - "*for the discovery of the interaction of disorder and fluctuations in physical systems from atomic to planetary scales*". The existence of massive black holes thus became a necessary condition for the inhomogeneity of space declared in Einstein's theory, which allows for chaos and fluctuations in the physical systems of celestial and quantum mechanics.

II. RESONANCE STABILITY OF OSCILLATORY SYSTEMS OF CELESTIAL AND QUANTUM MECHANICS

The laws of motion of the planets of the solar system, discovered by Kepler (1571-1630) based on the results of measurements made by him directly from the observations of the planets, later found their equivalent expression in the form of solutions of differential equations in the form of Newton, which are a solution to the inverse problem of celestial mechanics.

The inverse problem was reduced to the calculation of the elliptical (Keplerian) motions of the planets with the allocation of the main component of the motion, which is a mathematical description of the translational motion of idealized bodies and can be expressed purely kinematically, without considering the causes of motion (mass, forces generating it, etc.), in terms of space and time only.

From the point of view of mathematics, translational motion in its final result is equivalent to parallel translation - a special case of motion in which all points of space move in the same direction for the same distance. In the general case, translational motion is considered as a set of rotations, that have not ended.

In this case, rectilinear motion is understood as a limiting case, as a rotation around a center of rotation infinitely distant from the body.

The principle of relativity states that the process in both cases will proceed in exactly the same way and be described by the same functions of coordinates and time, since it is generally deterministic. Since the perturbing forces are random and small compared to the attraction of the central body, then on large time scales in real orbits the main, stable component of motion dominates, which is determined by the equations of the Kepler-Newton laws and is not sensitive to random effects of perturbing forces. A.M. Molchanov [4] formulated a hypothesis about the existence of a resonant structure (full resonance) of the solar system, according to which evolutionarily mature oscillatory systems are inevitably resonant, and their state is determined, like quantum systems, by a set of integers. Orbital resonance, according to this hypothesis, is provided by small dissipative forces: tidal forces, braking from interstellar dusty matter, etc. These dissipative forces are very small, orders of magnitude smaller than weak perturbations due to planetary interactions. However, acting for millions or more years, they lead the motions of the planets to stationary resonant orbits.

Resonances in celestial mechanics are expressed by the relation:

$$n_1 w_1 + n_2 w_2 + ... + n_k w_k = 0, \qquad 2.1$$

Here $w_{1,} w_{2,} ... w_{k,}$ are the revolution frequencies (or average angular velocities) of the corresponding planets around the Sun, or satellites of the planet around it, or planets (satellites) around their axis; $n_1 ... n_k$, are integers, positive or negative.

The fundamental difference between resonant dissipative celestial systems is the anomalously high level of kinetic energy stored at the stage of their formation, whether they are planetary systems or rotating neutron stars - pulsars. Thus, the solar system, for hundreds of years of observations, has been stably and predictably reproducing the regular spin-orbital motions of the planets in accordance with the laws of Kepler-Newton-Galileo.

A pulsar is a unique galactic object with pronounced spin-resonance properties. The quality factor of its resonant oscillatory system exceeds the quality factor of the resonant planets of the solar system by several orders of magnitude. Therefore, a pulsar as a source of periodic electromagnetic radiation propagating in the form of an electromagnetic radio wave front in galactic space has every reason to claim the role of a physical standard of the fourth coordinate - time in three-dimensional space. And, as a result, to bring the metrics of four-dimensional space-time in line with the kinematic laws of physical processes in inertial coordinate systems.

The estimated quality factor of a pulsar as a source of electromagnetic radiation is about $10^{15}/10^{21}$, and the stored energy of rotation of the pulsar $E_{rot} = I\Omega^2$ ~$10^{45}/10^{52}$ erg is quite enough for $E_{rot}=I\Omega\Omega$ ~$10^{31}/10^{34}$ $_{эрг \cdot с}$-^{1} observed periodic emission of pulsars remained coherent during the entire interval $10^6/10^7$ years characteristic of neutron stars. Our observations of pulsars at the BSA FIAN radio telescope in Pushchino are consistent with these estimates [7].

III. PHYSICAL GENERALIZATION OF INERTIAL COORDINATE SYSTEMS IN THE TIME DOMAIN

There is a limiting speed of propagation of any kind of action. This speed is numerically equal to the speed of light in free space.

The propagation equation for a wave front of any nature coincides with the propagation equation for a light wave front.

This equation is a generalization of Maxwell's equation in some frame of reference, which is inertial in the sense of mechanics and in which, along with this, the equations for the propagation of an electromagnetic wave front are valid [1]:

$$\frac{1}{c^2}\left(\frac{dw}{dt}\right)^2 - \left[\left(\frac{dw}{dx}\right)^2 + \left(\frac{dw}{dy}\right)^2 + \left(\frac{dw}{dz}\right)^2\right] = 0 \qquad (3.1)$$

In this expression, the inertial reference frame is characterized by the following two principles:

1. In the inertial system, all points of the body in the absence of forces describe the same trajectory according to the laws of Kepler (direct problem) and Newton (inverse problem), while maintaining the parallelism of any segment to itself, which in its end result is equivalent to a parallel transfer that determines the inertiality in in the usual mechanical sense.
2. In an inertial system, the equation for the propagation of an electromagnetic wave front determines the inertiality with respect to the field as well.

The basic postulate of the theory of relativity asserts the independence of phenomena from the unaccelerated motion of a closed system within which they occur. Thus, the translational movement of the material system as a whole does not affect the course of physical processes occurring within the system.

The principle of relativity is confirmed by the totality of our knowledge about nature. In the field of mechanics, it has long been known as Galileo's principle of relativity. The theory of relativity can be built on the basis of two postulates: the principle of relativity of Galileo and the principle of independence of the speed of propagation of an electromagnetic wave from the speed of the source. The second of these principles was taken into account from the very beginning, since the law of propagation of the front of an electromagnetic wave was taken as the basis for constructing the theory of relativity. The independence of the speed of light from the speed of the source directly follows from this law: *there is a limiting speed of propagation of any kind of action. This speed is numerically equal to the speed of light in free space.*

V.A.Fok, who analyzed in detail the physical foundations of the theory of relativity, gives estimates of possible alternative approaches. In principle, it is possible, for example, to transmit signals with the help of extremely fast particles and the matter waves corresponding to them in the sense of quantum mechanics. It is also conceivable (although practically unrealizable - V.A. Fok), the use of gravitational waves, the existence of which follows from Einstein's theory of gravitation.

The formulated principle, which affirms the existence of a general limit for the speed of transmission of any actions and signals, gives the speed of light a universal value, not related to the particular properties of the agent transmitting signals, but reflecting the objective property of space and time. This principle is in logical connection with the principle of relativity. If there were no single limiting speed, and various agents, for example, light and gravity, would propagate in emptiness (free space) at different speeds, then the principle of relativity would be violated with respect to at least one of these agents.

The differential equation (3.1) is a generalization of Maxwell's equation in some frame of reference, which is inertial in the sense of mechanics and in which the propagation equations for the front of an electromagnetic wave are also valid. In such a system, the body moves according to the laws of Kepler (direct problem) and Newton (inverse problem) - in accordance with the concept of inertiality in the usual mechanical sense. In an inertial system, the equation for the propagation of an electromagnetic wave front also determines the inertiality with respect to the field.

The physical generalization of inertiality with respect to the field, displayed on the left side of the equation, is achieved by the so far missing solution of the inverse problem of celestial mechanics in the time domain. V.A.Fok solved this problem purely theoretically by introducing a harmonic coordinate system for calculations in space at infinity. We have adopted a pulsar as a physical source of electromagnetic radiation - a galactic source of periodic radio emission that satisfies the principle of inertia with respect to the field. The propagation of the wave front of the electromagnetic radiation of a pulsar in time is completely determined by the measurable parameters of its rotation, and the stored energy of rotation of the order of $10^{45}|10^{52}$ erg with its gradual scattering of $10^{31}|10^{34}$ $_{\text{эрг}\cdot\text{с}}{}^{-1}$ neutron stars within an interval of $10^{6}|10^{7}$ years.

IV. AXIOMATICS OF COORDINATE PULSAR TIME SCALES IN INERTIAL FRAMES OF REFERENCE

The physical generalization of inertiality with respect to the field, displayed on the left side of equation (3.1), is achieved as a result of the so far lacking solution of the inverse problem of celestial mechanics in the time domain.

We have proved that the only and sufficient source by which the pulsar rotation parameters are determined is the measured time of arrival of the pulsar radiation pulses observed with a radio telescope. V.A.Fok previously solved this problem theoretically by introducing a harmonic coordinate system for calculations in space at infinity. We have adopted a pulsar as a physical source of electromagnetic radiation - a galactic source of periodic radio emission that satisfies the principle of inertia with respect to the field. The propagation of the front of a wave of electromagnetic radiation of a pulsar in time is completely determined by the measurable parameters of its rotation.

Einstein, when creating the general theory of relativity (the theory of gravitation), suggested that not only mechanical motion, but also any physical processes under the same initial conditions proceed in exactly the same way in the gravitational field and outside it, but in an accelerated reference frame (equivalence principle). However, it should be taken into account that this principle is local, i.e., the identity of the gravitational field to the accelerated reference frame is valid only in a limited region of space, in which the gravitational field can be considered uniform and constant in time. Therefore, during measurements, conditions inevitably arise for violating this principle and, as a result, discrepancies between the measured and calculated values appear, in particular, in the form of residual deviations of the moments of arrival of radiation pulses of the pulsar at the radio telescope. The process of timing pulsars is thus fundamentally dependent on an accurate description of everything that affects the time of arrival (ToAs).

In order to avoid distortions of the measured ToAs, we took into account the fact that the intervals between radiation pulses depend on the parameters of the monotonically slowing down rotation of the pulsar, the period *P* and its derivatives *P* and *P* .

There is a combination of rotation parameters *P* and its derivatives *P* and *P*, unique for each pulsar, the sequence of measured radiation intervals can be expressed analytically in terms of the observed rotation parameters, as an index of neutron star deceleration $n=2\ pp/p^2$.

Thus, each pulsar has its own combination of rotation parameters, which is generalized by the neutron star deceleration index.

According to our timing data at the BSA FIAN radio telescope, the numerical values of the deceleration index are limited by the limits n = –(0.9 ± 0.2), which corresponds to monotonic deceleration of the rotation of neutron stars (*P* > 0, *P*> 0).

For comparison, according to the well-known work of Hobbs, G., Lyne, A.G., Kramer, M. (2010) [6], *the value of the braking index estimated from the residual timing deviations is in the range –287986 < n < +36246 with its average value nav = –1713.*

Out of a sample of 366 pulsars, 193 pulsars have a positive second derivative of the rotation frequency, while the remaining 173 have a negative one.

V. PHYSICAL PULSAR TIME SCALES

The only unique combination of measurable parameters of pulsar rotation, corresponding to its monotonic slowdown in time, cannot be established within the framework of Einstein's theory of gr`1avitation, which does not extend directly to the time domain of real physical processes. The braking index is thus independent of the time and location of observations and is a fixed measurable quantity for each pulsar in a rather narrow range of physical values corresponding to the coherent periodic radiation of the pulsar.

The time scale in ephemeris astronomy is considered as some material system that has a continuous and stable movement and represents a certain measurable parameter P, which changes as a function of an independent time variable (V.K. Abalakin, 1978) [5].

The time of passage of the front of the wave of electromagnetic radiation of the pulsar is fixed on the pulsar time scale PT within any selected interval and is expressed as a digital series of intervals as a function of the pulsar rotation period for the selected initial epoch and its derivative (1):

$$PT_{tj} = \frac{1}{P_0^*} \int_{tj}^{tj} P(t)dt \qquad (1)$$

$$P(t) = P_0^* + P^* t; \ -\infty < t < +\infty \tag{2}$$

The differential pulsar scale (2) is a sequence of daily increments of the pulsar rotation period within any observation duration.

$$P_i = P_0^* \pm \Delta P_i \,; \ -\infty < i < +\infty \tag{3}$$

The time scale is set to any arbitrarily chosen origin of time, determined by a fixed value of the period for this epoch, and is axiomatically transferred to any other epoch of the future (for i > 0) or the past (for i < 0) within any chosen interval (3).

VI. DETECTION OF QUANTUM STATES OF ELEMENTARY PARTICLES

The quantum world consists of many small blocks, each of which is an indivisible elementary particle - an atom. The size of a simple hydrogen atom is approximately: 1A = 100 pm = m. The detection of quantum states consists in detecting the interference of two or more coherent waves when they are superimposed on each other.

The time interval for the passage of the pulsar electromagnetic wave front within the hydrogen atom is $0.33 \cdot 10^{-18}$ s. Hence the required resolution of the time scale is $\Delta t < 10^{-18} s$.

