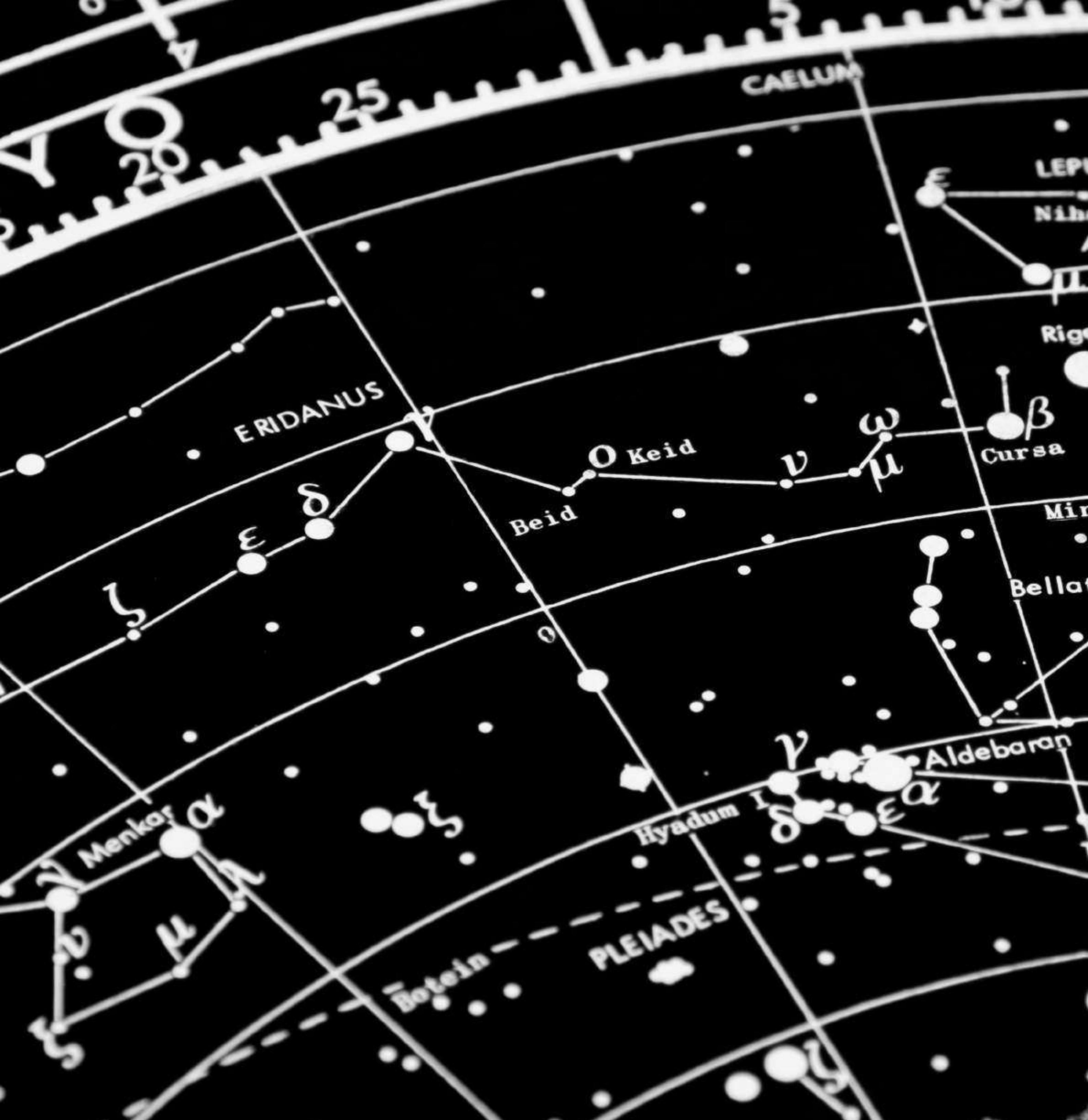

IMAGE: A MAP OF THE STARS OF THE ORION CONSTELLATION

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Reveal the Space-Time Weapon

Sean Yuxiang Wu & Lü Wu

ABSTRACT

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I. INTRODUCTION

"We have a weapon that no one has a clue what it is. And this is the most powerful weapon in the world, which is more powerful than anyone even close." (1) President Donald Trump said on April 9, 2025.

On April 14, 2025, Michael Kratsios, Director of the White House Office of Science and Technology Policy, stated that "Our technologies permit us to manipulate time and space. They leave distance annihilated, cause things to grow, and improve productivity." (2)

So, what exactly is this thing that "can manipulate time and space", or this "time-space weapon"? It sounds like a prop from science fiction, very mysterious, very grand, very scary, and incredibly powerful. What secrets does it contain?

Let us explore and think about it in this article.

Ideas

The sentence "manipulate time and space" contains a verb and two nouns.

The verb is "manipulate," to manipulate and control. The nouns are "space" and "time." So, we must first figure out:

What exactly is space?

What is time?

Then we can understand what kind of things can manipulate time and space?

Let's discuss these questions one by one.

1.1 Space

Please close your eyes and think about it: What is the "space" you perceive as?

Searching for "space", we can see a wide variety of definitions. All of them make sense, but none of them touch upon the essence of space. Let's take a quick look at one of them:

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"Space is a three-dimensional continuum containing both position and direction. In classical physics, physical space is usually conceived of as having three linear dimensions. Modern physicists generally consider physical space to be part of an unbounded four-dimensional continuum called spacetime that extends over time."

This is to treat space as an ordinary object with volume. It also adds many things that have nothing to do with space. But it does not touch upon the essence of space.

We use "cosmic subtraction" to define space. Subtract the universe in front of us, remove the sun, the earth, the Milky Way, the nebula, and all other celestial bodies from the universe one by one; remove light, wind, animals and plants, even thoughts, everything from the universe, and the remaining empty container with nothing in it is space.

This "space" is a place where all the things stored in it can be taken out one by one, and these taken out things can be put back one by one.

Basically, space is an empty volume with nothing in it. Anything can be placed in it according to certain rules.

So, we define space like this:

Space concept consists of the following three parts:

- Ultimate space,
- Subspace (also called space when without confusion, a part of the ultimate space),
- Subspace isolation (also called isolation, Some kind of existence used to distinguish sub-space from ultimate space.)

The ultimate space is "the place that can accommodate anything after taking away everything from the universe in front of us". This includes the so-called Big Bang and everything it produced, including all celestial bodies, electromagnetic waves, thoughts, visible and invisible things, all things with or without mass, including everything in the space outside the Big Bang..... In short, everything is taken away, and what is left is a place that has nothing but can accommodate anything.

The ultimate space is so huge that we don't know its beginning and end. Even if people use the so-called Big Bang imagination, they can't find the edge of the ultimate space, nor can they know the beginning and end of the ultimate space. Therefore, the ultimate space can be tied to time, but it is meaningless. If time is tied to the ultimate space, then it is impossible for people to know the beginning and end of this time. People can't use the scale of time to measure unknown things, such as the place outside the so-called Big Bang universe.

The ultimate space has no form and no mass, so people have no way to change its state, and certainly can't bend it! What are you bending? Bend "nothing"?

Although the ultimate space is right in front of us, we cannot touch it. The ultimate space is all around us, and we are in this space we can only feel it, but cannot touch it. Everything, including imagination, is contained in the ultimate space. The process of all things and their processes occur, develop, born, and die in the ultimate space.

The huge ultimate space itself is a whole. Changing any part of it requires changing the whole. However, due to the hugeness of the ultimate space, it is impossible to obtain that huge energy to change its state. It is impossible to bend, split, decompose, or fold it, etc.

We say the ultimate space doesn't have time, because we don't know the start or end time of it. Or if one argues, ultimate has a time. Then, what does this time exist for? It can't tell us any useful information. It also can't change, can't separate into small pieces of time. Anything that exists inside the ultimate space has its own existing time.

A subspace is a smaller space separated out from the ultimate space according to something existing in the space. It is part of the ultimate space, so it has all the properties of the ultimate space. If there is no this "something" object, there is no such existing time. In the absence of confusion, subspace can be called space.

All subspaces constitute the ultimate space.

The most important thing to note here is not to regard objects existing in space as space itself! Light travels along curved geodesics, and it is the geodesics that are curved, not space. Why are geodesics curved? Because something inside the area of the space caused the curve of the geodesics. It is not space that is curved, but some factors such as gravity that make the geodesics curved. Confusing "space" with "matter existing in space" is a major failure of contemporary physics! It is a mockery of the great scientists who buried their heads and followed Einstein.

Subspace isolation, or simply isolation, is the boundary that separates subspace from ultimate space.

If ultimate space is likened to a huge treasure chest, subspace is one of the various grids in it, and the edge of each grid is subspace isolation.

Some of these subspace separations are tangible, some are intangible, some are completely or partially overlapping, and some are completely independent. "Separation" can be a certain substance, a certain mathematical division, or a certain idea. It can be a certain physical means, or perhaps an intangible product of some thought or imagination... or even a combination of the above.

"Isolation" also has its own form, defined by the things in it.

When discussing space, be careful not to attach the segmentation attribute of "isolation" of subspace to the space itself.

Houses are common subspaces isolated by materials such as building materials. "Thousands of miles of sharing the same moon-lady" is an imaginary subspace isolated by moonlight. "Parallel space" is a subspace that is used as "isolation" by imagination and has not been confirmed to exist.

Depending on the purpose of our research, the division of subspaces can also be varied and diverse, and even completely or partially overlapped.

For example, if there is a magnet in space, then there are many ways to divide the subspace related to this magnet: one is to divide the subspace that contains the volume of the magnet according to its volume, and the other is to divide the magnetic field space according to the some extent range of influence of the magnetic field of this magnet. These two subspaces are of course different. If the magnet is moved to another position, the subspace itself remains unchanged and is still at the same originally located position, but it is no longer the subspace that contains the magnet; and the new subspace that contains the magnet is already in the new subspace position where the magnet is located now.

The same thing happens if we replace the magnet with a celestial body. If something like a geodesic appears because of the existence of this celestial body, then when the celestial body moves to another place, the geodesic will also move with it. The original space is now empty, and nothing has changed there.

Generally speaking, we talk about subspace rather than ultimate space. For example, the space between the Earth and the Moon. Ultimate space is the entirety of space, while subspace is a part of ultimate space. The ultimate space contains everything we know and do not know, but it is not full. Its subspace, because it is part of the ultimate space, has the ability to contain anything. But having this ability does not mean that we can put anything into this subspace at any time. It is conditional. For example, in the subspace occupied by the earth, you cannot put another planet into this subspace.

1.2 Time

As is customary, we should first introduce the various wonderful theories of time that our ancestors have had. Let's choose a definition from the dictionary. The simplified definition of "time" in the Oxford Dictionary is as follows:

As a noun:

1. The existence and events of the past, present and future as a whole continue indefinitely. 2. A point in time measured in hours and minutes after midnight or noon.

As a verb:

1. To plan, schedule, or arrange the time for (something) to happen or be done.
2. To measure the time taken by (a process, activity, etc., or a person who performs the process, activity, etc.).

The most important thing is the first meaning as a noun. Please take a closer look: "The existence and events of the past, present and future as a whole continue indefinitely." Doesn't it feel a bit mysterious, fancy, grand and incomprehensible?

We can search for various definitions of "time" on the Internet, and then think about it: What is the exact definition of "time?"

When people can't even clearly explain what the basic independent "time" is, but mix time and space together to use. Isn't it ridiculous?

III. OUR CONCEPT OF TIME

We believe that time has two faces: scale time, and the trajectory time of things existence, referred to as existence time. Here the meaning of “things” includes all kinds of events.
We use “existence time” to describe “things,” and use “scale time” to measure the “existence time.”
Scale Time

Scale Time is a ruler created and used by Earthlings to measure time. The time concept of Earthlings first comes from the natural rotation phenomenon of the Earth. Scale time is gradually developed by Earthlings according to the rotation of the Earth as day, year, hour...... Scale time is a noun that describes the length of time, and a verb that measures the length of time. These are both very easy to understand concepts.

So, what kind of time can the scale time be used to measure? Of course, it cannot be used to measure an empty space, as we have discussed before. We can only use the scale time to measure the existence time of the object that exists in space!

How do we measure the time of things that exist in space? When something—let us call it thingA— does not exist, we certainly cannot measure it. Once thingA is born, we can label it with the time of existence and start to measure it with scale time.

From the above reasoning process, we can see that "scale time" is a measurement ruler tool defined by humans, and used to measure the "existence time" of the sequential process of the occurrence and development of an object.

In terms of application, strictly speaking, "scale time" is just a unit of measurement to measure time, a standard ruler. It is only used to measure the existence time, and does not belong to the "existence time" itself. The "scale time" commonly used by humans now is just a ruler of existence time defined by earthlings, which strictly measures the process of things in order according to the occurrence, development, evolution, and end of things, and is an unchangeable ruler of existence time.

The traditional definition of scale time is: to use a specified unit of measurement to measure the existence of an object in an irreversible order from the beginning to the end, in the order in which the object occurs and develops. This is the definition of scale time generally accepted by ordinary people, and almost all human beings use this definition of "time" to communicate.

This is like when a person is born, time is used to record his birth, and then as he grows up, he is constantly measured using a time scale that everyone accepts until he dies. When this person grows to 5 years old, the measured time can only reach 5 years, and this person has only existed for 5 years. With each additional year, the measured time of this person increases by one year, and his existence also increases by one year. This way of calculating the existence time of things is irreversible. Even if the person's biological age grows from birth to 10 years old, and then regresses from 10 to 1 year old, the measurement of time is still 0 to 20 years old. When this person dies, the measured time can mark his death time, and the measurement after death will calculate how long this person has been dead according to the agreed time unit. But the living existence time of this person ends at the time he dies.

The actual units of "scale time" are defined by earthlings. They define the units of scale time such as year, month, day, and hour, etc. based on the relative motion of the earth and the sun. If we encounter an alien who has just arrived from a distant planet, he will not know our concepts about time scales, such as year, month, day, and so on. Because it is the Earthling who defines them according to the relative movement of the Earth and the Sun, they are native products. Aliens are not familiar with the Earth. At the beginning of their arrival, they don't know these relationships. Say "one year" to a new arriving alien, or "a light-year," he will not know what you are saying. When people on Earth talk about the measurement units related to "scale time", they understand what the other person is talking about.

For example:

Question: What time is it now?

Answer: 12 o'clock.

The meaning of the definition of the concept of "time" of the Earth is closely related to "scale time." Terms such as years, months, days, etc. not only represent the concept of time, but also act as a recognized scale to measure and use. The more mankind evolves, the more precisely the time scale is defined. On the contrary, if thousands of years ago, somebody said "0.001 seconds," no one would know what he was saying. At that time, there was no demand for using such a short time.

When scale time is used to measure objects, several important concepts preside: "moment" and "time interval." "Moment" refers to a certain point in time, such as now, noon, and so on. A time interval can be determined by comparing two different moments, the length of which between is the value determined by these two moments, such as 8:00 to 12:00.

However, "existence time" can only be measured and described sequentially using "scale time."

Existence Time

"Existence time" is the "time" of something's existence in our usual sense.

Existence time can be explained as "the historical trajectory of something's existence from the beginning to the end", or simply called "existence time".

According to the understanding of space described above, the ultimate space is too huge for humans, and humans cannot find the beginning and end time of the ultimate space. And all subspaces are part of the ultimate space, so of course we cannot know the beginning and end of the ultimate space from them.

Subspace is part of the ultimate space. It is not separated from the ultimate space, but separated by things contained in the ultimate space. Therefore, subspace, like the ultimate space, cannot be measured by time.

Why do we need to divide subspaces? Because subspace is defined by something that exists in some location of the ultimate space. We need to describe this something in the subspace. Subspaces are separated from the ultimate space according to certain conditions of what they contain, and have a certain degree of isolation, so some subspaces can be traced back to the occurrence, change, or end of the division of the subspace. The time of the subspace is attached to the events and things that exist in the subspace. Therefore, we naturally know:

Existence time is only related to things that are in space, and has no direct relationship with space itself. Every event or thing in the universe has different periods of occurrence, development, and end. If we want to describe something, we need to know the specific time trajectory of these different periods relative to the "now" moment.

Existence time applies to all different things. When the process of something speeds up, it's because the rate of existence time of that thing has changed, not because "time" in general has changed (which implies a uniform change in the rate of time for all things). Each specific thing has its own unique existence time and its own unique rate of progress time. Because there is a unified, standard scale of time to measure the rate of all different existence times, we can know the speed of change in the progress of each thing. A person's pulse is normally 65 beats per minute; after running, his pulse becomes 90 beats per minute. Note that the change in this number is because the individual's rate of time has changed. Only under a constant time scale can we know the change in this person's existence time, that is, the change in the speed of the universally recognized time trajectory. If the time measured by the scale of time changes with different events, can we still accurately measure anything?

Therefore, space has no time attribute and has no direct relationship with time. Time acts on things within the subspace, not on the subspace itself.

The subspace constantly adjusts its position as the things contained in it move. When things move to a new position, the subspace moves to the new position. In fact, the isolated movement of the things moves to the new position, and the space itself remains part of the unchanging ultimate space and never moves.

From a philosophical perspective, existence time is an "absolute background" of the existence of things and an indispensable element in describing the process of things. Everything in the universe is performed against this background. Existence time accompanies everything and exists in all activities in the universe, but never deliberately expresses itself. It is unique and omnipresent. When we need it, it appears; when we don't need it, it hides. It can be said that it is such a special background material, or it can be said that it is a ubiquitous concept that can be used everywhere.

Existence time must be linked to things in the subspace. Scale time is a measure of the sequential progress of the existence time of things contained in the subspace.

Consider this question: If multiple twins were traveling on the same spaceship, would they age at the exact same rate? What if one of the twins developed a disease?

Above are common sense. Now we get into some ideals even we have not sensed before. They will let us understand the time concept from a brand-new way.

IV. UNIT OBJECT AND UNIT OBJECT FLOW

Existence time is actually a historical description of the existence of an object that has passed. And the real existence of a thing is at the "now" moment. The future time of a thing is just an expectation or prediction of the development of the thing, an expectation or prediction of the thing that has not yet happened, and does not really exist in the real world.

The Relationship between Existence and Time from the Perspective of a Person's Growth Process Assume that the 7 small pictures in Fig. 1 are a diagram of Li Si's life growth process.

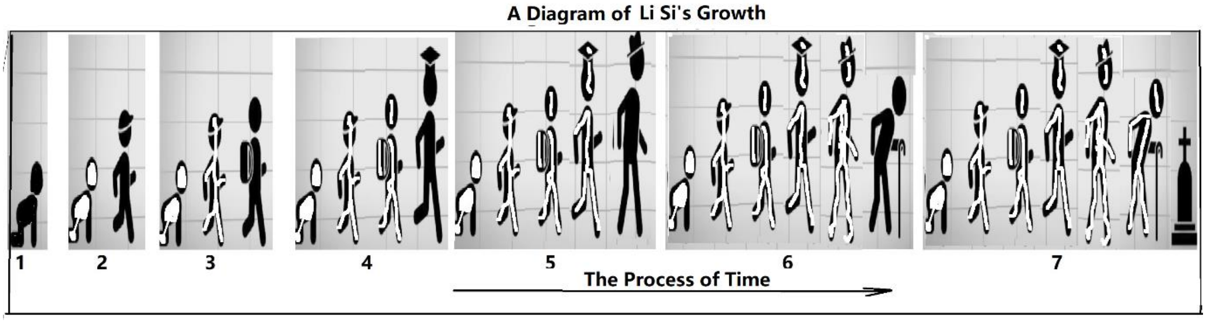


Fig. 1: Diagram of Li Si's Growth

In Frame 1 of Fig. 1, Li Si enters this world, and thus, his existence time begins to exist for Li Si. In Frame 2, Li Si grows into a child. The front figure of frame 2 represents Li Si as a child in the present moment, while the back figure represents Li Si as a baby at the last moment before the present moment (also the previous present moment in Frame 1). Because this baby no longer exists, but merely a historical memory of Li Si, he is depicted as a hollow image.

In frame 3, Li Si's "Now" has progressed a step further and he has become an elementary school student. The baby and child Li Si have become historical memories and no longer exist. In frame 3, two virtual images are used to represent his historical existence.

Similarly, from Frame 1 to Frame 7, the "Now" moment in each successive frame represents Li Si's actual past existence; as time passes, the "Now" in each frame rolls into the past, and the "Now" in the next frame becomes Li Si's true existence. This rolling replacement of the "Now" moment into the future continues endlessly, gradually accumulating traces of Li Si's historical existence into an increasingly rich history, as clearly seen in the gradual accumulation of virtual images from Frame 1 to Frame 7.