The resolution of the pulsar time scale 10^{-25} s therefore satisfies the condition for detecting discrete states of physical quantum systems within the physical boundaries of their existence in space. Thus during the passage of the wave front, the electromagnetic field of the pulsar, interacting with the electromagnetic field of microparticles, fixes their discrete mechanical states with an accuracy of up to 10^{-25} s resolved by the pulsar time scale.

Since the Schrödinger equation, which plays the same role in quantum mechanics as Newton's equations in classical mechanics (the inverse problem of celestial mechanics), is invariant under Lorentz transformations based on the Galilean relativity principle, this implies the existence of a number of quantum mechanics operators and the existence of quantum mechanical invariants associated with Galilean transformations, similar to those in celestial mechanics.

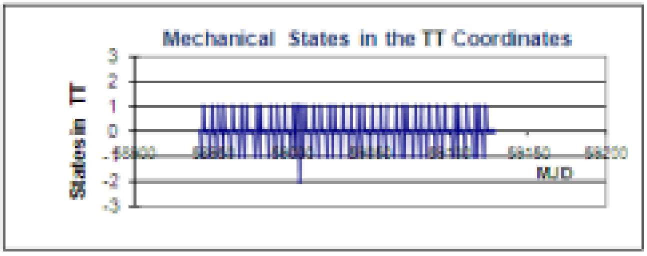
Thus, the wave front of electromagnetic radiation of a pulsar, which determines the pulsar time scale, when interacting with microparticles with wave properties, fixes the change in time of their mechanical states, and according to the observed sequence of changing discrete states, fixes their movement along a certain trajectory.

In quantum mechanics, quantum oscillations in molecules, atoms, and nuclei can occur only for a fixed set of discrete energies in the spectrum of levels of a quantum oscillator. The levels of discrete energies are equidistant and are determined by Planck's constant. The value of ω, which determines the fundamental tone of a quantum oscillator, is related to its potential energy by the classical relation /

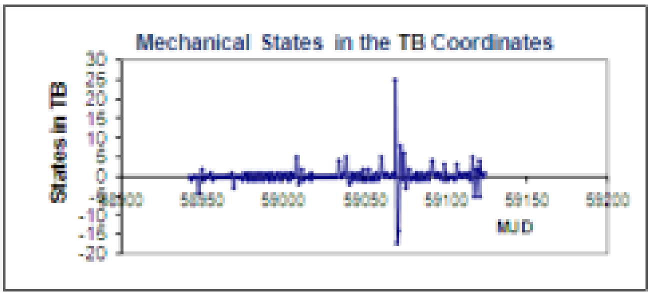
$$mw^2x^2/2 = kx^2/2$$

Quantum harmonic oscillators as energy emitters have energies not as continuous variable motions, like the motion of bodies in celestial mechanics, but as regularly ordered discrete values of the energy of wave processes, which are multiples of the energy quantum ħυ; (υ= ω/2π).

From long-term observations of the B0950+08 pulsar at the BSA FIAN radio telescope in Pushchino, we took daily timing data in the topocentric (TT) and barycentric (TB) coordinate systems. Pulsar B0950+08 in a supernova remnant (age 1.7×10^7 years) in the vicinity of the solar system (distance 0.26 kpc) can be attributed to older stars, the lifetime of which, together with the solar system, is estimated to be approximately 4.6×10^9 years, as well as Sun [9].



a) in topocentric coordinates TT



b) in barycentric coordinates TB

Figure 1: Measured mechanical states of microparticles

Discrete states of microparticles are multiples of a constant value, which is the same in any inertial coordinate system. Considered by M. Planck, quantum harmonic oscillators as energy emitters have energies not as continuous variable movements, like the movement of bodies in celestial mechanics, but as regularly ordered discrete values of the energy of wave processes that are multiples of the energy quantum $\hbar\upsilon$; ($\upsilon= \omega/2\pi$).

Quantum oscillations in molecules, atoms, nuclei can occur only for a fixed set of discrete energies in the spectrum of levels of a quantum oscillator. The levels of discrete energies are equidistant and are determined by Planck's constant. The value of ω, which determines the fundamental tone of a quantum oscillator, is related to its potential energy by the classical relation $mw^2x^2/2=kx^2/2$.

By analogy with a photon - a fundamental massless particle moving at the speed of light, transferring local electromagnetic interaction in the quantum world of microparticles, a neutron star - a pulsar also performs the function of transferring electromagnetic interaction, but, unlike a photon, it is not limited only by the quantum world of microparticles, but extends to the material world as a whole, including the solar system, the Galaxy, etc.

Thus, the wave front of electromagnetic radiation of a pulsar, which determines the pulsar time scale, when interacting with microparticles with wave properties, fixes the change in time of their mechanical states, and according to the observed sequence of changing discrete states, fixes their movement along a certain trajectory.

VII. TO THE PHYSICAL GENERALIZATION OF THE MATERIAL WORLD AS A WHOLE

The physical processes of the material world of quantum and celestial mechanics are inseparable in their conditioning and sequence.

The quantum world - beginningless, omnipresent, eternal and indivisible, became the basis of the material world of the Universe as a whole, laying the foundation for it and remaining in its beginningless state forever.

The axiomatics of measurable spatial states of matter means the omnipresence of the material world of the Universe as a whole.

The axiomatics of measurable temporal states of matter means the perpetuity of the material world of the Universe as a whole.

Axiomatic transformations of the space-time states of the material world of the Universe as a whole strictly correspond to the fundamental physical principles - the integral laws of conservation of energy momentum and angular momentum.

The metric variety of the material world and physical processes occurring in celestial and quantum mechanics is finite and gen eralizable a s a whole.

There are no physical reasons for the crisis of celestial and quantum mechanics. There is a problem of correspondence of mathematical theories of time-space to the objective laws of nature.

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Prof. I.F. Malov for critical discussions on theoretical approximations and physical consistency in observational pulsar astrometry;

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Extraction and Characterization of Flavonoid Kaempferol-3-Glucuronide from Mango for Treatment of Different Types of Cancers

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ABSTRACT

Mango (Mangifera indica) is one of the most popular tropical terrestrial fruit bearing plants. Different phytochemicals especially, polyphenols (specifically flavonoids), terpenoids, xanthones are present in sufficient amount in different parts of mango, so that mangos are considered as nutraceuticals in treatment of different diseases.

Among thousand varieties of mango, eight varieties are selected for our study from our college garden. Bioactive components from these unripe mangos have been extracted. The estimation of total phenolic content with these extracts shows that 'himsagar' variety of mango contains highest concentration of polyphenols in different pH and incubation time at 37°C.

UV-spectrometric screening for flavonoids present in unripe mango pulp of 'himsagar', reveals that this specific variety of mango contains kaempferol-3-glucuronide. The presence of this phytochemical has been confirmed by the application of UV shift reagents such as NaOH, NaOAc, NaOH+ NaOAc and NaOAc +H3BO3 reagents.

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ABSTRACT

Mango (Mangifera indica) is one of the most popular tropical terrestrial fruit bearing plants. Different phytochemicals especially, polyphenols (specifically flavonoids), terpenoids, xanthones are present in sufficient amount in different parts of mango, so that mangos are considered as nutraceuticals in treatment of different diseases.

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UV-spectrometric screening for flavonoids present in unripe mango pulp of 'himsagar', reveals that this specific variety of mango contains kaempferol-3-glucuronide. The presence of this phytochemical has been confirmed by the application of UV shift reagents such as NaOH, NaOAc, NaOH+ NaOAc and NaOAc +H3BO3 reagents.

In-silico study with the compound kaempferol-3-glucuronide, identifies the aldehyde reductase (AKR1B1) as target protein for drug design for different diseases. This gene is expressed in increased amount in head and neck cancer cells compared to normal cells. From a pan-cancer point of view, this specific gene is also differentially expressed in twelve types of cancer cells. By using gene network analysis and molecular docking studies with target gene AKR1B1 and phytochemical kaempferol-3-glucuronide, present in unripe mango pulp, the efficacy of this flavonoid has been discussed elaborately for the treatment of various types of cancers.

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I. INTRODUCTION

Mango is probably the most popular tropical terrestrial fruit bearing plant. Various parts of this tree have been used as an important herb in Ayurveda, since time immemorial. Mango comes from the Malayalam word- Maanga. The earliest evidence of the mango in India comes from sixty million years old fossils found in Damalgiri in Meghalaya. Thus it can be rightfully concluded that the lush fruit is India's gift to the world. Various parts of a mango fruit are known to be enriched with various nutraceutical compounds such as Mangiferin, kaempferol, protocatechuic acid and beta carotene. Studies show that these compounds have anti-diabetic and anti-carcinogenic effects [1]. Thus, study of phytochemicals present in mango is essential to identify, extract and ultimately utilize the nutraceuticals. Different phytochemicals present in mango plants are phenolic acids, xanthones and polyphenols (mainly flavonoids) [2]. Two types of phenolic acids are found in mango plant; they are: 1) Hydroxybenzoic acid derivatives 2) Hydroxycinnamic acid derivatives. Three types of polyphenols are present in mangoes [3] such as flavonoids (in high concentration), stilbenes and lignans. Flavonoids namely catechins, quercetin (mainly flavanols-glycosides of quercetin like glucose, galactose, rhamnose, xylose, arabinose), kaempferol, rhamnetin and anthocyanins and tannic acids, are obtained from mangoes. Furthermore, several other chemical compounds such as xanthones, mangiferin, dimethyl mangiferin, homomangiferin, mangiferin gallate, isomangiferin and isomangiferrin gallate are also available in mangoes [4].

Cancer is the second leading cause of death globally, accounting for an estimated 9.6 million deaths per year. The problem with cancer medication is that it also kills healthy cells in the patient's body. But certain phytochemicals show tremendous promise to act as nutraceuticals in treating detrimental diseases like cancer [5] and diabetes [6]. Mango is a traditional household fruit which is used to quench one's thirst of enjoying this lush fruit on a hot summer afternoon. Though mango has been used in ayurveda since time immemorial but its application in today's medicinal world is almost nil, may be due to lack of research in this area.

It was found that various phytochemicals like flavonoids, xanthones, polyphenols are present in considerable quantities in the edible part of mangoes. These phytochemicals are proven to interfere in biological action of our cells which gives beneficial outcomes for the individual. Certain phytochemicals like flavonoids, mangiferin, polyphenols show anticarcinogenic [5], antidiabetic properties [6] and even act as antioxidants [7] reducing the oxidative stress of the cells. Flavonoids are the most promising group of plant secondary metabolites which have proven to decrease the rate of tumorigenesis in various types of cancer. Chemically flavonoids are often hydroxylated in positions 3,5,7, 2, 3'', 4' and 5' in the basic flavonoid skeleton structure as shown in Figure 1.

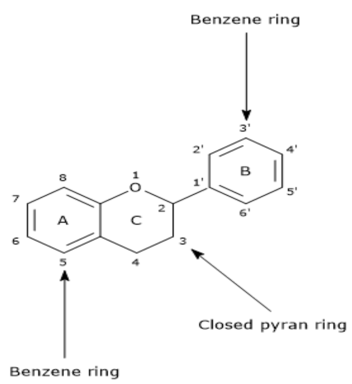

Figure-1: Basic flavonoid skeleton

Flavonoids are classified as flavone, isoflavone, flavon-3-ol, flavanone, anthocyanidin and flavonol according to their chemical structures (Figure 2).

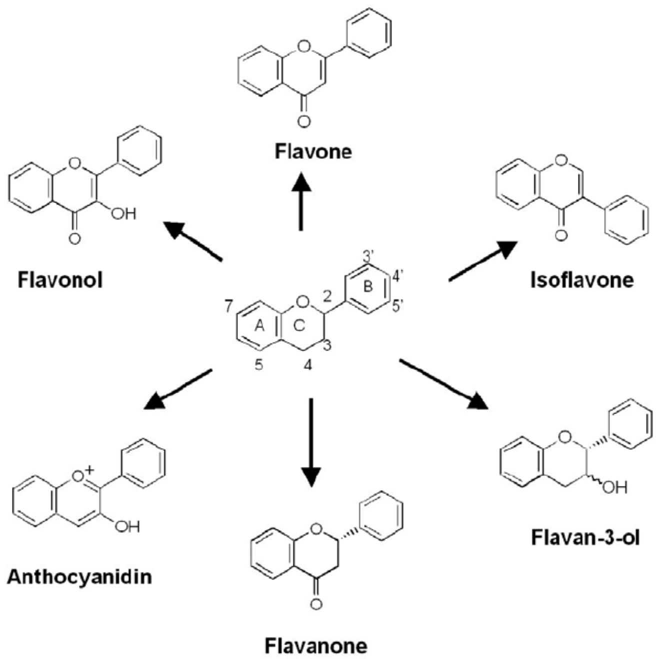

Figure-2: Various derivatives of flavonoids

Most flavones and flavonols exhibit two major absorption bands in UV region. Band I (320-385 mm) represents the B ring absorption and Band II (250-285mm) represents the A ring absorption (A and B rings are shown in Figure 1). Functional groups attached to the flavonoid skeleton may cause a shift in adsorption bands are observed in UV spectrophotometric studies with flavonoids [8], such as at 367nm for kaempferol (3, 5, 7, 4' hydroxyl groups), 371 nm for quercetin (3, 5, 7, 3',4' hydroxyl groups), 374nm for myricetin (3, 5, 7, 3', 4', 5' hydroxyl groups). Flavonoids can be identified by comparing the observed absorption maxima with that of the already reported by other research groups.