After Li Si's death, his time continues to move forward, frame by frame, through the "Now" moment, accumulating ever more historical traces and memories.

1. Summarizing the evolution of Li Si's life, we can begin to appreciate the traces of time and the mystery of the existence of objects:
2. Only after Li Si appeared and existed in the world, did times related to Li Si come into being;
3. Li Si exists only in the present moment, the "Now" moment;
4. Li Si's past is the historical trajectory of Li Si's past and his no longer existing history. This trajectory becomes longer as Li Si's existence increases;
5. Li Si's future doesn't yet exist; it's merely the next moment that Li Si's "Now" can reach. We expect Li

Si's "Now" to smoothly and normally progress to his next future "Now," but it's entirely possible that Li Si's existence at this "Now" moment undergoes a dramatic change, such as the transition from frame 6 to frame 7 in Fig. 1. During this change, Li Si's life disappears, but his information and various other states persist. All the information about Li Si's life from frames 1 to 7 becomes a historical record. Li Si himself continues to exist in a different state in the rolling river of time.

Let's have a more in-depth discussion from a more general perspective.

4.1 Unit Object

We define a Unit Object as something that can exist independently. A man, a car, a planet, etc. are all Unit Objects. The "Li Si" we discussed earlier is also such a unit object.

Using the conclusions drawn from Li Si's above discussion to describe unit objects, we know that the time and existence related to unit objects are: unit objects exist at the "Now" moment; the past of unit objects is its historical trajectory, but it no longer exists in reality; the future of unit objects is the expectation of the rolling evolution of objects from the "Now" to the next moment, and it also does not exist at the "Now" moment.

A unit object is not just a single thing; it is more often a unit object composed of a collection of multiple unit objects. A typical example is that a human is composed of countless molecules and atoms. A large unit object can be broken down into smaller unit objects, or many unit objects can be aggregated into a single unit object. A unit object has various levels of complexity, and can be inclusive or combined. A typical example is when we view the Earth as a unit object, the countless unit objects on Earth, such as humans, animals, and so on, are all contained within this unit object - the Earth.

Various celestial bodies, including the Earth, are composed of countless different units. On the one hand, these celestial bodies themselves can also be considered individual units.

The world is a collection of units, composed of countless different units.

Units make up the world. The laws governing these units are the fundamental laws of the universe. When we discuss the nature of units, their duration should also conform to the laws governing their existence: that is, they exist only in the present moment.

The world exists only in the present moment.

4.2 Definition of the Law of Unit Object Existence

Law of Unit Object Existence: In the process of evolution from birth, through the past, to the present, and into the future, a Unit Object exists only in the moment of "Now."

We call this existence time as UOT.

Specifically:

A unit object begins its own time memory from the moment of its birth, the "Now" moment. The past of a unit object is the time memory of the events of its evolution, which has passed and no longer exists.

The future of a unit object is a prediction of the state that does not yet exist, an expectation of a possible future state.

The "Now" moment of a unit object is the moment of its true existence. This "Now" moment continuously rolls forward into the future, leaves a no longer existing trajectory behind.

4.3 The Temporal Progression of a Unit Object

Consider dividing the growth process of a unit object into intervals, such as "a year" or "a day." We call the day this unit object exists "today," the day that just passed "yesterday," the upcoming day "tomorrow," and the day after "tomorrow" "the day after tomorrow." Similarly, we call the countless days after tomorrow "the future," and the time from "yesterday" to the day this unit object was born "the past." Therefore, all past time is the historical trajectory of the unit object 's past existence, a time when it once existed but no longer exists. The future is not the time when this unit object existed, but rather an unrealized expectation of its future existence. The only time that a unit object truly exists is the present moment!

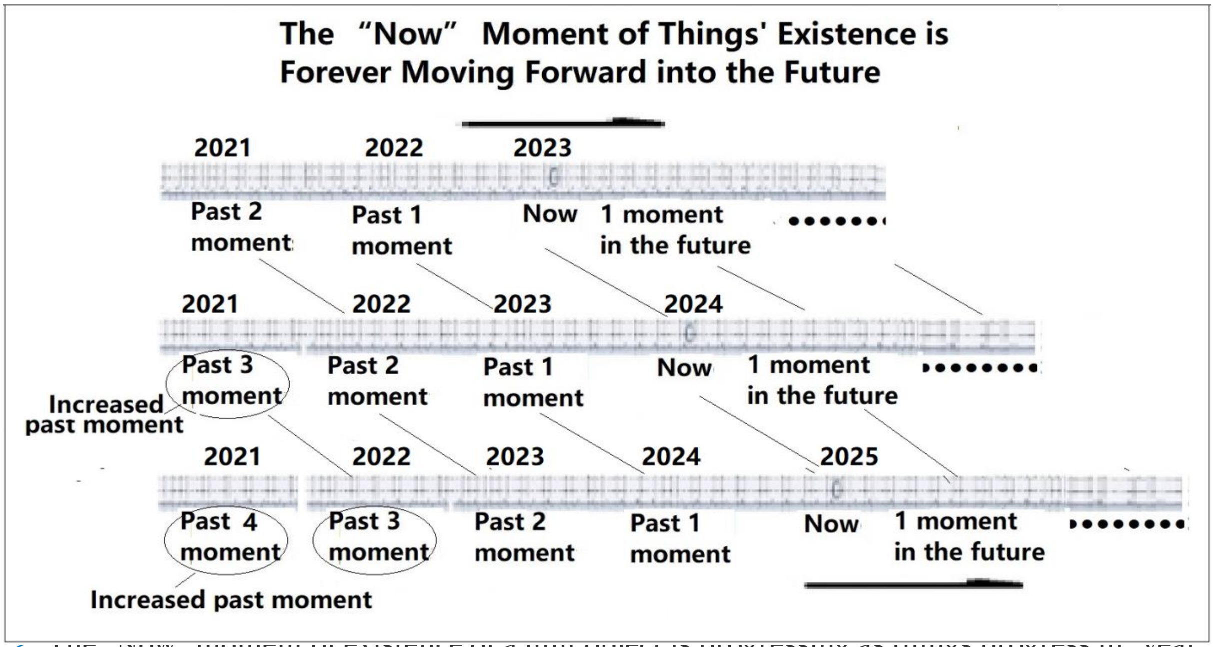


Fig. 2: The "Now" moment of existence of a unit object is progressing as things progress in "year moments"

Fig. 2 illustrates a unit object, born in 2021, progressing year by year from the "Now" moment of 2023 to 2024, one moment in the future. The unit object's overall time advances one moment into the future. The history of this unit object also adds the "past 3 moment" in the left edge of the middle row in the diagram. The bottom row of the diagram shows the unit object progressing from 2024 to 2025. With each step, the "Now" moment advances one moment into the future, and the final history of this unit object adds a record of a past moment. Thus, the unit object evolves moment by moment, accumulating history. The result of this evolution is that the unit object always exists in the "Now" moment, its future is always just one possible development, never exists, and its past leaves an increasingly long historical trajectory from the "Now" moment to its birth. This is the law of evolution of unit objects.

4.4 The growth rhythm of unit objects

Linking the time of unit objects with the rate of their evolution would easily blur the distinction between time and the rate of their development. Time is a standard measure used by all humans, but the time associated with individuals varies from one object to another.

To this end, we propose the new concept of the tempo of development of unit objects to distinguish it from concepts such as time dilation. For example, if an astronaut returns younger than his twin brother who stayed on Earth after a spacecraft travels, we cannot say that time has slowed down for him (Because time is only a constant measure). Rather, we should say that his aging tempo has been slowed down by the impact of the flight.

Conversely, if there is no unchanging time scale, how can we know how fast objects change? Only by measuring with an unchanging time scale can we measure the different rhythms of objects happening and developing.

The scale of time is constant, but the paces of growth of unit objects change with changes in their environmental conditions. These changes shouldn't be interpreted as changes in the velocities of time for those unit objects. For example, if Li Si's heart rate is 200 beats per minute while running, compared to 60 beats per minute at other times, we shouldn't say that the velocity of time has changed while Li Si is running. Instead, we should understand that it's the unchanging scale of time that measures the change in Li Si's heart rate as the intensity of his exercise changes.

Wouldn't it be more reasonable to say that each unit object has its own growth rhythms, and unit objects are not exactly the same. Just as my heart rate is 60 beats per minute, while the average person's is 74 beats per minute, there are always some equal and some different rhythms. When certain conditions change (such as running), the rhythms of these people's hearts change. This cannot be attributed to a change in time itself affecting each unit object, but rather to the influence of something else (in this case, running). It's not that time itself has changed, but rather that the relative rhythm of the unit object has changed. When people use time to describe this change, they are measuring it on an unchanging time scale, thus knowing the relative rhythm of each unit object. If time changes with the changes in the rhythms of different unit objects, then people have no accurate scale to measure the rhythms of unit objects, and therefore cannot measure the differences in the rhythms of different unit objects, or under different conditions.

How long is the "Now" moment?

How long is the "Now" moment?
This is a profound question worthy of more in-depth study.

The "Now" moment refers to the actual duration of a unit object's existence, and it's not easily determined. For example, if Li Si's arm is broken in a car accident at a certain minute, should the "Now" describing Li Si's accident be defined as "minute"?

We haven't figured out the relationship between the "Now" moment and the rhythm of things. We also haven't yet figured out how to determine the actual duration of the "Now" moment for a unit object. There are profound implications here, and interested readers are encouraged to join us in this study.

4.5 Unit Object Flow

A unit object flow is a flowing group composed of a large number of identical unit objects. Rivers, starlight, and so on are examples of unit object flows. Rivers are composed of a large number of flowing individual water molecules, while starlight is composed of a large number of moving individual photons. Unit objects and unit object flows are two complementary concepts that we define.

Previous research on unit object flow has been largely incomplete. Unit object flows refer to groups composed of many units. Previously, unit object flows were studied and discussed from a holistic perspective, treating them as unit objects, leading to some debatable arguments.

We specifically define unit objects and unit object flows to study individual objects and groups of similar objects separately, in order to further understand the nature of existence and time.

World line graphic expression of unit object flow: The propagation, existence and historical trajectory of unit object flow in spacetime.

The world line graphical representation of unit object flow is completely different from the world line representation of the aforementioned unit object.

Fig. 3 below shows observations and studies of multiple sample photons emitted by StarA at different times. The photons emitted at different times form a vast ball of light centered on StarA, filled with countless photons.

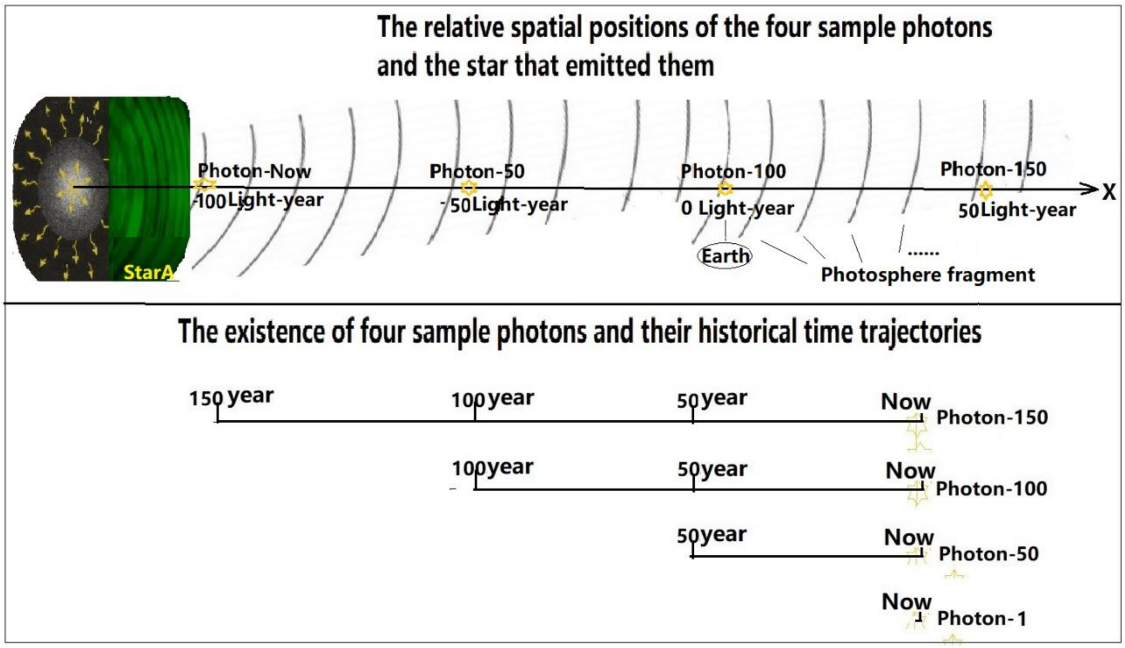


Fig. 3: Preliminary study of the distance and historical time trajectories of four samples of a unit object flow (here, the photon stream of StarA).

The image on the left of the upper frame in Fig. 3 represents a StarA located 100 light-years from Earth. Assuming that StarA is 200 years old. From its birth, it continues emitting photons to the surrounding space in all directions. Drawing an X-axis in one direction as shown in the upper frame of

Fig. 3. We put the names of the photons we are studying at the upper of X-axis, and the distance between the photon and the Earth at lower of X-axis.

Suppose the Earth is 100 light-years away from Star A. Drawing an X-axis in one direction. This axis is densely packed with photons emitted by StarA at various times. The Earth, where is the observer's position is, is used as the coordinate origin. Four of these photons, emitted at different times—150 years ago, 100 years ago, 50 years ago, and now—are selected as sample photons and are named Photon-150, Photon 100, Photon-50, and Photon-Now. The upper frame of Fig. 3 shows the evolution of Photon-150's position.

When this photon was emitted from StarA 150 years ago, it was at Photon-Now's position in the diagram. Fifty years later, it was at Photon-50's position, and Photon-100 appeared at Photon-Now's position, and so on. Finally, these four sample photons are now at the "Now" positions shown in Fig. 3 lower frame.

In this unit photon flow, each unit photon rolls forward from the present moment to a new position at the next moment, while the old position is occupied by the later moment emitted photon from Star A. That is, after the unit photon rolls forward to the next position, the old position is filled by the later emitted and arrived unit photon. This is totally different from the progress of a single unit object.
The historical trajectory time of the unit object flow is shown in the time trajectory diagram in the lower frame of Fig. 3. For each unit photon in the unit photon flow, its historical trajectory and "Now" moment of existence are no different from those of normal unit object events. From this figure we can see that it is not easy to combine time and distance together to draw in one time-space figure. Interested readers can refer to our try in (1-4), and other related work (5-11).

Compare the above description in Fig.4 to the famous light cone in Fig.4 below:

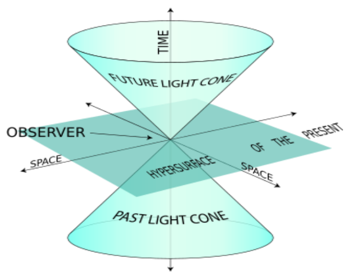


Fig. 4: The latest image of a light cone from Wikipedia (11)

Simply compare this diagram with the previous unit object flow diagram 4, where the observer is at the "present" position, namely, Earth's location. For StarA observed from Earth, only the light emitted by StarA 100 years ago can illuminate Earth at the "present" moment; other past and future light cones do not exist. Of course, there are many problems with this, so we won't address them one by one here.

A further point requires discussion: these descriptions are all about the "light" emitted by celestial bodies, not the celestial bodies themselves. A celestial body cannot move at the speed of light. So, focusing only on "light" without discussing the celestial body that emits this "light" cannot correctly describe the relationship between space and time. What is the significance of the light cone?

Three-dimensional light images, such as the light cone, cannot represent anything in the universe that is not "light." All light originates from a light source. Does studying light mean that we can study all light sources? What if Li Si doesn't emit light? Can you use the light-cone to describe any celestial body itself, not its light?

Compare the above description to the orbit of the unit Earth around the Sun in Fig. 5 below. In its orbit around the Sun, the unit Earth always exists only at a single point on its trajectory. But the Sun's countless photons are always filled with the whole surrounding space.

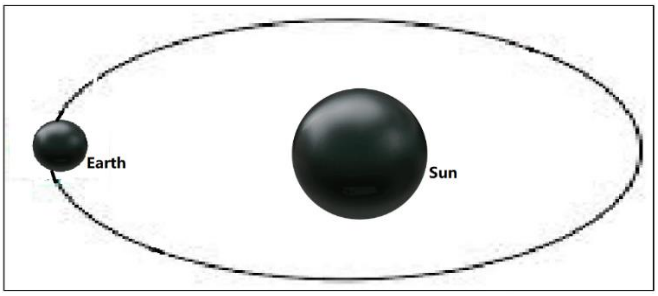


Fig. 5: There is only one Earth or one Sun at any given moment in the moving Earth's orbit around the Sun

This is the fundamental difference between unit objects and unit object flows: a unit object has only one real moment of existence, namely "Now", while a unit object flow is composed of a large number of unit objects that exist at every moment, emitted at different times and last a very long time.

By the way, again, talking about spacetime, space is an empty location host everything. How can an empty space have time? The unit object located in the space has time, not space itself has time. Ultimate space is so huge, the beginning and ending time can't be known by anybody. So, add time, even scale time to this empty space is meaningless.

V. THE EXISTENCE TIME OF UNIT OBJECT AND THE UNIT OBJECT FLOW —THE COMPLETE PICTURE OF TIME

The existence of a moving unit object is a fragmented sequence of constantly evolving moments of existence. From the perspective of a unit object's existence, it is an accumulation of a continuously progressing, "present," fragmented historical trajectory.

How do we calculate the existence of a static unit object? Is its fragmented existence based on its developmental laws, its rate of aging, or other factors?

The progressive rate of the existence of a unit object is somewhat like the concept of differential equations; we haven't fully grasped it yet. Even if a unit object is static, it only exists in the "present" moment; the past is history, and the future has not yet arrived.

We refer to the existence time of a single unit as UOT. UOT is the actual "present" fragment of time in which the unit truly exists. The historical trajectory of the existence time of a unit is referred to as UOTH. It does not include UOT.

The time fragment of UOT is not constant. First, for different units, each has its own UOT. And for the same unit, its duration may change as various conditions change.

The existence time of a unit flow is completely different from UOT. It is a continuous whole of unit objects over a very long period. Because it calculates the time of a large collection of unit objects, its existence time is also a collection of a large number of UOTs. Simultaneously, it is also a description of the continuous evolutionary historical trajectory of this entire UOT collection. We refer to the existence time of a unit flow as UFT.

Thus, we have seen the complete picture of time, which can be summarized as follows:

1. Scale-time is the only ruler created by humankind to measure time. Based on the relative changes in the movements of the sun and earth, and other materials at our disposal, humans have defined different intervals of time, such as years, months, days, and hours, to measure UOT and UFT. This scale spans the ages, describing everything at hand regarding the state of time. It uses the year 1 AD as its origin, depicting the existence of all things within this scale.

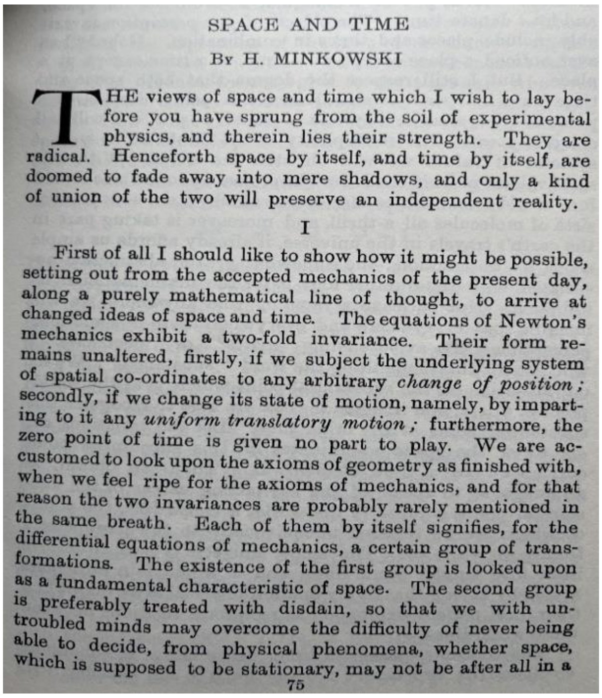
Scale-time is merely a ruler of time. Of course, this ruler was gradually created by humankind, and therefore its existence in various states can be traced. However, these existence times describe the evolutionary state of the scale itself during its formation process and have no special relation to any object it measures.