The absence of 3- hydroxyl groups (3-OH) in flavones distinguishes them from flavonols. Flavanones have a saturated heterocyclic ring. So, the C ring with no conjugation, present in between the two A and B ring. Flavanones exhibit a strong Band II absorption maxima between 270 nm and 295 nm and only a shoulder for Band I at 326 and 327 nm. Band II with one peak at 270 nm for monosubstituted B ring but as two peaks or one peak at 258 nm with a shoulder (272 nm) one shoulder when di, tri, or O-substituted B ring is present. Natural flavonoids exist in plants in O- glycosides or c-glycosides [9]. O-glycosides are formed by attaching sugar in the hydroxyl oxygen. C- glycosides are sugar moieties combined directly to the flavonoid backbone. O-glycosides can hydrolyze into corresponding aglycones that show similar biological properties as aglycones. When glycosides are formed, the glycosidic linkage is normally located in positions 3 or 7 and the carbohydrates are 1- rhamnose, D-glucose, glucose rhamnose, galactose or arabinose.

With the help of various freely accessible databases, the target enzymes of the flavonoids were determined and then the types of cancer in which the enzyme showed an enhanced activity was determined as a part of the in silico study.

The main objective of this work was to identify the phytochemicals present in the pulp of several unripe mango cultivars and to study their effects on various enzymes involved in tumorigenesis as a part of in silico study.

II. MATERIALS AND METHODS

Thousands of varieties of Mango cultivars are present in India- the ones used in our study are - a) chousa , b) rani pasand, c) saranga ,d) kishen bhog , e) himsagar , f) begum-phuli, g) bel-kusum and h) langra. In this project the mesocarp (pulp) of the unripe mango fruit is used. The objective of the experiment is to identify anticarcinogenic phytochemicals present in the edible part of the mango fruit. Thus mesocarp (the edible part) of the fruit was used.

2.1 Sample collection

On 6.4.22, 8 varieties of unripe mango (Mangifera indica) were collected from the campus of Gurudas College library building. The varieties of the sample collected (and identified by Prof. Gautam Pahari, Dept. of Botany, Gurudas College) are as follows: -

Table 1: Sample number and varieties of mango

Sample no.	Name of the variety of mango
Sample 1	Chousa
Sample 2	Rani Pasand
Sample 3	Saranga
Sample 4	Kisan Bhog
Sample 5	Himsagar
Sample 6	Begum-phuli
Sample 7	Bel-kusum
Sample 8	Langra

2.2 Sample preparation

On 7.4.22 all the 8 varieties of sample were washed and then their flesh was peeled off and the pulp was cut into smaller sections and grinded using mortar pestle and stored in eppendorfs.

2.3 Extraction of Bioactive components

On 13.4.22, 8 test tubes were labeled and 2 ml of ethanol was added in each of them. 300mg of each sample was added to the respectively labeled test tubes and were mixed well and allowed to stand undisturbed for a few hours.

2.4 Phytochemical Screening of extract

a. Test for Carbohydrates

Molisch test was performed for all the 8 samples. 2-3 drops of Molisch reagent was added to a small amount of the analyte in a test tube and mixed well. Next a few drops of conc. sulphuric acid was added dropwise along the walls of the test tube to facilitate the formation of a layer. The development of a purple ring at the layer formed by the conc. acid was found in all the 8 test tubes. Thus it was concluded that carbohydrate was present in all the 8 samples.

b. Test for phenols

Ferric chloride test for phenol:

5% concentrated ferric chloride solution was poured drop wise in 1 mL of diluted extract solution. The appearance of a greenish blue color confirms phenol is present.

c. Test for flavonoids

The aqueous filtrate from each plant extract was combined with 3 mL of dilute ammonia. The solution was then treated with 1 mL of concentrated sulphuric acid (H_2SO_4). Flavonoids were detected in each extract by the presence of yellow color.

2.5 Phytochemical analysis

Variation of Concentration of Polyphenolic Compounds for different varieties of mango with varying pH and time at 37°C was observed. 4 test tubes for each sample were taken. 2 of them were labeled for pH 4 and the remaining were labeled for pH 7. The two different pH were chosen to mimic the pH of our body during digestion. pH 4 represents the pH of gastric juice while pH 7 represents the pH of the rest of the gastro-intestinal tract (GIT). One of the test tubes of pH 4 was kept for 30 minutes at 37°C water bath and the other test tube was kept at 120 minutes at 37°C water bath for each of the samples. Same process was repeated for pH 7 labeled test tubes for each of the samples. pH capsules were used for making the pH 4 and pH 7 solutions and it was ensured that the pH of the solutions were correct with the help of pH meter. Concentration of the solutions were measured using a colorimeter via Folin-Ciocalteu test.

2.6 UV-Spectrophotometric screening for determination of Flavonoids present

9g for each of the eight ethanolic samples UV-spectrophotometric screening was done within the range of 200-450 nm. The peaks and absorbance were noted for determination of the flavonoids that may be present in our samples. To further narrow down our search to determine the flavonoids present in mango pulp extracts, various UV shift reagents such as, NaOH, NaOAc were added into the ethanolic extract and the absorption spectra were analyzed. Different absorption maxima (peaks and shoulders) were observed during UV spectrometric analysis. Due to the presence of UV shift reagents, bathochromic and hypsochromic shifts were noted in different samples and subsequent conclusions were drawn.

2.7 In-silico study for pharmacological activity of flavonoids present in mango extract

In- silico Study for pharmacological activity of flavonoids present in mango extract, was executed by webtools such as Therapeutic Target Database (TTD). The target gene for identified phytochemical is selected from Therapeutic Target Database (TTD) (https://idrblab.net/ttd/) [10]. Biological activity has been determined for the target gene from KEGG (https://www.genome.jp/kegg/pathway.html) [11] database.

III. RESULTS

3.1 Phytochemical Screening of Mango pulp extract

Phytochemical screening confirmed the presence of phytochemicals like phenols, alkaloids, flavonoids, alkaloids and tannins in the ethanolic extract of mangoes as shown in Table 2.

Table 2: Qualitative phytochemical screening of unripe mango pulp extract

Sl. No.	Tests	Mango pulp ethanolic extract
1.	Phenols	+
2.	Flavonoids	+
3.	Alkaloids	+
4.	Tannins	+
5.	Carbohydrates	+

3.2 Variation of Concentration of polyphenols from different varieties with varying pH and time at 37°C

The phenolic content of the test varieties of Mango was measured by using the Folin–Ciocalteu reagent (FCR) which is sensitive to polyphenols. The reagent gives as a blue color complex on reaction with the polyphenols. The F-C assay principle is based on the transfer of electrons. In alkaline medium, these act as reducing equivalents from phenols to phosphomolybdic/ phosphotungstic acid complexes that gives the blue colouration. The concentration of polyphenols in different varieties of mango are present in tabular form and graphical form in Table 3 and Figure 3.

Table 3: Variation of concentration of polyphenols for different varieties with varying pH and time at 37°C

Sl.No.	Sample	Time (minutes)	Concentration at pH 4 (mg/ml)	Concentration at pH 7 (mg/ml)
1.	Chousa	30	1.8	1.28
		120	1.88	1.75
2.	Rani Pasand	30	1.88	1.49
		120	1.88	2.15
3.	Saranga	30	1.11	1
		120	0.73	1.05
4.	Kishen Bhog	30	0.73	1.23
		120	0.78	1.18
5.	Himsagar	30	8.3	4.4
		120	4.4	4.4
6.	Begum-Fuli	30	1.29	0.42
		120	0.83	0.92

7.	Bel-kusum	30	4.4	4.4
		120	4.4	4.4
8.	Langra	30	1.63	1.23
		120	0.83	0.83

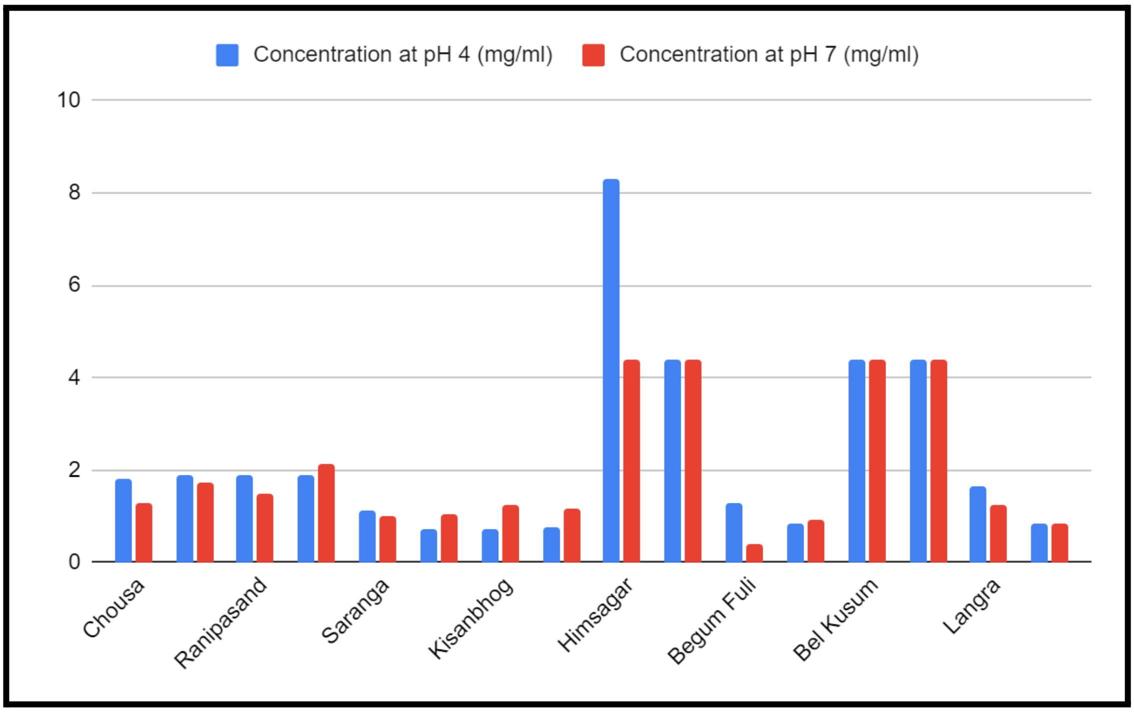

Figure 3: Graphical representation of Variation of Concentration of polyphenols for different varieties with varying pH and time at 37°C

3.3 UV-Spectrophotometric screening for determination of Flavonoids present

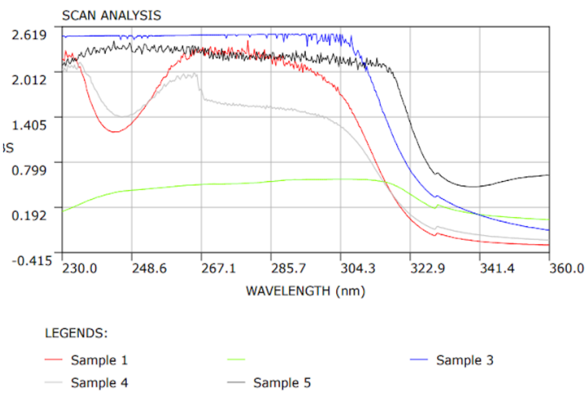

Figure 4: Graphical representation of the UV absorption peaks obtained during collection of UV spectrophotometric data for sample numbers 1,2,3,4,5

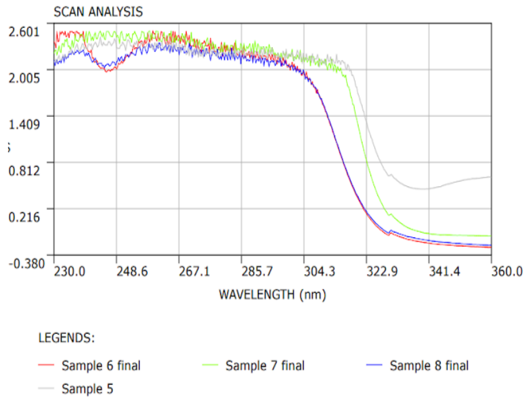

Figure 5: Graphical representation of the UV absorption peaks obtained during collection of UV spectrophotometric data for sample numbers 6,7,8,5

Table 4: Absorption maxima observed in ethanolic solution and the appropriate inference for eight samples

Sample Number	Bands Obtained with Reagent: Ethanol	Inference
1	265 (BAND II)	Might be due to the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone.
	299 (BAND I) 330 (SHOULDER)	Might be due to the presence of (2S,3S)-Dihydrokaempferol-3-O-β-D-glucoside or (2R,3R)-Dihydrokaempferol-3-O-β-D-glucoside.
2	267 (BAND II)	Might be due to the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone.
	289 (BAND I) 330 (SHOULDER)	Might be due to the presence of (25,35)-Dihydrokaempferol-3-O-β-D-glucoside or (2R,3R)-Dihydrokaempferol-3-O-β-D-glucoside.
3	268 (BAND II)	Might be due to the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone.
	295 (BAND I)	No related inference
4	268 (BAND II)	Might be due to the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone.
	296 (BAND I) 330 (SHOULDER)	Might be due to the presence of (25,35)-Dihydrokaempferol-3-O-β-D-glucoside or (2R,3R)-Dihydrokaempferol-3-O-β-D-glucoside.
5	263 (BAND II)	Might be due to the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone.
	302 (BAND I)	No related inference

	354 (SHOULDER)	
6	267 (BAND II)	Might be due to the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone.
	291 (BAND I)	No related inference
7	267 (BAND II)	Might be due to the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone.
	295 (BAND I)	No related inference
8	267 (BAND II)	Might be due to the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone.
	295 (BAND I)	No related inference

Table 5: Absorption maxima observed after addition of the UV shift reagent sodium hydroxide and the appropriate inference for presence or absence of Dihydrokaempferol-3-O-β-D-glucoside

Sample Number	Bands Obtained With Reagent: Sodium Hydroxide	Inference
1	360 (BAND I)	This shift from 299 nm to360 nm may confirm the presence of Dihydrokaempferol-3-O-β-D-glucoside
2	351 (BAND I)	This shift from 289nm to351 nm may confirm the presence of Dihydrokaempferol-3-O-β-D-glucoside
4	386 (BAND I)	This shift from 296 nm to386 nm may confirm the presence of Dihydrokaempferol-3-O-β-D-glucoside

Table 6: Absorption maxima observed after addition of the UV shift reagent sodium acetate and the appropriate inference for presence or absence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone

Sample Number	Band Obtained With Reagent: Sodium Acetate	Inference
2	303 (BAND II)	This shift from 267 nm to 303 nm may confirm the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone

3	340 (BAND II)	This shift from 268nm to 340 nm may confirm the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone
4	245 (BAND II)	No related inference
5	295 (BAND II)	This shift from 263 nm to 295 nm may confirm the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone
6	296 (BAND II)	This shift from 267 nm to 296 nm may confirm the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone
7	295 (BAND II)	This shift from 267 nm to 295 nm may confirm the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone
8	295 (BAND II)	This shift from 267 nm to 295 nm may confirm the presence of 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone

Table 7: Comparison of UV spectrophotometric data for all 8 samples in the presence of different UV shift reagents

Reagents	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Sample 7	Sample 8
Ethanol	265(II) 299 (I) 330 (sh)	267 (II) 289 (I) 330 (sh)	268 (II) 295 (I)	268(II) 296 (I) 330 (sh)	263 (II) 302 (I) 354 (sh)	267 (II) 291 (I)	267 (II) 295 (I)	267 (II) 295 (I)
Sodium hydroxide	360 (I)	351 (I)	386 (I)	386 (I)	386 (I)		311 (I)	387 (I)
Sodium Acetate		303 (I)	340 (I)	245(II)	295(II)	296 (I)	295 (I)	295 (I)

Since sample 5 contains highest amount of phenols, so this sample is selected for further analysis.

Table 8: Absorption maxima observed and the appropriate inference from sample 5

Sample Number	Reagents	Bands Obtained	Inference
5	Ethanol	263 (BAND II) 354 (SHOULDER)	This indicates presence of a free 4'- OH which further might confirm the presence of Kaempferol-3-glucuronide.

5	Sodium hydroxide	356 (BAND I)	This indicates presence of a free 4'- OH which further might confirm the presence of Kaempferol-3-glucuronide.
5	Sodium acetate	295 (BAND II)	This indicates presence of a free OH in the 7th position which further confirm the presence of Kaempferol-3-glucuronide.

So, it can be concluded from the Table 5 that samples 1, 2, 4 may contain Dihydrokaempferol-3-O-B-D-glucoside. From the Table 6 it can be concluded that samples 1, 2, 3, 5, 6, 7, 8, may contain 5-Hydroxyflavone, 7-Hydroxyflavone or 5,7-Dihydroxyflavone. From Table 8 it can be concluded that the sample 5 may contain Kaempferol-3-glucuronide.

3.4.1 In-silico study for pharmacological activity of flavonoids present in mango extract

In silico study was done for the determined compound Kaempferol-3-glucuronide.

3.4.1 Determination of target from Therapeutic Target Database (TTD)

From the TTD [10] database, it was determined that the enzyme that is affected by Kaempferol-3-glucuronide is an aldose reductase. The gene name of target enzyme was found out to be AKR1B1(aldo-keto reductase family 1 member B1).

3.4.2 Functional annotation of AKR1B1 from KEGG database:

It catalyzes the NADH-dependent reduction of a wide variety of carbonyl - containing compounds to their corresponding alcohols with a broad range of catalytic efficiencies. According to KEGG pathway database [11], this target is relevant in the following pathways such as Pentose and glucuronate interconversions, Fructose and mannose metabolism, Galactose metabolism, Glycerolipid metabolism.

3.4.3 Involvement of target gene in cancer

Pan cancer analysis was executed to identify the role of AKR1B1 gene in cancers. For the target gene AKR1B1, a differential gene expression was observed in tumor, normal and metastatic tissues, which was obtained from the TNM plot [12]. It was seen that its expression is higher in metastatic and tumor cells than in normal cells (Figure 6).

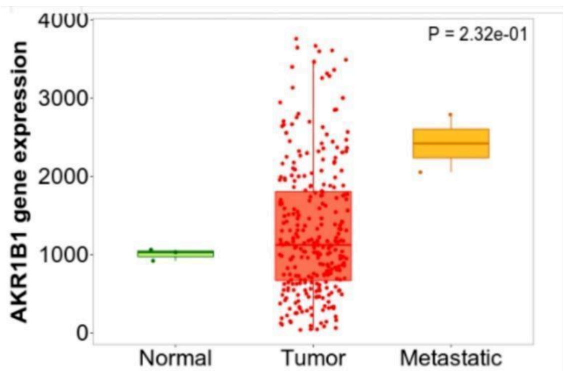

Figure 6: Comparison of AKR1B1 gene expression in normal, tumor and metastatic cells

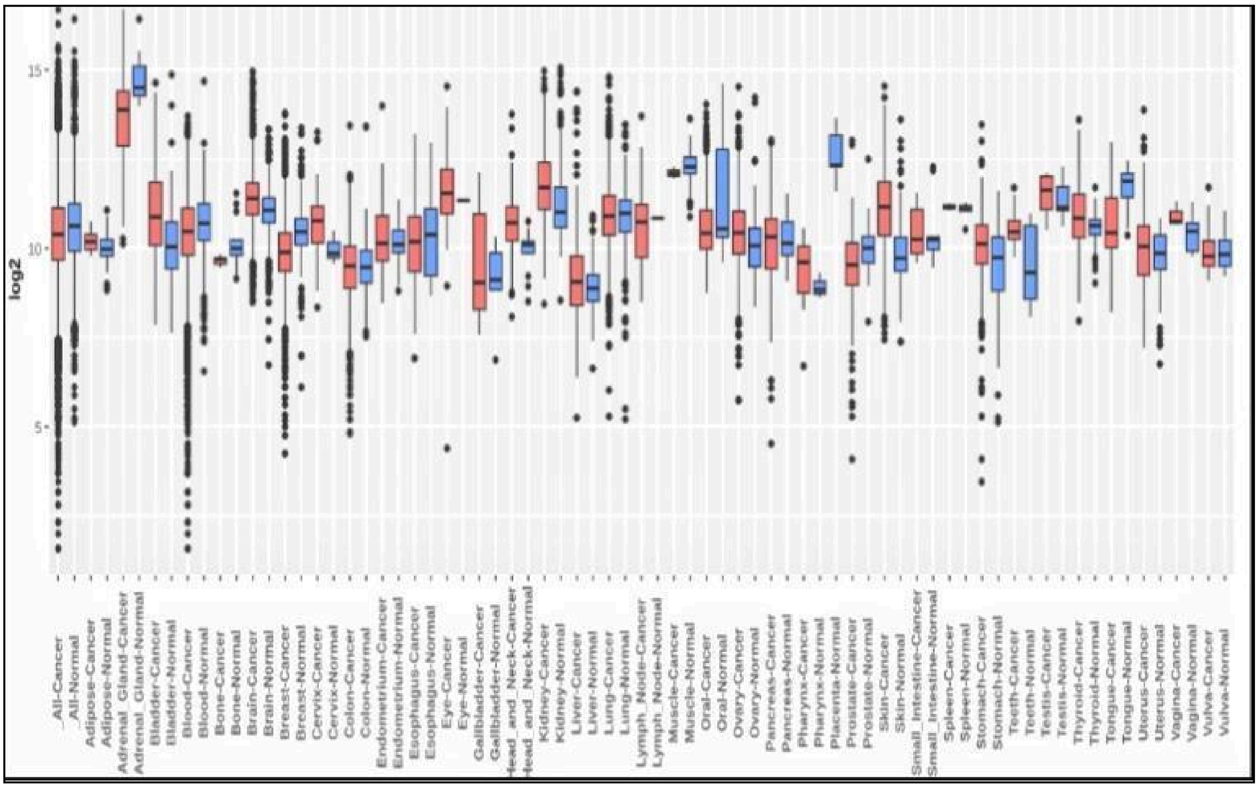

Figure 6(a): AKR1B1 gene expression in head and neck cell cancer is much more compared to normal cells in head and neck

Comparing AKR1B1 gene's involvement in various cancers it was found out that AKR1B1 gene expression in head and neck cell cancer is much more compared to normal cells in head and neck.

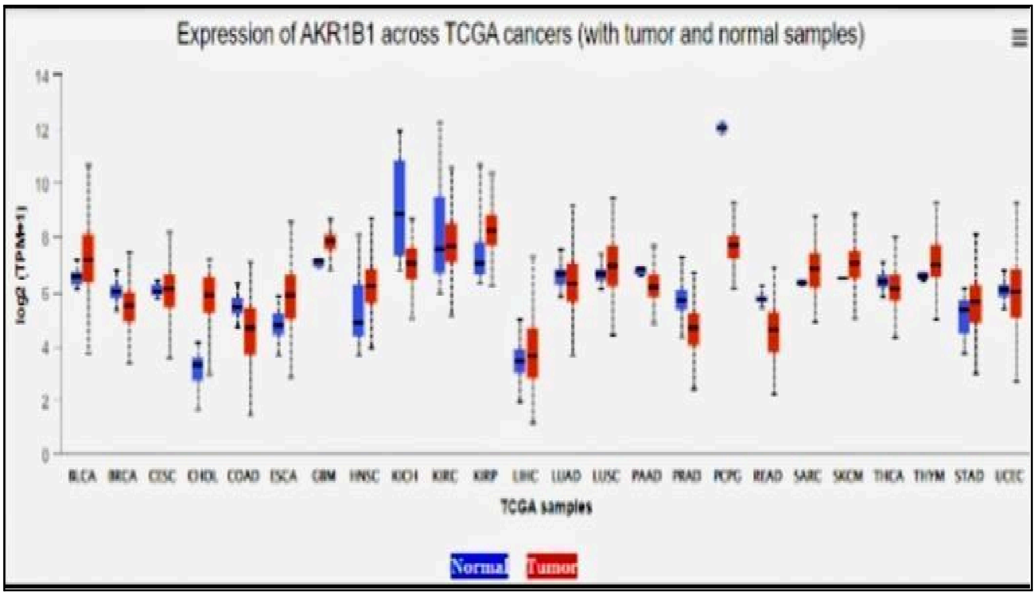

Figure 6(b): AKR1B1 gene expression in head and neck cell cancer is much more compared to normal cells in head and neck (TCGA cancers) [13].

Further analysis of expression of AKR1B1 gene, shows that (Figure 6(b)), in five other types of cancer cells its expression is higher in tumor cells compared to normal cells. They are Liver Hepatocellular Carcinoma (LIHC), Cervical Squamous cell Carcinoma (CESC), Sarcoma, Kidney Renal Papillary cell Carcinoma (KRPCC), and Skin Cutaneous Melanoma (SCM). In all the above mentioned cancer types, correlated genes with AKR1B1 were determined and then analyzed for at least five common genes to form a string correlation. Through rigorous screening, it was found out that AKR1B1, ATP6V1F, GLA, and G6PD were found to be common in HNSC and Sarcoma.