2. Existence time is a specific description of the existence of various people or things. With the birth of a thing, the existence time of things related to that thing also appears and continues to increase.
3. Based on our research on unit object and unit object flow, existence time must be divided into the existence time of Unit Object (UOT) and the existence time of Unit Object Flow (UFT).
4. A UOT does not perish. Even if the unit object associated with that UOT no longer exists, the UOT that recorded that unit object continues to exist.
5. UFT describes a group of events consisting of a large number of identical unit entities. The number of unit entities within these groups is often unimaginably large. Previously, people often treated unit object flow time and unit object time as equivalent. However, there are actually many differences between the two. Previously, people were not clear about the fundamental difference between scale time and existence time, nor did they distinguish between UOT and UFT, or even have a clear concept of the difference between the two. Therefore, it was impossible to clearly explain the existence time of things.

With these basic concepts in mind, we can easily identify some concepts that have been distorted by our predecessors. For example, the relationship between space and time. How can an eternally existing, empty space be interchanged with the existence of time attached to things? Something like a light cone describes a unit flow, a collection of a large number of photons. It cannot describe the existence of a unit object.

On the concept of binding time and space together to discuss space-time. In scientific research, we try to separate various related factors and study them clearly, and then study the things that have been studied separately together. Space and time should also be studied in the same way.

However, in recent years, the physics community has been discussing space and time together. Because this is what Einstein advocated. And Einstein's idea was copied from Minkovsky's 1908 paper "Space and Time"



SPACE AND TIME

By H. MINKOWSKI

THE views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

I

First of all I should like to show how it might be possible, setting out from the accepted mechanics of the present day, along a purely mathematical line of thought, to arrive at changed ideas of space and time. The equations of Newton's mechanics exhibit a two-fold invariance. Their form remains unaltered, firstly, if we subject the underlying system of spatial co-ordinates to any arbitrary *change of position*; secondly, if we change its state of motion, namely, by imparting to it any *uniform translatory motion*; furthermore, the zero point of time is given no part to play. We are accustomed to look upon the axioms of geometry as finished with, when we feel ripe for the axioms of mechanics, and for that reason the two invariances are probably rarely mentioned in the same breath. Each of them by itself signifies, for the differential equations of mechanics, a certain group of transformations. The existence of the first group is looked upon as a fundamental characteristic of space. The second group is preferably treated with disdain, so that we with untroubled minds may overcome the difficulty of never being able to decide, from physical phenomena, whether space, which is supposed to be stationary, may not be after all in a

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Fig. 6: Minkovsky's 1908 paper "Space and Time"

Here the first paragraph from Minkovsky is such: "The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

First of all, Minkovsky and Einstein did not understand what is the real meaning of "space" and of "time," because there was no concept of UOT and UFT before, how can they understand what is the real meaning of time? And "space-time?"

I don't understand how time can appear in a blank space. Please tell me the meaning of time appearing in a blank space. I also don't know how time can be combined with empty space. Does it make sense to combine time with the empty "nothing"? If we insist on combining time with the ultimate space, what is the meaning of this connection? This combination does not know the starting point and end point of the ultimate space, nor can it describe a small space that has not been defined by subspace isolation, so it has no practical value.

Time only makes sense when combined with something that exists in space. In other words, time is connected to space through things that exist in space. Time never connects to space directly. Time can never be directly connected to space.

About Space-Time Weapon

In summary, the direct combination of time and space is meaningless. Combining the "emptiness" of space with time is to combine time with nothing. Can this combination succeed? Does this combination have any meaning?

From this, we can see that "time-space weapons" are a fantasy without understanding the nature of space and time. There is no scientific content and no practical value.

"Space" cannot be manipulated because "emptiness" cannot be manipulated. "Time" is essentially a rate of the trajectory of the course of events related to something existing in space, and this rate only can be changed by the change of the thing itself.

Trump and Kratsios may want to manipulate the rate of evolution of things in space. I don't know how this sci-fi weapon works. But obviously no one can manipulate the empty space. The scale time is a ruler that can't be changed. The existence time is attached to something existing in space. Each thing has its own existence with a specific development rate for this thing. Its kind of a description of the thing, not the life or essence of the thing. How can one change it with space?

A unit object exists only at "Now" moment. Trump and Kratsios don't even have the ideal of this existence moment, how can they manipulate it?

On the other hand, if there really is such a space-time weapon, why build new fighters and research new weapons? Just manipulate this space-time weapon to kill all the guys you don't like! What's the point of negotiating? Just order them to surrender.

VI. CONCLUSION

Space relates to the existence and time of things within it. That is, space cannot directly relate to the existence time of any thing; the relationship between them is completely independent. Therefore, space cannot influence time. Time, in turn, cannot influence empty space. Space is devoid of everything and cannot influence anything. Do not confuse the interactions between things existing within space, or the interactions arising during the evolution of things themselves, and impose them on the "empty" space itself.

Time, on the other hand, exists because of the existence of things. Without things, there is no existence time for those things; things must exist in some subspace. Only after things exist can their existence time be measured in terms of scale time. If the rate of evolution of this existence time changes, it is due to changes in the things themselves, and is obtained by measuring in terms of scale time and comparing it with the existence rates of other units of things.

Existence time is essentially descriptive, describing different evolutionary states of things. It cannot, in turn, influence the evolution of things.

When we examine existence time from the perspective of unit things and the flow of unit things, we arrive at entirely different conclusions. Unit things that can only exist in the "now" moment cannot be manipulated. How long is the "now" moment exactly? We don't know yet; further research is needed. But we do know for certain that the existence time of unit things is only this "now" moment; the past is history, and the future has not yet arrived. There is no previous research on the existence of such things. In other words, the concept of time has never been clearly understood by humankind.

Given this, what ability do humans have to manipulate time, which they themselves don't even fully understand?

Humans cannot mani ulate time and s ace.

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What's Inside of a Black-Hole in AdS Space

Najmuj Sahadat Sarkar

ABSTRACT

In this paper, i will so the Interior geometry of a worm hole, that's connect one black to a white hole using the theory of Complexity. I ask a question: "what inside of a blackhole in AdS space! If i looked at the complexity curve there i shall get three different part! First one is C-ramp, the linear growth of Complexity with time (from a initial state or a zero state of complexity!) and then i shall get the second part, C-plateau: which defines that "super equilibrium", where the complexity stops growing with time, for a very-long, very-long periods of time! And then a sudden change in complexity Occurs! The complexity drops completely or partially with time to get into the initial state! Which is known as quantum recurrence! The essence of my paper is that, some how from the "complexity curve", we may come-upon with an idea that: What actually happens inside of a "Blackhole" or inside of a "Wormhole"! And Thats Say's much more about the Interior geometry of black-holes! So, Let's get into details!.

Keywords: blackhole, wormhole, whitehole, penrose, diagram, c-ramp, c-plateau, recurrence, quantum, mechanics.

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I. INTRODUCTION

In this paper i wanna talk about (I decided not to give a broad introduction on "Complexity and Black-holes", just focus on some particular problem, which have been concern to me sometimes) a problem on what the black holes do, how do they behave (The interior, when i speak about black hole, i usually mean the Interior of the black hole.) at exponential amount of time. Exponential time means exponential amount of entropy! So there is a very good reason to believe that something different, something perhaps you may call a break-down of "Standard Classical General Relativity", happens inside of a blackhole (at exponential time)! So how do we describe it??? That's i think not known, not well known (I mean, we don't know well) what happens and, i like to discuss the basic idea, when-ever i wrote any paper! I'm not a rigorous mathematician! But, i have some usual sense, between what i write and what i think! In this particular problem, i don't know how long i write or talk about??? Because, it's a big and interesting problem, about the relationship between quantum mechanics and gravity! So, here we have a penrose diagram for black hole (See fig: a)! Lower part of this figure called "Whitehole", and the upper part is called the "Blackhole" region! And i fill-up the diagram with space-like surfaces! And those space-like surfaces are anchored with the AdS-boundary!

This figure has (see fig: a) two sided static spatial boundaries, that means it can be described by the two copies of Confotmal Field Theory! The region between to the left-horizon and right-horizon, you can think of as a Wormhole, connecting two sides! An worm-hole just a growing volume! You can see it from the "figure: a"! The volume of the worm-hole at the middle is "Zero"! Then it's growing and growing, in fact if you looked at the corners (what's going on), it's growing for very long time to enormously large, classically it just grows forever or tends to infinity! And on the white-hole side, the opposite thing happens, from a very large volume it's shrink-down! And the puzzle that i confused for five and a half year; what the meaning of this entire process??? Because, if you stand outside from the black hole and observe it, the black-hole completely stationery! And it's completely clear that it's doesn't have to do, growth and shrink of entropy during a "Boltzmann Fluctuation" or anything like that (it doesn't have to do with entropy!). So it's have to do something else! I think we now, have a large amount of evidence of some form of complexity is responsible for this kind of process! So, what do we know from this space like surfaces??? The spatial volume of them grows linearly with time and the coefficient A_h confirms, that linear growth (A_h: is the area of the horizon)!

$$V=A_h \cdot t$$

That the one thing we know! Other thing that i strongly suspect is that, the complexity of a quantum state (The quantum state of the boundary theory) also grows linearly for some period of time:

$$C=S \cdot T \cdot t$$

And the coefficient which confirms that linear growth is proportional to the entropy (S) of blackhole × the temperature (T)! That is actually a guess, not a rigorous statement! But it's seems to work very well. So if we put this two-formula together and use a little bit of information about the geometry; we find the complexity given by the volume of the worm-hole (V) divided by Newton Constant (G) times Ads-radius!

$$C=V/[G \cdot l(ads)]$$

So, complexity proportional to the volume! Now what did we mean by complexity??? Normally the usual way, to think about complexity: How many gates or how many elementary operation does it takes to produce a certain quantum state (From a reference state which is in some sense very simple)??? So the evolution of state, is also proportional to the number of gates or operations! In other words, we can Say complexity is the number of circuit elements or how hard it is to bring the state back into some simple state! Equivalently In terms of worm hole, how complicated, how difficult it is, to restore, to shrink the worm-hole back to something very small! It's doesn't matter which way you think about "The Complexity"! It's like taking some worm-hole very big and shrunken-down for some purpose to smallest possible "worm-hole", then the complexity measure in terms of "How hard to do that"!!!

So, now let's focus on the Complexity-curve!

It's a conjecture curve! Which we certainly don't know for sure that it's exactly right! But i will assume that; it close to being right! The complexity as a function of time. First, we start with the "Zero Complexity State", i mean the "TFD"/"Thermal Field Double" state, then it's (complexity) start to increase linearly! I called it linear increase complexity ramp or "C-ramp.