IV. DISCUSSION

Phytochemical screening of mango pulp extract confirmed the presence of phenols, flavonoids, alkaloids, tanins and carbohydrates in all eight samples. Moreover, that the concentration of polyphenol present in mango pulp extract is highest in sample no. 5 i.e. in himsagar variety of mango for both the pH4 and pH7 conditions. Between these two pHs , at pH 4 condition, that is in pH of stomach, the amount of polyphenol is present in higher amount (8.3 mg/ml) compared to that of pH 7 (4.4 mg/ml).

UV spectrophotometric screening of mango pulp extract shows drastic variation in absorption spectra among all samples. Sample no. 1 shows two bands at 299 nm (band I) and 265 nm (band II) with a shoulder at 330 nm. Similarly, sample number 2, 4 also show the three bands in almost same wavelength. Furthermore, for sample no. 5 in addition to two bands at 302nm (band I) and 263 nm (band II), a shoulder band is observed at 354 nm. For three samples with sample no. 6. 7, 8, two bands in UV spectra are observed in 295 nm (band I) and 267 nm (band II) respectively. So, it can be concluded that in addition to 5 hydroxyflavone, 7 hydroxyflavone and 5,7 dihydoxyflavone, a unique flavonoid, 2,3 dihydrokaempferol is also present in different samples such as sample no. 1,2, and 4. For these three samples the bathochromic shift of band I from 299 nm to 360 nm, indicates the presence of dihydrokaempferol-3-O-B-D-glucoside in those samples, in presence of NaOH solution. Moreover, due to presence of shift reagent sodium acetate, the band II, shifts from 276 nm to 295 nm, confirms the presence of 5 hydroxyflavone, 7 hydroxyflavone and 5,7 dihydoxyflavone in different samples.

From the in silico study it can be concluded that Kaempferol-3-glucuronide decreases the catalytic activity of the enzymes coded by the following genes AKR1B1, CLIP4, ATP6V1F, G6PD and GLA. Thus this phytochemical decreases the tumorigenesis in the cancer cell types (Liver Hepatocellular Carcinoma (LIHC), Cervical Squamous Cell Carcinoma (CESC), Sarcoma, Kidney Renal Papillary Cell Carcinoma (KRPCC), and Skin Cutaneous Melanoma (SCM)), where these genes are overexpressed.

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Products of Multiplication, Composition and Differentiation on Weighted Bergman Spaces on the Unit Ball

Chao Zhang

ABSTRACT

We define differentiation operator on H(B) by radial derivative, then we study the boundedness and compactness of products of multiplication, composition and differentiation on weighted Bergman spaces on the unit ball.

Keywords and Phrases: composition operator; multiplication operator; differentiation operator; weighted bergman space.

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Products of Multiplication, Composition and Differentiation on Weighted Bergman Spaces on the Unit Ball

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ABSTRACT

We define differentiation operator on H(B) by radial derivative, then we study the boundedness and compactness of products of multiplication, composition and differentiation on weighted Bergman spaces on the unit ball.

Keywords and Phrases. composition operator; multiplication operator; differentiation operator; weighted bergman space.

I. INTRODUCTION

Let $\mathbb{D}$ be the open unit disk in the complex plane. Let $\mathbb{B} = \{z \in \mathbb{C}^n : |z| < 1\}$ be the unit ball of $\mathbb{C}^n$, and $\mathbb{S} = \partial\mathbb{B}$ its boundary. We will denote by dv the normalized Lebesgue measure on $\mathbb{B}$.

Recall that for $\alpha > -1$ the weighted Lebesgue measure dv_α is defined by

$$dv_\alpha(z) = c_\alpha(1 - |z|^2)^\alpha dv(z),$$

where

$$c_\alpha = \frac{\Gamma(n+1+\alpha)}{n!\Gamma(1+\alpha)}$$

is a normalizing constant so that dv_α is a probability measure on $\mathbb{B}$.

Let $\mathbf{H}(\mathbb{B})$ denotes the space of holomorphic functions on $\mathbb{B}$. Take $1 \le p < \infty$.

Then $f \in \mathbf{H}(\mathbb{B})$ is said to be in the weighted Bergman space $\mathbf{A}^p_\alpha(\mathbb{B})$ if

$$\|f\|^p_{\mathbf{A}^p_\alpha} = \int_{\mathbb{B}} |f(z)|^p dv_\alpha(z) < \infty.$$

Let φ be an analytic self-mapping of $\mathbb{B}$, then the composition operator on $\mathbf{H}(\mathbb{B})$ is given by

$$C_\varphi f = f \circ \varphi.$$

Recently, there have been an increasing interest in studying composition operators acting on different spaces of analytic functions, for example, see [2,3] for details about composition operators on classical spaces of analytic functions.

Let D be the differentiation operator defined by

$$Df = f', \quad f \in \mathbf{H}(\mathbb{D}).$$

Hibschweiler and Portnoy [3] defined the linear operators DC_φ and $C_\varphi D$ and investigated the boundedness and compactness of these operators between Bergman spaces using Carleson-type measure. S. Ohno [4] discussed boundedness and compactness of $C_\varphi D$ between Hardy spaces. Recall the multiplication operator M_ψ defined by

$$M_\psi f = \psi f, \quad f \in \mathbf{H}(\mathbb{D}).$$

A. K. Sharma defined [5] products of these operators in the following six ways:

$$\begin{aligned}
(M_\psi C_\varphi D f)(z) &= \psi(z) f'(\varphi(z)),\\
(M_\psi D C_\varphi f)(z) &= \psi(z)(\varphi'(z)) f'(\varphi(z)),\\
(C_\varphi M_\psi D f)(z) &= \psi(\varphi(z)) f'(\varphi(z)),\\
(D M_\psi C_\varphi f)(z) &= \psi'(z) f(\varphi(z)) + \psi(z)(\varphi'(z)) f'(\varphi(z)),\\
(C_\varphi D M_\psi f)(z) &= \psi'(\varphi(z)) f(\varphi(z)) + \psi(\varphi(z)) f'(\varphi(z)),\\
(D C_\varphi M_\psi f)(z) &= \psi'(\varphi(z)) f(\varphi(z))\varphi'(z) + \psi(\varphi(z)) f'(\varphi(z))\varphi'(z).
\end{aligned}$$

for $z \in \mathbb{D}$ and $f \in \mathbf{H}(\mathbb{D})$.

There are a lot of papers researching these products, see [6,7,8]. Since those results focus on $\mathbb{D}$, naturally, we consider similar questions on $\mathbb{B}$. Of course, the method we used is different from the case on $\mathbb{D}$.

For $f \in \mathbf{H}(\mathbb{B})$, we define the differentiation operator on $\mathbf{H}(\mathbb{B})$ by radial derivative. Recall that for $z \in \mathbb{B}$ and $f \in \mathbf{H}(\mathbb{B})$,

$$Rf = \sum_{j=1}^{n} z_j \frac{\partial f}{\partial z_j}(z) = \lim_{r \to 0} \frac{f(z + rz) - f(z)}{r}, \quad r \in \mathbb{R}.$$

One can see that for $z \neq \varphi^{-1}(0)$,

$$|R(f \circ \varphi)(z)| = \frac{|(Rf)(\varphi(z)) \cdot R\varphi(z)|}{|\varphi(z)|}.$$

Then we also have six ways of products of these operators on the unit ball:

$$\begin{aligned}
(M_\psi C_\varphi R) f(z) &= \psi(z) \cdot (Rf)(\varphi(z)),\\
(C_\varphi M_\psi R f)(z) &= \psi(\varphi(z)) \cdot (Rf)(\varphi(z)),\\
|(M_\psi R C_\varphi f)(z)| &= \frac{|\psi(z) \cdot R\varphi(z) \cdot (Rf)(\varphi(z))|}{|\varphi(z)|},\\
(C_\varphi R M_\psi f)(z) &= (R\psi)(\varphi(z)) \cdot f(\varphi(z)) + \psi(\varphi(z)) \cdot (Rf)(\varphi(z)),\\
(R M_\psi C_\varphi f)(z) &= f(\varphi(z)) \cdot R\psi(z) + R(f(\varphi(z))),\\
(R C_\varphi M_\psi f)(z) &= R(\quad(\varphi(z))) \cdot f(\varphi(z)) + R(f(\varphi(z))) \cdot \psi(\varphi(z))
\end{aligned}$$

for $z \neq \varphi^{-1}(0)$.

In this paper, we characterize the boundedness and compactness of $M_\psi R C_\varphi$, $M_\psi C_\varphi R$ and $R C_\varphi M_\psi$ on the weighted Bergman spaces on the unit ball.

2. $M_\psi R C_\varphi$

For $a, b \in \mathbb{B}$, we will denote $\beta(a, b)$ the distance with the Bergman metric on $\mathbb{B}$. For $r > 0$, let the Bergman metric ball

$$D(a, r) = \{z \in \mathbb{B} : \ \beta(a, z) < r\}.$$

For a point $\zeta \in \mathbb{S}$ and $t > 0$, the non-isotropic metric ball with center ζ and radius t is

$$Q_t(\zeta) = \{z \in \mathbb{B} : |1 - \langle z, \zeta\rangle| < t\}.$$

The following Lemma is Theorm 50 of [9].

Lemma 2.1 Suppose $0 < p \leq q < \infty$, α is real, and λ is a positive Borel maesure on $\mathbb{B}$. Then for any nonnegative integer m with $\alpha + mp > -1$ the following conditions are equivalent.

(a) There is a constant $C > 0$ such that

$$\int_{\mathbb{B}} |R^m f(w)|^q d\lambda(w) \leq C\|f\|^q_{\mathbf{A}^p_\alpha}$$

for all $f \in \mathbf{A}^p_\alpha(\mathbb{B})$.

(b) For each (or some) $s > 0$ there is a constant $C > 0$ such that

$$\int_{\mathbb{B}} \frac{(1 - |z|^2)^s}{|1 - \langle z, w\rangle|^{s+(n+1+\alpha+mp)q/p}} d\lambda(w) \leq C$$

for all $z \in \mathbb{B}$.

(c) There is a constant $C > 0$ such that

$$\lambda(Q_t(\zeta)) \leq Ct^{(n+1+\alpha+mp)q/p}$$

for all $t > 0$ and $\zeta \in \mathbb{S}$.

(d) For each (or some) $r > 0$ there is a constant $C > 0$ such that

$$\lambda(D(a, r)) \leq C(1 - |a|^2)^{(n+1+\alpha+mp)q/p}$$

for all $a \in \mathbb{B}$.

Theorem 2.2. *Let $0 < p \leq q$ and $\alpha, \beta > -1$. Let φ, ψ be a holomorphic maps on $\mathbb{B}$ and $\frac{\psi R\varphi}{|\varphi|} \in \mathbf{A}^q_\beta(\mathbb{B})$. Define a finite positive Borel measure μ on $\mathbb{B}$ by*

$$\mu(E) = \int_{\varphi^{-1}(E)} \left(\frac{|\psi(z) \cdot R\varphi(z)|}{|\varphi(z)|}\right)^q dv_\beta(z)$$

for all Borel sets E of $\mathbb{B}$. Then the following are equivalent:

(1) $M_\psi RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ boundedly into $\mathbf{A}^q_\beta(\mathbb{B})$.

(2)

$$\mu(D(a, r)) = O((1 - |a|^2)^{\frac{q(n+1+\alpha+p)}{p}} \text{ as } |a| \to 1.$$

Proof. Suppose (1) holds. Since $\frac{\psi R\varphi}{|\varphi|} \in \mathbf{A}^q_\beta(\mathbb{B})$, by the definition of μ, we get (see [10, p.163])

$$\begin{aligned}\|M_\psi RC_\varphi(f)\|^q_{\mathbf{A}^q_\beta} &= \int_{\mathbb{B}} \Big(\frac{|\psi(z)\cdot(Rf)(\varphi(z))\cdot R\varphi(z)|}{|\varphi(z)|}\Big)^q dv_\beta(z)\\ &= \int_{\mathbb{B}} |Rf(w)|^q d\mu_(w)\\ &= \|Rf\|^q_{\mathbf{L}^q(\mu)}.\end{aligned}$$

Since $M_\psi RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ boundedly into $\mathbf{A}^q_\beta(\mathbb{B})$,

$$\|Rf\|^q_{\mathbf{L}^q(\mu)} = \|M_\psi RC_\varphi(f)\|^q_{\mathbf{A}^q_\beta} \leq C\|f\|^q_{\mathbf{A}^p_\alpha}$$

holds for all $f \in \mathbf{A}^p_\alpha(\mathbb{B})$. From Lemma 2.1, one can see that

$$\mu(D(a,r)) = O((1-|a|^2)^{\frac{q(n+1+\alpha+p)}{p}} as\ |a| \to 1.$$

Conversely, if (2) holds, also by Lemma 2.1, we have

$$\|M_\psi RC_\varphi(f)\|^q_{\mathbf{A}^q_\beta} = \|Rf\|^q_{\mathbf{L}^q(\mu)} \leq C\|f\|^q_{\mathbf{A}^p_\alpha}.$$

Then, $M_\psi RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ boundedly into $\mathbf{A}^q_\beta(\mathbb{B})$.

The following lemmas were obtained in [11] and [9] respectively.

Lemma 2.3. let $r > 0$, $p > 0$, $\alpha > -1$, then there is a constant C such that

$$|f(z)|^p \leq \frac{C}{(1-|z|^2)^{n+1+\alpha}} \int_{D(z,r)} |f(w)|^p dv_\alpha(w)$$

for all $f \in \mathbf{H}(\mathbb{B})$ and all $z \in \mathbb{B}$.

Lemma 2.4. Suppose $p > 0$, $n+1+\alpha > 0$, then there exists a constant $C > 0$ (depending on p and α) such that

$$|f(z)| \leq \frac{C\|f\|_{\mathbf{A}^p_\alpha}}{(1-|z|^2)^{\frac{n+1+\alpha}{p}}}$$

for all f in $\mathbf{A}^p_\alpha(\mathbb{B})$ and $z \in \mathbb{B}$.

Theorem 2.5. Let $0 < p \leq q$ *and* $\alpha, \beta > -1$. *Let* φ, ψ be a holomorphic maps on $\mathbb{B}$ and $\frac{\psi R\varphi}{|\varphi|} \in \mathbf{A}^q_\beta(\mathbb{B})$. Define a finite positive Borel measure μ on $\mathbb{B}$ by

$$\mu(E) = \int_{\varphi^{-1}(E)} \Big(\frac{|\psi(z)\cdot R\varphi(z)|}{|\varphi(z)|}\Big)^q dv_\beta(z)$$

for all Borel sets E of $\mathbb{B}$. Then the following are equivalent:

(1) $M_\psi RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ compactly into $\mathbf{A}^q_\beta(\mathbb{B})$.

(2)
$$\mu(D(a,r)) = o((1-|a|^2)^{\frac{q(n+1+\alpha+p)}{p}} as\ |a| \to 1.$$

Proof. First suppose that $M_\psi RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ compactly into $\mathbf{A}^q_\beta(\mathbb{B})$. Let $a \in \mathbb{B}$ and consider function

$$f_a(z) = \frac{(1-|a|^2)^{\frac{n+1+\alpha}{p}}}{(1-\langle z,a\rangle)^{\frac{2(n+1+\alpha)}{p}}}.$$

Clearly $\|f_a\|_{\mathbf{A}^p_\alpha} \cong 1$ and f_a converges to zero uniformly on compact subsets of $\mathbb{B}$ as $|a| \to 1$. Since $M_\psi RC_\varphi$ is compact, so for gives $\varepsilon > 0$, we can find $0 < r_0 < 1$ such that $\|M_\psi RC_\varphi(f)\|^q_{\mathbf{A}^p_\alpha} < \varepsilon$ for $|a| > r_0$. Thus

$$\varepsilon > \int_{\mathbb{B}} |Rf_a(z)|^q d\mu(z) \geq \int_{D(a,r)} |Rf_a(z)|^q d\mu(z)$$

for $|a| > r_0$. Since $1-|a|^2 \cong |1-\bar{a}z|$ when $z \in D(a,r)$, so

$$|Rf_a(z)| = \frac{2(n+1+\alpha)(1-|a|^2)^{\frac{n+1+\alpha}{p}}\langle z,a\rangle}{p(1-\bar{a}z)^{\frac{2(n+1+\alpha)+p}{p}}} \cong \frac{2(n+1+\alpha)|a|^2}{p(1-|a|^2)^{\frac{n+1+\alpha+p}{p}}}.$$

Then

$$\mu(D(a,r)) = o((1-|a|^2)^{\frac{q(n+1+\alpha+p)}{p}}$$

as $|a| \to 1$.

Conversely, assume that (2) holds. Let $\{f_k\}$ be a sequence in $\mathbf{A}^p_\alpha(\mathbb{B})$ such that $\|f_k\|_{\mathbf{A}^p_\alpha} \leq M_\psi$ and $\{f_k\} \to 0$ uniformly on compact subsets of $\mathbb{B}$. To show that $M\ RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ compactly into $\mathbf{A}^q_\beta(\mathbb{B})$, it is sufficient to prove that

$$\|M_\psi RC_\varphi(f_k)\|^q_{\mathbf{A}^q_\beta} = \|Rf_k\|^q_{L^q(\mu)} \to 0 \ as\ k \to \infty$$

From Lemma 2.3,

$$\int_{\mathbb{B}} |Rf_k|^q d\mu \leq C \int_{\mathbb{B}} \frac{1}{(1-|a|^2)^{n+1+\alpha}} \int_{D(a,r)} |Rf_k(z)|^q dv_\alpha(z) d\mu(a).$$

Note that $\chi_{D(a,r)}(z) = \chi_{D(z,r)}(a)$ and $1-|a|^2 \cong 1-|z|^2$ when $a \in D(z,r)$. At the same time, $f_k \in \mathbf{A}^p_\alpha(\mathbb{B})$ if and only if $Rf_k \in \mathbf{A}^p_{\alpha+p}(\mathbb{B})$, then by lemma 2.4,

$$|Rf_k(z)| \leq \frac{\|Rf_k\|_{\mathbf{A}^p_{\alpha+p}}}{(1-|z|^2)^{\frac{n+1+\alpha+p}{p}}} \leq \frac{C\|f_k\|_{\mathbf{A}^p_\alpha}}{(1-|z|^2)^{\frac{n+1+\alpha+p}{p}}}.$$

Then, by an application of Fubini's theorem, we have

$$\begin{aligned}
\|M_\psi RC_\varphi(f)\|^q_{\mathbf{A}^q_\beta} &\leq C' \int_{\mathbb{B}} |Rf_k(z)|^q \frac{\mu(D(z,r))}{(1-|z|^2)^{n+1+\alpha}} dv_\alpha(z) \\
&\leq C'\|f_k\|^{q-p}_{\mathbf{A}^p_\alpha} \int_{\mathbb{B}} |Rf_k(z)|^p \frac{\mu(D(z,r))}{(1-|z|^2)^{\frac{q(n+1+\alpha+p)-p^2}{p}}} dv_\alpha(z) \\
&\leq C' M^{q-p}\Big(\int_{|z|\leq r_0} |Rf_k(z)|^p \frac{\mu(D(z,r))}{(1-|z|^2)^{\frac{q(n+1+\alpha+p)-p^2}{p}}} dv_\alpha(z) \\
&+ \int_{|z|>r_0} |Rf_k(z)|^p \frac{\mu(D(z,r))}{(1-|z|^2)^{\frac{q(n+1+\alpha+p)-p^2}{p}}} dv_\alpha(z)\Big) \\
&= I + II.
\end{aligned}$$

Now (2) implies that for a give $\varepsilon > 0$, there is $0 < r_0 < 1$ such that

$$\begin{aligned} II &= C'M^{q-p}\int_{|z|>r_0}|Rf_k(z)|^p\frac{\mu(D(z,r))}{(1-|z|^2)^{\frac{q(n+1+\alpha+p)-p^2}{p}}}dv_\alpha(z) \\ &\le \varepsilon C'M^{q-p}\int_{|z|>r_0}|Rf_k(z)|^p(1-|z|^2)^p dv_\alpha(z) \\ &\le \varepsilon C'M^{q-p}\|f_k\|^p_{\mathbf{A}^p_\alpha} \\ &\le \varepsilon C'M^q. \end{aligned}$$

Since $f_k \to 0$ uniformly on compact subsets of $\mathbb{B}$,

$$\begin{aligned} I &= C'M^{q-p}\int_{|z|\le r_0}|Rf_k(z)|^p\frac{\mu(D(z,r))}{(1-|z|^2)^{\frac{q(n+1+\alpha+p)-p^2}{p}}}dv_\alpha(z) \\ &\le \varepsilon C_1C'M^{q-p}\int_{\mathbb{B}}\mu(D(z,r))dv_\alpha(z) \\ &\le \varepsilon C_1C_2C'M^{q-p}\int_{\mathbb{B}}\mu(\mathbb{B})dv_\alpha(z) \\ &= \varepsilon C_1C_2C_3C'M^{q-p}. \end{aligned}$$

for k large enough. Thus

$$\lim_{n\to\infty}\|M\psi\, RC_\varphi f_k\|^q_{\mathbf{A}^q_\beta} = 0,$$

and hece $M\ \ RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ compactly into $\mathbf{A}^q_\beta(\mathbb{B})$.

Lemma 2.6. [9, Theorem 54] Let $0 < p < q < \infty$ *and* α be any real number, and let λ be a positive Borel measure on $\mathbb{B}$. Then for any nonnegative integer m with $\alpha + mp > -1$ the following conditions are equivalent.

(a) There is a constant $C > 0$ such that

$$\int_{\mathbb{B}}|R^m f(w)|^q d\mu(w) \le C\|f\|^q_{\mathbf{A}^p_\alpha}$$

for all $f \in \mathbf{A}^p_\alpha(\mathbb{B})$.

(b) For any bounded sequence $\{f_j\}$ in $\mathbf{A}^p_\alpha(\mathbb{B})$ with $f_j(z) \to 0$ for every $z \in \mathbb{B}$,

$$\lim_{j\to\infty}\int_{\mathbb{B}}|R^m f_j(z)|^q d\lambda(z) = 0.$$

(c) For any fixed $r > 0$, define the function

$$\widehat{\lambda}(z) = \frac{\mu(D(z,r))}{(1-|z|^2)^{n+1+\alpha+mp}}, \quad z \in \mathbb{B},$$

then $\widehat{\lambda}(z) \in \mathbf{L}^{\frac{p}{p-q}}(v_{\alpha+mp})$.

(d) For any fixed $s > 0$, define the function

$$B(\lambda)(z) = \int_{\mathbb{B}}\frac{(1-|z|^2)^s d\lambda(w)}{|1-\langle z,w\rangle|^{n+1+s+mp}}, \quad z \in \mathbb{B},$$

then $B(\lambda)(z) \in \mathbf{L}^{\frac{p}{p-q}}(v_{\alpha+mp})$.

(d) For any fixed $s > 0$, define the function

$$B(\lambda)(z) = \int_{\mathbb{B}} \frac{(1-|z|^2)^s d\lambda(w)}{|1-\langle z, w\rangle|^{n+1+s+mp}}, \quad z \in \mathbb{B},$$

then $B(\lambda)(z) \in \mathbf{L}^{\frac{p}{p-q}}(v_{\alpha+mp})$.

Theorem 2.7. Let $0 < p \leq q$ *and* $\alpha, \beta > -1$. *Let* φ, ψ be a holomorphic maps on $\mathbb{B}$ and $\frac{\psi R\varphi}{|\varphi|} \in \mathbf{A}^q_\beta(\mathbb{B})$. Define a finite positive Borel measure μ on $\mathbb{B}$ by

$$\mu(E) = \int_{\varphi^{-1}(E)} \Big(\frac{|\quad (z) \cdot R\varphi(z)|}{|\varphi(z)|} \Big)^q dv_\beta(z)$$

for all Borel sets E of $\mathbb{B}$. Let $G(z) = \frac{\mu(D(z,r))}{(1-|z|^2)^{n+1+\alpha+p}}$. Then the following are equivalent:

(1) $M_\psi RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ boundedly into $\mathbf{A}^q_\beta(\mathbb{B})$.
(2) $M_\psi RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ compactly into $\mathbf{A}^q_\beta(\mathbb{B})$.
(3) $G \in \mathbf{L}^{\frac{p}{p-q}}(v_{\alpha+p})$.

Proof. (1)$\Longleftrightarrow$(3). Suppose (1) holds. By the computation before,

$$\|M_\psi RC_\varphi f\|^q_{\mathbf{A}^q_\beta} = \|Rf\|^q_{\mathbf{L}^q(\mu)}.$$

Since $M_\psi RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ boundedly into $\mathbf{A}^q_\beta(\mathbb{B})$, we can find a positive constant C such that

$$\|Rf\|^q_{\mathbf{L}^q(\mu)} \leq C\|f\|^q_{\mathbf{A}^p_\alpha}.$$

Then by Lemma 2.1 and Lemma 2.6, $M_\psi RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ boundedly into $\mathbf{A}^q_\beta(\mathbb{B})$ if and only if $G \in \mathbf{L}^{\frac{p}{p-q}}(v_{\alpha+p})$.