And, according to the formula it takes place for a exponential time (as expected)! I will say; 2^n (2 the power n, n: is the number of q-bits that describe the system)! But complexity does not increases as we expected, since the complexity bounded above 2^n–limit (see the fig: b)! So the complexity is now flattened-out, i call it: complexity plateau or "C-plateau" (see the fig: b)! That complexity plateau stay's for enormous amount of time! It's kind of equilibrium but it's kind a super equilibrium, way way beyond the thermal equilibrium! And it last's for very very long time, double exponential in the number of q-bits [$t \sim \exp(2^n)$]! But on that time-scale, "recurrence" can happen! Yes it do happen! Recurrence are more accidental return of the system into original state! Of course it's much more likely that you have a partial recurrence ; but I'm interested now in the full recurrence! The full recurrence is go all the way back, and as i said it's incredibly a rare phenomena! But one of the reason, I'm interested in; especially on recurrence is because it's my claim that a quantum recurrence is describe exactly by the penrose diagram that you see before! The white-hole, the black-hole that is exactly the quantum recurrence! It's last a amount of tme: 2^n (simple exponential); but it take's place in a "C", long long "C" of time (Grand Range Of Time) that this quantum recurrence is happen double exponential time [exp(2^n): rarely]! Now, what do i think??? I think that it's only during this quantum recurrence that the classical description of worm-hole actually makes sense! So i would, kept that view: "The Full Krusskal Diagram" (including black-hole, white-hole) is really a picture of quantum recurrence! And the life of the black-hole is mainly spent (I'm talking about eternal black-hole, not the evaporating one, means eternal black-hole in AdS–space) in this Complexity-equilibrium, or this "Complexity/C-plateau"! Where noting much happen! It's just sit there, and doing very interesting phenomena that i can be describe classically and the view i gonna proposed in a moment!

So the question is: "what happened to the worm-hole at the transition point, between the complexity ramp and complexity plateau???

So what is happening, i mean what is happening according to the "bulk worm-hole" point of view? What happens to worm-hole at exponential amount of time??? And you can also think it in terms of what going on during the long period of equilibrium on the "plateau"?

How do we describe it? I can give you some possible number of answers; firstly "Just Keep Growing"! The just keep growing theory says:

"Nothing happens there, it just keep growing!!!

The whole idea using complexity as a marker for worm-hole size just breaks-down! And the worm hole doesn't care all about complexity, just keep growing. I think it's worng point of view, but though it's just one possible answer!

Anyway, it can be called the "pseudo-complexity. But the other possible answer is; it's stop growing, that intact you can't extend complexity equal volume or C~V, for a exponential amount of time. So if that's true, then is it transition sudden or gradual. In either case, what is the geometry of the interior on the "C-plateau"??? Is there any geometry or it's just a terrible quantum-mechanical mess of complexity??? Actually the geometry doesn't even makes any sense! That's are the questions that i like to answer!!! Or, may be just speak about some partial answers! The partial answers are less like to be report; but i can do as much as i can! Okay, let's denote a state of a worm-hole with volume: V by a ket state: $|V\rangle$.

I want to think the evolution of the state as V-increases (Volume increase)! I wanna think it as a sequence of state in which the volume changes by an amount which is perceptible and measurable! Which means that, the wave function is the function of volume, goes through a sequence almost orthogonal states!

The time it takes for a state go itself to something, which is almost orthogonal i called the "Haranov-abounded" time. As i learn it from "Haranov"! And we can think of the evolution as a sequence of states which are classically distinguishable from each other, or at least distinguishable from each other. And that sense they define a clock! The volume defines kind of clock variables! But eventually you, run out of states! Why that??? Because the hilbert space of the black-hole has finite dimensionality! The dimensionality is roughly~2^n of the black-hole, means 2^n [n: the number of q-bits]! And eventually you run out of orthogonal states! If the theory was integrable, you might expect what happens???

What happens with an ordinary-clock who's states are periodic??? And if you ho some certain amount of time, you get to just before mid-night and just after mid-night [the clock jumps back one clock on exponential time scale, see the figure: c]! That's you would expect if the energy levels were exactly equally spaced! Then the system must be integrable! But that's does not how the chaotic system behave! Chaotic system [just tell you, the way chaotic system are expect to behave, is: "once you reach the exponential amount of time (so you run out of states), the new state of the system beyond that becomes a linear superposition of all the previous states]! On other words, try to make a Wormhole longer than the exponential, you get defeated, and instead, you make a linear super-position of states, of all of the worm-hole lengths with more or less equal probability! Now, that is something i have not seen any paper on worm-hole complexity, i just learnt it by doing thought experiment by myself, but it's may be well known to many people, i think!!!

$$|\Psi\rangle=\sum f(V)\bullet|V\rangle$$

$$f^{*}(V)\bullet f(V)\approx e^{-s}$$

Here, N runs from 1 to 2^n. Okay! now this is firstly represent the volume of worm-hole ; and after a certain amount of time, we see what happens: become a superposition of many many different classical state of the worm-hole! Is that mean, there is a sudden breakdown of classical general relativity???

The classical states don't mean anything! There is no geometry just the super-position of geometry??? I don't think so!!! I think this is somewhat misleading! I will tell you why, as i go long! But i come back that later (just keep it in mind)! I think, the rightthing to understand this "Complexity Geometry"! By complexity geometry, i mean the thing L. Susskind and his collaborators invented! I don't go through the details on it, i just use it in a way to describe "black hole". But if you want to know more about it please visit "The 2nd Law of Complexity" by L.Susskind. And what i really gonna do is a "toy-model" of it; thats come from Brown-Susskind, and A.Hao "toy-model"!

The "toy-model" is useful highly rigorous, but easy to understand! Okay, before we talk about that, lets talk about "complexity in unitary matrices". Unitary matrices have two-rows! One row represent the unitary operators!

Unitary Matrices = Unitary Operators

$$=U_{ij}|i\rangle\langle j|$$

"U" of course the element of SU(2^n) [U∈SU(2^n)]!

The unitary operator acts on the space of n~Q'bits (n~S)! So, one thing we do is to make unitary operators! The other thing we do with the same unitary matrices, that they represent the maximum entangled states! I think i don't need to explain that. U_{ij}, where i, j may be some basis, sometimes called computational basis! So, what is Complexity geometry??? Complexity geometry is simply a geometry on the space of unitaries: SU(2^n). But there is many geometry such on space; and there is very standard one "Bi-invariant Geometry" [I mean Left-Right invariant geometry]; and that's not the one which represent the complexity. The geometry that represent the complexity; is a geometry which you can start with a usual Matrix [Bi-invariant one] and then stretched those directions; which corresponds to difficult direction to move in. The difficult direction of move-in, once which is generated by Hamiltonians in which each-term involves simultaneously many many Q-bits.On other words, non-k'local Hamiltonian! You stretched the directions, make them very costly in length, the directions which you think of hard-directions, and leave small other directions. That pretty much, that i can speak about: "The notion of a penalty factor"! Penalty factor is a stretching factor, where you stretched out the geometry in a great deal of length; And the directions you want to think of as highly complex. So, what we know about such "Complex Geometry"??? First of all it's homogeneous! Homogeneous means same as every where! And i didn't inherent the homogeneity from the group structure SU(2^n).

But it also right invariant [as supposed to "Bi-invariant", and if you don't know what it mean just ignore it]! It has a diameter (diameter means, the largest distance between points) of order 2^n. Now, you may surprised by 2^n. And thats represents simply the maximum complexity! The largest distance between points; or the distance to the identity operator represent the complexity of unitary! The volume of Complexity geometry is much much bigger, the volume is about~exp(2^n). So the diameter is the logarithm of volume "D=log(V)".

Now that is a characteristics of negatively curved spaces, as you move out the volume grows exponentially with the diameter. The another feature of complexity geometry is they negatively curved, or has some particular sectional curvature. Sectional curvature is the curvature of 2d surfaces! All the geometry/most of the geometry are negatively curved.

That's means; when you start out with geodesics, they deviates from each other rapidly or exponentially, and that is a sign of quantum chaos. Means, complexity geometry is negatively curved. And finally "the cut locus".

This is a guess, and also a conjecture about what a complexity geometry should do. The cut locus, is far away from the identity, as it could possibly be, which means the diameter "D" of geometry. I haven't told you what cut locus is, so now I'm going to tell you. If you know the graph theory, you might begin to suspect that this sequence of five things here is very similar to the definition of a "expanded graph"! Diameter-Volume connection, negatively curved space and so forth, it's a good analogy but if you don't know what a expanded graph is, i not gonna use it. So now, what is the meaning of a cut-locus??? Let me tell you, what cut locus is! Because i thing it's play a central role of my story! The cut-locus of a point is the collection of all cut-points related to that point. So what is a cut-point? Take a geodesic straight at point "p" (the cut-locus of point "p" or the cut points associated with "p")! Take a geodesic in the geometry, in the complexity geometry or any other geometry, and take a geodesic through it, or starting at it! Now lets label that geodesic a(t). a: just represent a point, and t is just the parameter along the curve. It could be the time, but let's assume it's a some kind of parameter. Could be the length of the curve itself [The length of time you may consider]. You can expect that, for some sufficiently short time that geodesic that i have drawn (see the figure: d) is the shortest geodesic connecting p with a(t)! You can always be sure, for some length of time any given geodesic is the shortest geodesic that Connect's the point a. But at some point typically that a labeled t_c (c: for cut), the another geodesic of equal length $\gamma(t_c)$ will emerged and it's cross a(t)! Let's imagine, there is some Short's of obstacles, it may be the topological obstacles or a big hill in the geometry that you have go around (see fig: d) There may other geodesic in addition the big geodesic (which goes around the hill). But typically for short times, that 2nd geodesic that point "p" with a(t) could be longer than the primary geodesic I'm started with! But as you let "t" increase, typically you would come to a point where some other geodesic $\gamma(t_c)$ will have the same length as the primary geodesic, beyond that $\gamma(t)$ is shorter than the primary geodesic! In other words; you reach a point, in which the shortest geodesic connecting "p" with "a", is a member of family $\gamma(t)$: that point is called "cut-point". And the "cut-locus" is the collection of all cut-points that you get by sending any number of geodesic from the point "p". The cut-point represent a transition, in the behavior of shortest geodesic. If the shortest geodesic represent the complexity; then the cut-locus or the cut-points are the points; in which the character of the shortest geodesics discontinuously changes; the length of the shortest. The length of the shortest geodesic doesn't discontinuously change, it's contentious but the first derivative of it's does!