It is clear that (2) implies (1).

It remains to verify that (3) implies (2). Assume that

$$\|f_k\|_{\mathbf{A}^p_\alpha} \leq C$$

and $f_k \to 0$ uniformly on compact subsets of $\mathbb{B}$. It is sufficient to show that

$$\lim_{n\to\infty} \|M_\psi RC_\varphi f_k\|^q_{\mathbf{A}^q_\beta} = 0.$$

By the computation in the Theorem 2.5, we have

$$\begin{aligned} \|M_\psi RC_\varphi f_k\|^q_{\mathbf{A}^q_\beta} &\leq C\int_{\mathbb{B}} |Rf_k(z)|^q \frac{\mu(D(z,r))}{(1-|z|^2)^{n+1+\alpha}} dv_\alpha(z) \\ &= C\int_{\mathbb{B}} |Rf_k(z)|^q G(z) dv_{\alpha+p}(z). \end{aligned}$$

Let $\varepsilon > 0$. Then the hypothesis of (3) implies that there exists $0 < r_0 < 1$ such that

$$\int_{|z|>r_0} (G(z))^{\frac{p}{p-q}} dv_{\alpha+p}(z) < \varepsilon^{\frac{p}{p-q}}.$$

It follows by Holder's inequality that

$$\begin{aligned} & \int_{|z|>r_0} |Rf_k(z)|^q G(z) dv_{\alpha+p}(z) \\ \leq & \left(\int_{\mathbb{B}} |Rf_k(z)|^p dv_{\alpha+p}(z)\right)^{\frac{q}{p}} \left(\int_{|z|>r_0} (G(z))^{\frac{p}{p-q}} dv_{\alpha+p}(z)\right)^{\frac{p-q}{p}} \\ \leq & \varepsilon \|Rf_k\|^q_{\mathbf{A}^p_{\alpha+p}} \\ \leq & \varepsilon C \|f_k\|^q_{\mathbf{A}^p_{\alpha}} \\ \leq & C\varepsilon. \end{aligned}$$

Since $f_k \to 0$ uniformly on compact subsets of $\mathbb{B}$, by Cauchy's estimate, $|Rf_k| < \varepsilon$ for all $|z| < r_0$ and for all $n > n_0$. Thus

$$\int_{|z|\leq r_0} |Rf_k(z)|^q G(z) dv_{\alpha+p}(z) \leq \varepsilon^q \int_{|z|\leq r_0} G(z) dv_{\alpha+p}(z).$$

for all $n > n_0$. Since $\frac{\psi R\varphi}{\varphi} \in \mathbf{A}^q_\beta(\mathbb{B})$ and thus

$$G(z) \leq C\mu(D(z,r)) \leq C\mu(\mathbb{B}) < \infty$$

thus

$$\int_{|z|\leq r_0} G(z) dv_{\alpha+p}(z) \leq C \int_{\mathbb{B}} \mu(D(z,r)) dv_{\alpha+p}(z) \leq C.$$

Then

$$\int_{|z|\leq r_0} |Rf_k(z)|^q G(z) dv_{\alpha+p}(z) \leq C\varepsilon$$

for $n > n_0$. Hence, $M \;\; RC_\varphi$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ compactly into $\mathbf{A}^q_\beta(\mathbb{B})$.

3. $M_\psi C_\varphi R$

Similar to the proof in section 2, we have the following results about $M_\psi C_\varphi R$, here we omit the details.

Theorem 3.1. Let $0 < p \leq q$ *and* $\alpha, \beta > -1$. *Let* φ, ψ be a holomorphic maps on $\mathbb{B}$ and $\psi\varphi \in \mathbf{A}^q_\beta(\mathbb{B})$. Define a finite positive Borel measure μ on $\mathbb{B}$ by

$$\mu(E) = \int_{\varphi^{-1}(E)} | \quad (z)|^q dv_\beta(z)$$

for all Borel sets E of $\mathbb{B}$. Then the following are equivalent:

(1) $M_\psi C_\varphi R$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ boundedly into $\mathbf{A}^q_\beta(\mathbb{B})$.

(2)

$$\mu(D(a,r)) = O((1-|a|^2)^{\frac{q(n+1+\alpha+p)}{p}} \; as \; |a| \to 1.$$

Theorem 3.2. Let $0 < p \leq q$ *and* $\alpha, \beta > -1$. *Let* φ, ψ be a holomorphic maps on $\mathbb{B}$ and $\psi\varphi \in \mathbf{A}^q_\beta(\mathbb{B})$. Define a finite positive Borel measure μ on $\mathbb{B}$ by

$$\mu(E) = \int_{\varphi^{-1}(E)} |\ \ (z)|^q dv_\beta(z)$$

for all Borel sets E of $\mathbb{B}$. Then the following are equivalent:

(1) $M\ \ C_\varphi R$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ compactly into $\mathbf{A}^q_\beta(\mathbb{B})$.
(2)

$$\mu(D(a,r)) = o((1-|a|^2)^{\frac{q(n+1+\alpha+p)}{p}} as\ |a| \to 1.$$

Theorem 3.3. Let $0 < p \leq q$ *and* $\alpha, \beta > -1$. *Let* φ, ψ be a holomorphic maps on $\mathbb{B}$ and $\psi\varphi \in \mathbf{A}^q_\beta(\mathbb{B})$. Define a finite positive Borel measure μ on $\mathbb{B}$ by

$$\mu(E) = \int_{\varphi^{-1}(E)} |\ \ (z)|^q dv_\beta(z)$$

for all Borel sets E of $\mathbb{B}$. Let $G(z) = \frac{\mu(D(\ \ z,r))}{(1-|z|^2)^{n+1+\alpha+p}}$. Then the following are equivalent:

(1) $M\psi\, C_\varphi R$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ boundedly into $\mathbf{A}^q_\beta(\mathbb{B})$.
(2) $M\psi\, C_\varphi R$ maps $\mathbf{A}^p_\alpha(\mathbb{B})$ compactly into $\mathbf{A}^q_\beta(\mathbb{B})$.
(3) $G \in \mathbf{L}^{\frac{p}{p-q}}(v_{\alpha+p})$.

4. $RC_\varphi M_\psi$

In this section, we characterize the boundedness and compactness of $RC_\varphi M\psi$ by using Carleson measures.

Recall that a positive Borel measure μ on $\mathbb{B}$ is called Carleson measure for $\mathbf{A}^p_\alpha(\mathbb{B})$ if there exists a constant $C > 0$ such that

$$\int_{\mathbb{B}} |f|^p d\mu \leq C \int_{\mathbb{B}} |f|^p dv_\alpha$$

for all $f \in \mathbf{A}^p_\alpha(\mathbb{B})$.

Similarly, a positive Borel measure μ on $\mathbb{B}$ is called a vanishing Carleson measure for $\mathbf{A}^p_\alpha(\mathbb{B})$ if

$$\lim_{k\to\infty} \int_{\mathbb{B}} |f_k|^p d\mu = 0$$

whenever $\{f_k\}$ is a bounded sequence in $\mathbf{A}^p_\alpha(\mathbb{B})$ that converges to 0 uniformly on compact subsets of $\mathbb{B}$.

Theorem 4.1. Let $1 \leq p < \infty$, $\alpha > -1$. Let φ be a holomorphic self-map of $\mathbb{B}$ with $\frac{R\varphi}{|\varphi|} \in \mathbf{A}^p_\alpha(\mathbb{B})$ and $\ \in \mathbf{A}^p_\alpha(\mathbb{B})$ such that $R\psi \in \mathbf{A}^p_\alpha(\mathbb{B})$. Define a finite positive Borel measure $\mu_{\varphi,\alpha}$ on $\mathbb{B}$ by

$$\mu_{\varphi,\alpha}(E) = \int_{\varphi^{-1}(E)} \Big(\frac{|R\varphi(z)|}{|\varphi(z)|}\Big)^p dv_\alpha(z)$$

for all Borel sets E of $\mathbb{B}$. Let $d\mu(w) = |\psi(\omega)|^p d\mu_{\varphi,\alpha}(w)$. If for every (or some) $r > 0$, there is a constant $C > 0$ such that

$$\mu(D(a,r)) \leq C(1-|a|^2)^{n+1+\alpha+p} \tag{1}$$

holds for all $a \in \mathbb{B}$, then $RC_\varphi M\psi$ is bounded on $\mathbf{A}^p_\alpha(\mathbb{B})$ if and only if $|R\psi|^p d\mu_{\varphi,\alpha}$ is a Carleson measure on $\mathbf{A}^p_\alpha(\mathbb{B})$.

Proof. First suppose that $|R\psi|^p d\mu$ is a Carleson measure on $\mathbf{A}^p_\alpha(\mathbb{B})$. Then for $f \in \mathbf{A}^p_\alpha(\mathbb{B})$, by the definition of $\mu_{\varphi,\alpha}$, we get (see [10, p.163])

$$\begin{aligned}
\|RC_\varphi M\psi(f)\|^p_{\mathbf{A}^p_\alpha} &= \int_{\mathbb{B}} \Big(\frac{|(R\psi)(\varphi(z))\cdot R\varphi(z)\cdot f(\varphi(z))| + |\psi(\varphi(z))\cdot(Rf)(\varphi(z))\cdot R\varphi(z)|}{|\varphi(z)|}\Big)^p dv_\alpha(z) \\
&= \int_{\mathbb{B}} (|\psi(w)Rf(w)| + |f(w)R\psi(w)|)^p d\mu_{\varphi,\alpha}(w) \\
&\leq \int_{\mathbb{B}} |\psi(w)|^p|Rf(w)|^p d\mu_{\varphi,\alpha}(w) + \int_{\mathbb{B}} |f(w)|^p|R\psi(w)|^p d\mu_{\varphi,\alpha}(w).
\end{aligned}$$

Since $|R\psi|^p d\mu_{\varphi,\alpha}$ is Carleson measure on $\mathbf{A}^p_\alpha(\mathbb{B})$, then

$$\int_{\mathbb{B}} |f(w)|^p|R\psi(w)|^p d\mu_{\varphi,\alpha}(w) \leq C\|f\|^p_{\mathbf{A}^p_\alpha};$$

On the other hand, for $r > 0$, there exits a constant $C > 0$ such that

$$\mu(D(a,r)) \leq C(1-|a|^2)^{n+1+\alpha+p}$$

holds for $a \in \mathbb{B}$, then by Lemma 2.1,

$$\int_{\mathbb{B}} |\psi(w)|^p|Rf(w)|^p d\mu_{\varphi,\alpha}(w) = \int_{\mathbb{B}} |Rf(w)|^p d\mu(w) \leq C\|f\|^p_{\mathbf{A}^p_\alpha},$$

thus

$$\|RC_\varphi M\psi(f)\|^p_{\mathbf{A}^p_\alpha} \leq C\|f\|^p_{\mathbf{A}^p_\alpha},$$

Therefore, $RC_\varphi M\psi$ is bounded on $\mathbf{A}^p_\alpha(\mathbb{B})$.

For the converse, assume $RC_\varphi M\psi$ is bounded. Then there exists a constant $C > 0$ such that

$$\|RC_\varphi M\psi(f)\|^p_{\mathbf{A}^p_\alpha} \leq C\|f\|^p_{\mathbf{A}^p_\alpha}$$

for all $f \in \mathbf{A}^p_\alpha(\mathbb{B})$. Also, there exists a constant $M > 0$ such that $f \in \mathbf{A}^p_\alpha(\mathbb{B})$,

$$\begin{aligned}
\|RC_\varphi M\psi(f)\|^p_{\mathbf{A}^p_\alpha} &\geq M\int_{\mathbb{B}} |R(\psi f)(w)|^p d\mu_{\varphi,\alpha}(w) \\
&\geq M\int_{\mathbb{B}} |f(w)|^p|R\psi(w)|^p d\mu_{\varphi,\alpha}(w) - M\int_{\mathbb{B}} |\quad(w)|^p|Rf(w)|^p d\mu_{\varphi,\alpha}(w) \\
&\geq M\int_{\mathbb{B}} |f(w)|^p d\nu(w) - M\int_{\mathbb{B}} |Rf(w)|^p|\quad(w)|^p d\mu_{\varphi,\alpha}(w),
\end{aligned}$$

where $d\nu(w) = |R\psi|^p d\mu_{\varphi,\alpha}$. From (1) and lemma 2.1, there exists a constant $C > 0$ such that

$$\int_{\mathbb{B}} |Rf(w)|^p |\psi(w)|^p d\mu_{\varphi,\alpha}(w) \leq C\|f\|^p_{\mathbf{A}^p_\alpha}.$$

then exists a constant $K > 0$ such that

$$\int_{\mathbb{B}} |f(w)|^p d\nu(w) \leq K\|f\|^p_{\mathbf{A}^p_\alpha}.$$

Thus, $d\nu(w) = |R\psi|^p d\mu_{\varphi,\alpha}$ is a Carleson measure on $\mathbf{A}^p_\alpha(\mathbb{B})$.