II. DISCONTINUOUS FUNCTION

L[p, a(t)] IS CONTINUOUS AT CUT POINT
BUT (dL/dt) IS NOT!!!

And so, something starts happen to shortest geodesic at the cut locus. Why its interesting??? Because, if the length of geodesic represent complexity, and if complexity something have to do, with the Interior of the black holes, then the cut locus is the place, where the sudden sharp phenomena or transition can potentially take place! Let's go further! I have drawn here (see fig: d), i don't know what i called??? May be 2nd order cut.

Now again starting with "p" you go little ways, and you come to the cut-locus; with a "black-family" (incidentally notice that, the shortest geodesics after the cut-point not a single geodesics, anymore but a family of geodesics), and as you move along the main curve the sequence of "black-curves" are the shortest geodesics until you come to a 2nd point! At some 2nd point, the "geodesic" ho through different obstacles, might suddenly emerged as shortest geodesic! And so for a typical geometry you find a whole sequence of cut points like this, and rather complicated answer of this question of cut-points like this (see fig: e), and rather complicated answer of geodesic this question how the energy of the shortest geodesics vary as you increase t/time. It doesn't even have to vary monotonically incidentally. The shortest geodesic can get shorter for a period of time [even though the main-curve get longer]! So, that's sequence of curves represent the transition in the behavior of shortest geodesics (see fig: e). Okay, now i wanna come to the "Toy model of complexity geometry"! Apply, this idea for "cut-locus", to a model of complexity geometry. The model is first proposed by Brown, Susskind, and Z.Hao!

It seems to rather good job, even though it's tremendously simplified, simplified down to 2d. So it's kind a good job, represents a lot of what you expect from complexity. Okay, so what is the model??? The model is "The Poincarè disc".

2d, uniformly-negative curved surface mapped on to a disc (which is 2d)! On this disc i also want to put an inscribed polygon. The side of the polygon is geodesic. They are geodesic on a negatively curved space! And on the top of every-thing else we gonna make an identification (see Made with Xodo PDF Reader and Editor fig: f)! Make identification by identify the sides randomly. Now you have to been little bit of careful, there is one constraints, that you don't make conical singularities (it's a small constrain, you can satisfy it)! But upto that constraint's you are allowable to make random identification. So how many sides there? The number of sides:

$$\text{SIDES}=4\times[\text{GENUS Of The Riemann Surface}]$$
$$=\text{Area Of Total Riemann Surface.}$$
$$\approx\exp[2^n]$$

So the "Number of sides"~4 GENUS

~ AREA;"→all three of are enormously large, the exponential of 2^n! Now that's connect to the fact that the volume; of $SU(2^n)$ is exponentially large! And i wanna represent that volume, of $SU(2^n)$ is exponentially large. So:

DIAMETER=Log(Area)

$$=\text{Log}[\exp(2^n)]$$
$$=2^n$$

So, that's the model of complexity geometry! Only other statement is that the identity operator is identified by the center of the disc (see fig: f). Now, what is the diameter of the geometry??? The diameter of this geometry is the distance from the origin (from the identity of the polygon); and it's not hard to see this corresponds to the logarithm of the area. Means diameter~2^n. And that's also indicate the largest complexity (The largest distance from the origin is: 2^n)! Now, concentrate on the next picture! This picture (see the figure: g) has a line that connect the origin (or, the identity) to the boundary! Suppose it represent a trajectory, in which the complexity increases. The point a(t): That's the moving point, moves out, we can think of it as: e^⁻iHt.There is an important point about geodesics and e^⁻iHt. In general geometry; e^⁻iHt actually generated a curve in geometry; but not necessarily a geodesic. It's only if it is both geodesic and killing direction both match with the thing generated by e^⁻iHt. So that's the important thing to know. Following a particular geodesic; generated by some Hamiltonian, and just following it out on this model for the complexity geometry, we can see it in terms of a series of unitary operators, but of course the geometry is simple to really represent that!

$$a(t)=e^{-iHt}=U(t)$$

Complexity Grows Linearly.... (I.E Complexity Ramp)

So, as you move from the origin the complexity grows linearly as you move the distance from the origin! It grows linearly with the parameter we are talking about could be the distance itself, and obviously the distance grow linearly with the complexity! So what that representing? That, representing the "complexity-ramp"! The linear growth of complexity as you move away from the reference state. So, that's first of all.

And how long it's last? It last the diameter of the geometry, sometimes~2^n. What happens next??? Now, we use random identifications. The next picture is the random identification (see the figure: h) of a particular point end of "Mid-line" with some other point or the other side of the polygon! So the "outer-line" just make sure to guid your eye. Actually there is no any "outer-line" it just a jump! But strictly speaking, there actually no jump on the geometry either. Its just an identification! So let me called it a "Jump" (see the fig: h)! The jump across a new point. Where you reenter in the geometry; and since you out from the maximum distance, all you can do just go-in, a little bit. But because there are so many states; so many points out near the boundary; what tends to happen overwhelmingly probable that you not go very far!!! Just return to the polygon again. And there is double exponential probability for this to be happen. Means we won't get very far; just bounce around among the maximally complex states you like! If i plotted the growth of Complexity or the distance from the origin you would see the linear growth; you will get-up the polygon (i.e "outer line") and you will see in the graph that: complexity decreases for a short period of time and then sudden increases. Which i drawn in the figure here! Now, let's keep going!

In the next figure (see the figure: i) we go-out from the origin, we swing around, we hit the polygon, we jump to some new point, so what's actually happens??? Very quickly we exit the geometry again, go to some new point: go in, exit the geometry little bit, and so forth so on...! Almost endlessly (because there are so many points outside the boundary, that represents the maximum complex states; and you just bounce around them for exponential amount of time! But every once in a while by accident you are just happen to hit the polygon, just such a way; that you wind-up coming inward along a radial direction and get back to the origin or very close to the origin. That is "Quantum Recurrence". It takes double exponential amount of time. And just you can see; this model reproduces the "complexity-ramp", "complexity-plateau", and existence of very very infrequent "recurrence"! it's produce lot of other things about complexity, from which i get into now: "The Switch Back Effect"! Okay, now let's try to follow: "shortest-geodesic"! The shortest geodesic (see the figure: j), once you get-out the polygon and make a jump, the shortest geodesic will also be jump. you will have to hit the cut-locus. You can see that from the picture (see the fig: j); once you re'enter the geometry, the geodesic (outer geodesic) longer than the inner-geodesics which connect the origin more directly with moving point a(t). In other words, the polygon is the "cut-locus" (see the fig: j). On other words at the point in which the transition of the "C-plateau" happens. In particular let's looked at the geodesics, once which are more direct and shorter than "outer-geodesics"! Those are also, be generated by a kind of Hamiltonian flow, but in general they are not the Hamiltonian flow's which are generated by a time independent Hamiltonian (or any curve on the geometry can be represented as a flow or Hamiltonian evolution as a time dependent Hamiltonian). And so the expectation (if you know little bit about this things) is that a typical geodesic is just drawn from the class of geodesics, will be something generated by a time dependent Hamiltonian. So when you get to the point "a", [just re-enter the geometry] the black curve generated by a Hamiltonian H_a. By the time you get to the point "b", another Hamiltonian will generate the curve, that's connect to the pont "b". By the time you get to the "c" a diffrent Hamiltonian connect the point "c". And finally you get to the "d", you get a diffrent one. What's going on actually??? I claim that the worm-hole or the evolution of the worm-hole is contentiously changing [Is being generated instead of by a time-dependent hamiltonian; by a sequence of time dependent hamiltonian] between the point "a" and "d"! But then what happen when you get to the point "d" (see the fig: j)! When you get to the point "d", you get to the boundary again and you jump to some place else; a discontinuous transition happen, and that discontinuous transition means, the length of the geodesics doesn't change, but the whole character of the hamiltonian which generates it [The whole internal structure of it], will change discontinuously! This discontinuous changes (short time interval of contentious change), they are generated noies on the top of the complexity curve)! So what it say about worm-holes??? It Say's, as you make one of this jumps the whole internal structure of the worm-hole changes! Not in length; but the internal structure; you might think it in terms of "Tensor Networks", so the character of the Tensor networks jump from one tensor network to another! And if it's true, the hamiltonian which generate this flows is must time dependent. And that translate into the worm-hole is inhomogeneous. It's generated in a way which is dependent on way you are in worm-hole. And what it should do; it creates an inhomogeneous behavior to the worm-hole! In other way to say it; the worm-hole probably full of matter, with an inhomogeneous structure to it (see the figure: k)! When you get that point, suddenly the worm-hole become an inhomogeneous thing or stuff, it shrink little bit and, then grows back (C-plateau)! And then make's another jump (it makes another discontinuous jump)! After it makes another discontinuous jump it evolve a little bit contentiously and then jump again and jump again. So what I'm guessing: the post exponential time (after the exponential time), the worm-hole have a geometry, it's full of inhomogeneous matter! And theorist Henry.Lin Called it as a "Caterpillars"! Why a Caterpillar??

Because, it's long and segmented! But i think the word "caterpillar" is more technical on this context! That's I'm expecting the Interior of the black-hole to look like, after an exponential of time. I think it has a geometry, and the geometry might be reasonably smooth but it could different the kind of geometry we expect in early times! Let me do some summarization of thoughts! There are theree descriptions that i given you!

i. "You just keep growing theory": That really worm-holes don't have to do with complexity! They just growing growing and growing. And this theory known as: "pseudo-complexity"! I don't think it wrong. I think it's answers are some particular questions!

Q: How many gates did the natural evolution of the black-hole use to arrive at some Quantum state: $|\Psi(t)\rangle$???

ii. Quantum superposition of sub-exponential states! That's the picture, where you go beyond exponential, you run out of states! Behavior of the clock (instead of registering a coherent time), just grows into a state of superposition of all possible time!!! I think, that's a correct description. Mainly for the boundary description (The Quantum Mechanical Boundary Description), of the measurable things in the black-hole.

iii. Finally, the minimal geodesic description! The shortest circuit, or the shortest Tensor networks they all are the same thing! So the shortest geodesic; that the picture "caterpillar" picture! But what the question does it answer???

Q: It answer the question what is the minimal number of gates needed to shrink the worm-hole back to the reference state! (Reference state: very short worm-hole, or in terms of complexity it has almost zero complexity)!