The proof of the following lemma follows on similar lines as in [1, Proposition 3.11].

Lemma 4.2. Suppose $1 \leq p, q < \infty$. Let $T = RC_\varphi M\psi$. Let φ be a holomorphic mapping defined on $\mathbb{B}$ and $\in \mathbf{H}(\mathbb{B})$ be such that $T : \mathbf{A}^p_\alpha(\mathbb{B}) \to \mathbf{A}^q_\alpha(\mathbb{B})(\alpha > -1)$ is bounded. Then T is compact if and only if whenever $\{f_k\}$ is a bounded sequence in $\mathbf{A}^p_\alpha(\mathbb{B})(\alpha > -1)$ converging to zero uniformly on compact subsets of $\mathbb{B}$, then $\|Tf_k\|_{\mathbf{A}^q_\alpha} \to 0$.

Theorem 4.3. Let $1 \leq p < \infty$, $\alpha > -1$. Let φ be a holomorphic self-map of $\mathbb{B}$ with $\frac{R\varphi}{|\varphi|} \in \mathbf{A}^p_\alpha(\mathbb{B})$ and $\in \mathbf{A}^p_\alpha(\mathbb{B})$ such that $R\psi \in \mathbf{A}^p_\alpha(\mathbb{B})$. Define a finite positive Borel measure $\mu_{\varphi,\alpha}$ on $\mathbb{B}$ by

$$\mu_{\varphi,\alpha}(E) = \int_{\varphi^{-1}(E)} \Big(\frac{|R\varphi(z)|}{|\varphi(z)|}\Big)^p dv_\alpha(z)$$

for all Borel sets E of $\mathbb{B}$. Let $d\mu(w) = |\psi(\omega)|^p d\mu_{\varphi,\alpha}(w)$. If for every (or some) $r > 0$, there is a constant $C > 0$ such that

$$\lim_{|a|\to 1^-} \frac{\mu(D(a,r))}{(1-|a|^2)^{n+1+\alpha+p}} = 0$$

holds for all $a \in \mathbb{B}$ then $RC_\varphi M\psi$ is compact on $\mathbf{A}^p_\alpha(\mathbb{B})$ if and only if $|R\psi|^p d\mu_{\varphi,\alpha}$ is a vanishing Carleson measure on $\mathbf{A}^p_\alpha(\mathbb{B})$.

Proof. First suppose that $RC_\varphi M\psi$ is compact on $\mathbf{A}^p_\alpha(\mathbb{B})$. Then by using the similar argument as in Theorem 4.1, there exist a constant $C > 0$ such that for $f \in \mathbf{A}^q_\alpha(\mathbb{B})$,

$$\|RC_\varphi M\psi(f)\|^p_{\mathbf{A}^p_\alpha} \geq C\int_{\mathbb{B}} |R(\psi f)(w)|^p d\mu_{\varphi,\alpha}(w).$$

then

$$\begin{aligned}&\int_{\mathbb{B}} |f(w)|^p |R\psi(w)|^p d\mu_{\varphi,\alpha}(w)\\ \leq\ & C\|RC_\varphi M\psi(f)\|^p_{\mathbf{A}^p_\alpha} + C\int_{\mathbb{B}} |\psi(w)|^p |Rf(w)|^p d\mu_{\varphi,\alpha}(w).\end{aligned}$$

In the above inequality, take $f = k_z(w) \in \mathbf{A}^p_\alpha(\mathbb{B})$, where

$$k_z(w) = \frac{(1-|z|^2)^{\frac{n+1+\alpha}{p}}}{(1-\langle w,z\rangle)^{\frac{2(n+1+\alpha)}{p}}},$$

then

$$\begin{aligned} & \int_{\mathbb{B}} |k_z(w)|^p |R\psi(w)|^p d\mu_{\varphi,\alpha}(w) \\ \leq \; & C\|RC_\varphi M\psi\,(f)\|^p_{\mathbf{A}^p_\alpha} + C\int_{\mathbb{B}} |\psi\,(w)|^p |Rk_z(w)|^p d\mu_{\varphi,\alpha}(w) \\ = \; & C\|RC_\varphi M\psi\,(f)\|^p_{\mathbf{A}^p_\alpha} + C\int_{\mathbb{B}} |Rk_z(w)|^p d\mu. \end{aligned}$$

Since $RC_\varphi M\psi$ is compact on $\mathbf{A}^p_\alpha$ and the unit vectors k_z tends to 0 uniformly on compact subsets of $\mathbb{B}$ as $|z| \to 0$, by lemma 4.2, $\|RC_\varphi M\psi\,(f)\|^p_{\mathbf{A}^p_\alpha} \to 0$ as $|z| \to 0$. On the other hand, sice for every (or some) $r > 0$,

$$\lim_{|a|\to 1^-} \frac{\mu(D(a,r))}{(1-|a|^2)^{n+1+\alpha+p}} = 0,$$

by lemma 2.1,

$$\int_{\mathbb{B}} |Rk_z(w)|^p d\mu \leq \|k_z\|^p_{\mathbf{A}^p_\alpha}.$$

Then, we have

$$\lim_{|z|\to 1^-} \int_{\mathbb{B}} |k_z(w)|^p |R\psi(w)|^p d\mu_{\varphi,\alpha}(w) = 0.$$

thus, $|R\psi|^p d\mu_{\varphi,\alpha}$ is a vanishing Carleson measure on $\mathbf{A}^p_\alpha(\mathbb{B})$.

Conversely, suppose that $|R\psi|^p d\mu_{\varphi,\alpha}$ is a vanishing Carleson measure on $\mathbf{A}^p_\alpha(\mathbb{B})$. Let $\{f_k\}$ be a norm bounded sequence in $\mathbf{A}^p_\alpha(\mathbb{B})$ $(\alpha > -1)$ such that $\|f_k\|_{\mathbf{A}^p_\alpha} \leq 1$ and $\{f_k\} \to 0$ uniformly on compact subsets of $\mathbb{B}$. Now we prove that $RC_\varphi M\ \ $ is compact on $\mathbf{A}^p_\alpha(\mathbb{B})$. By Lemma 4.2, it is enough to show that $\|RC_\varphi M\ \ (f_k)\|_{\mathbf{A}^p_\alpha} \to 0$ as $k \to \infty$. Using the similar argument as before, we have

$$\|RC_\varphi M\psi\,(f_k)\|^p_{\mathbf{A}^p_\alpha} \leq C\int_{\mathbb{B}} |\psi\,(w)|^p |Rf_k(w)|^p d\mu_{\varphi,\alpha}(w) + C\int_{\mathbb{B}} |f_k(w)|^p |R\psi(w)|^p d\mu_{\varphi,\alpha}(w).$$

Since $|R\psi|^p d\mu_{\varphi,\alpha}$ is a vanishing Carleson measure on $\mathbf{A}^p_\alpha(\mathbb{B})$, then

$$\lim_{n\to\infty} \int_{\mathbb{B}} |f_k(w)|^p |R\psi(w)|^p d\mu_{\varphi,\alpha}(w) = 0.$$

Using the similar argument as before, we have

$$\lim_{n\to\infty} \int_{\mathbb{B}} |\ \ (w)|^p |Rf_k(w)|^p d\mu_{\varphi,\alpha}(w) = 0.$$

The proof is finished.

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Large Prime Gaps

Pham Minh Duc

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ABSTRACT

Let pn denote the nth prime. We prove that

$$\max_{p_{n+1}\leq X}(p_{n+1}-p_n) \ll X^{\frac{7}{12+\varepsilon}}$$

$$\max_{p_{n+1}\leq X}(p_{n+1}-p_n) \ll X^{1/2}\log X$$

for any sufficiently large X and any sufficiently small ε.

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for any sufficiently large X and any sufficiently small ε.

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I. INTRODUCTION

Let p_n denote the nth prime, and let

$$G(X) \coloneqq \max_{p_{n+1}\leq X}(p_{n+1}-p_n)$$

denote the the maximum gap between consecutive primes less than X. It is clear from the prime number theorem that

$$G(X) > \big(1+\mathrm{o}(1)\big)\log X,$$

as the average gap between the prime numbers which are $\leq X$ is $\sim \log X$. In 1931, Westzynthius proved that infinitely often, the gap between consecutive prime numbers can be an arbitrarily large multiple of the average gap, that is, $G(X)/\log X \to \infty$ as $X \to \infty$, improving upon prior result of Backlund and Brauer-Zeitz. Moreover, the strongest unconditional lower bound on $G(X)$ is due to Ford, Green, Konyagin, Maynard, and Tao, who have shown that

$$G(X) \gg \frac{\log X \log\log X \log\log\log\log X}{\log\log\log X}$$

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for sufficiently large X, with $\log_k X$ the k-fold iterated natural logarithm of X, whereas the strongest unconditional upper bound is

$$G(X) \ll X^{0.525}$$

a result due to Baker, Harman, and Pintz. Assuming the Riemann Hypothesis, Cramér showed that

$$G(X) \ll X^{1/2} \log X$$

My main theorem is the following further quantitative improvement.

Theorem 1: (Large prime gaps). For any sufficiently large X and any sufficiently small ε, one has

$$G(X) \ll X^{\frac{7}{12+\varepsilon}}$$

For any sufficiently large X and any sufficiently small ε, we have

$$X^{\frac{7}{12+\varepsilon}} \geq p_{n+1}^{\frac{7}{12+\varepsilon}} > p_n^{\frac{7}{12+\varepsilon}} > (\log p_n)^2 - \log p_n > G(X) \qquad (1)$$

(1) is correct when $(\log p_n)^2 - \log p_n > G(X)$ with $n \geq 5$

and $p_n^{\frac{7}{12+\varepsilon}} > (\log p_n)^2 - \log p_n$ when sufficiently large p_n

Indeed, consider $p_n = x$, consider the following limit

$$\lim_{x\to\infty} \frac{x^{\frac{7}{12+\varepsilon}}}{(\log x)^2 - \log x} = \infty$$

We try with $\varepsilon = 0$, (1) is correct when $p_n \geq 246$

Theorem 2: (Large prime gaps). For any sufficiently large X, one has

$$G(X) \ll X^{1/2} \log X$$

For any sufficiently large X, we have

$$X^{1/2} \log X \geq p_{n+1}^{\frac{1}{2}} \log p_{n+1} > p_n^{\frac{1}{2}} \log p_n > (\log p_n)^2 - \log p_n > G(X) \quad (2)$$

(2) is correct when $(\log p_n)^2 - \log p_n > G(X)$ with $n \geq 5$

and $p_n^{\frac{1}{2}} \log p_n > (\log p_n)^2 - \log p_n$ when $n \geq 1$.

Indeed, consider $p_n = x$, consider the following limit

$$\lim_{x\to\infty} \frac{x^{1/2}\log x}{(\log x)^2 - \log x} = \infty$$

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The Earth is Not Flat

Volker W. Thürey

ABSTRACT

Three arguments for a spherical earth are presented. The first is new, as far I know. The other two repeat arguments from the 'old greeks'. I think that it is necessary to record them. For the first argument, we need modern technique like telephones or mobiles. .

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Three arguments for a spherical earth are presented. The first is new, as far I know. The other two repeat arguments from the 'old greeks'. I think that it is necessary to record them. For the first argument, we need modern technique like telephones or mobiles.

INTRODUCTION

Since Nicolaus Copernicus (19 February 1473 – 24 May 1543) we know that the earth is spher ical, but many people believe in a flat earth. I think that it is nonsense, but I cannot change their minds. In this paper I describe three arguments which are not compatible with a flat earth. I don't use things like photography or spaceflight.

The first argument is very easy: Either it is bright or dark outside since it is day or night. Imagine that it is bright, i.e. it is day. Use a telephone or a mobile. Call somebody on the other side of the earth. (Perhaps you wake up the person). It will say that it is dark. If it is dark at your home, it would say that it is bright. This observation is not compatible with a flat earth, except the person lives on the other side of the earth disk.

The second argument refers to an old idea. You need two rods. With a pencil or a ballpoint you make a mark on each of the rods. The marks are placed in such way that on one side the first rod has the same length as on one side of the mark of the other rod. Now you bury one rod up to the mark. You bury the rod as vertically as possible. The same make you with the second rod. You bury it at a different parallel of latidude, for instance, at the equator. The parts above the marks have nearly the same length, i.e. the two rods have the same height. Then you measure the length of the shadows. They are different. This is not compatible with a flat earth.

The third argument uses partial lunar eclipses. The shadow of the earth is always a part of a circle. This observation is compatible with a flat earth, but it requires an earth disk which is always vertical to the moon. This seems to be impossible.

I know that these three arguments will not convince a 'flat-earther', but I think that it was necessary to write them down.

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