How you shorten worm-hole??? You may apply operations at the end and those operations simply decreases the size of the worm-hole from the boundary or from the horizon inwards! What is the minimum number of gates [not the number that if you time reverse the natural evolution] could possibly bring you back to the simple state??? That are the three different questions; with three different answers!!! (See the fig: l) But the remarkable thing is that; in sub-exponential time; this three description are same! We can come back to that! But the post exponential time, the three different description are different. I mean: "The Keep Growing Theory", "The Quantum Superposition", and "shortest-geodesic"! Okay, now which one is right??? Yes, some sense they all right! But, they answer different questions!

(a) So, let's start with "Just Keep Growing Theory" or "JKGT"! The "JKGT" gives absurd resulys. In certain sense absurd. You can see the absurdity must clear if you go to the "Quantum Recurrence"! If you go to a "Quantum Recurrence", the state return's into the simplest possible state! Namely the "TFD" or "Thermal Field Double"! And in TFD, the measurable entities, the correlations between the field on the left side and right side goes to large! Characteristics of a worm-hole is very short in size! Now if you jumped back into quantum recurrence, (close to the thermal field double), you may see: the Allice-Bob experiment is succeeded. So what is the Allice-Bob experiment??? Allice jumped into her side, Bob into his side, and they meet in the middle! But that won't happen if the worm-hole is very long.

As they meet in a traversable experiment, so that's the reason it called "Traversable worm-hole"! And in every possible way, the state of the black-hole is the Short worm-hole. But if you consider the "Keep Growing Theory: the volume and the length of the worm-hole increases exponentially! So, the "JKGT" or "Pseudo-complexity" doesn't make any reasonable description between what happens inside and what type of correlation acour in the boundary (or B•H—W•H side)!!!

(b) Now, what about the linear superposition theory??? "LST"—i think iit's right. It's answers are right but it actually gives nothing about the character of the interior. The reason i say that:

(i) Because, the mapping or the dictionary between boundary and the Interior of the black-hole is itself enormously complex! Because the bulk properties of the interior are connected with the boundary quantum mechanics by an insane complicated map, which some how random in nature! Or, atleast pseudo-random. Another reason is that, the linearity of superposition of states in the boundary theory, doesn't transform into the linearity of superposition of states in the bulk theory.

(C) What about the 3rd option??? The worm-hole geometry is governed by the minimal circuit/gates; minimal geodesics; minimal tensor network; that as far as i can tell; is consistent with all mensurable probes (even those design to probed the Interior)! And gives rise to a picture (see the fig: m) which is geometric but unusual to the way worm-hole evolve! And i think it's the only one; which gives rise to the reasonably geometric picture of the Interior!

III. CONCLUSION

So finally what i get? I get an overview here! Here is the "History Of Complexity"! And if you believe in "complexity-volume" connection [The volume of the "worm-hole"!]; long long periods of complexity equilibrium [e^e^s: double exponential], where the black hole sits most complex kinds of states, with the worm-hole just single exponential long fluctuations can chang it's character, from instant to instant!! But yes, it's punctuated by boltzmann type complexity fluctuations. By Boltzmann type complexity fluctuations, i mean: quantum mechanical recurrence [partial recurrence or even full recurrences] they are exceedingly rare! But it's only during those complexity recurrences, the classical General Relativity describe the interior! So only doing complete recurrences, this penrose diagram describe what's going on! In other words, the penrose diagram it-self is a description of recurrence [see fig:n]*!

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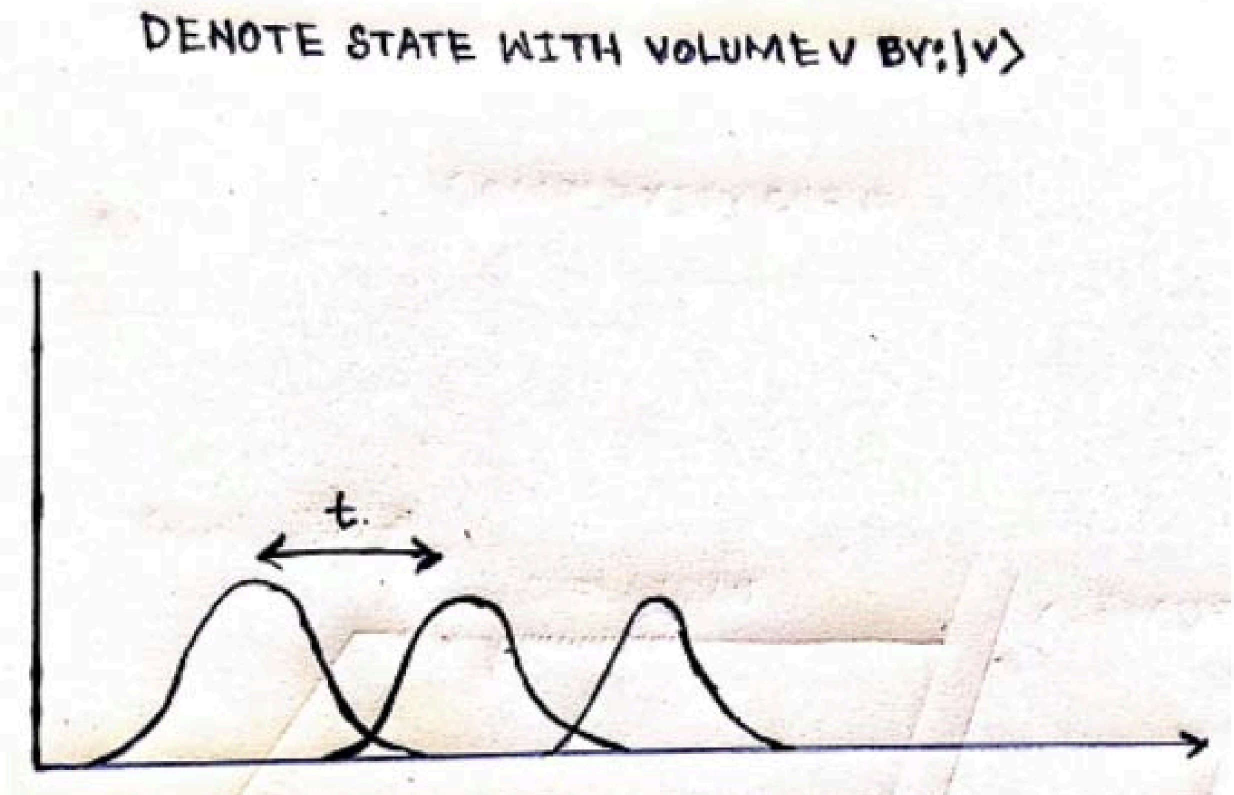
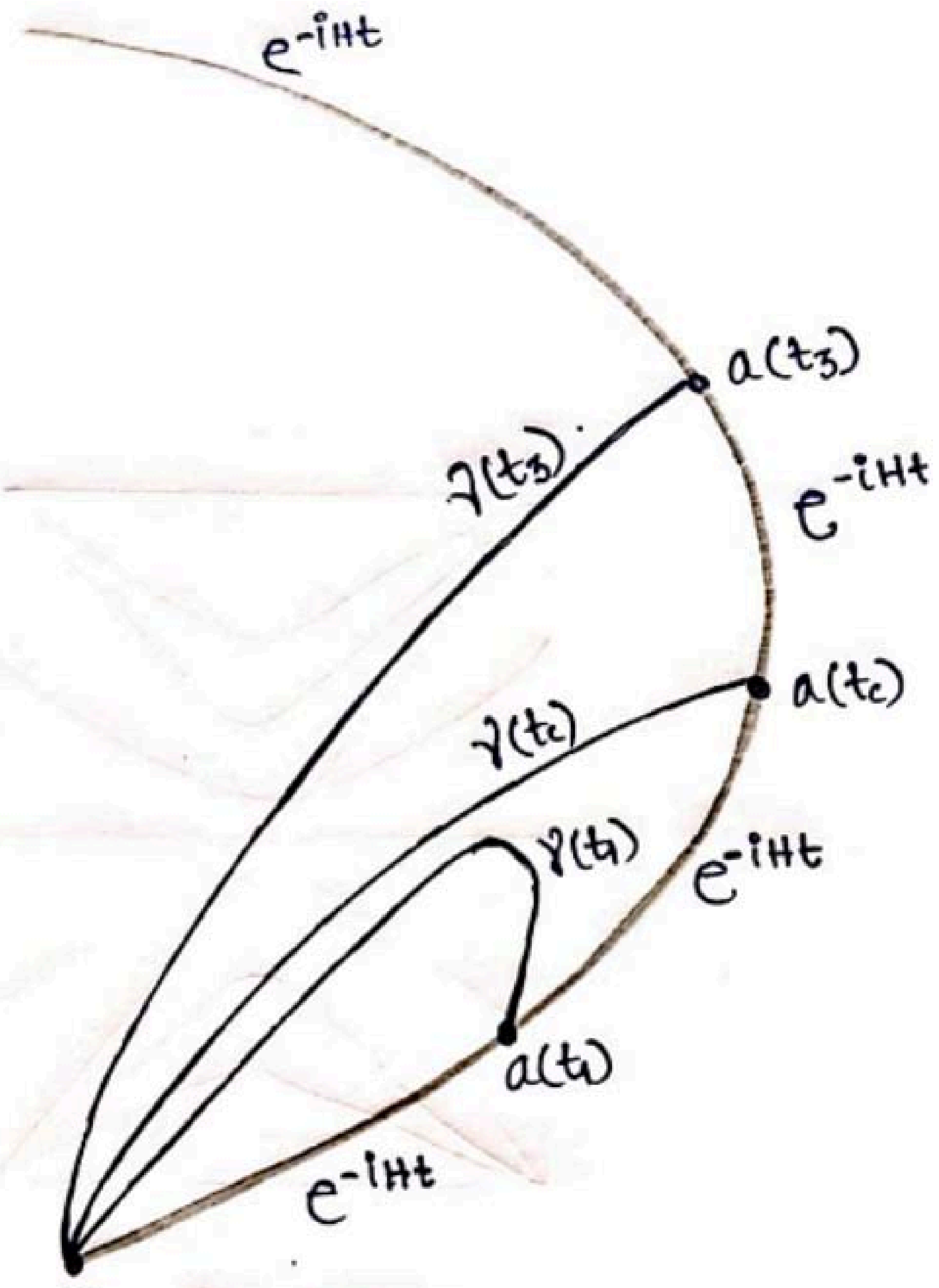


fig: (c)



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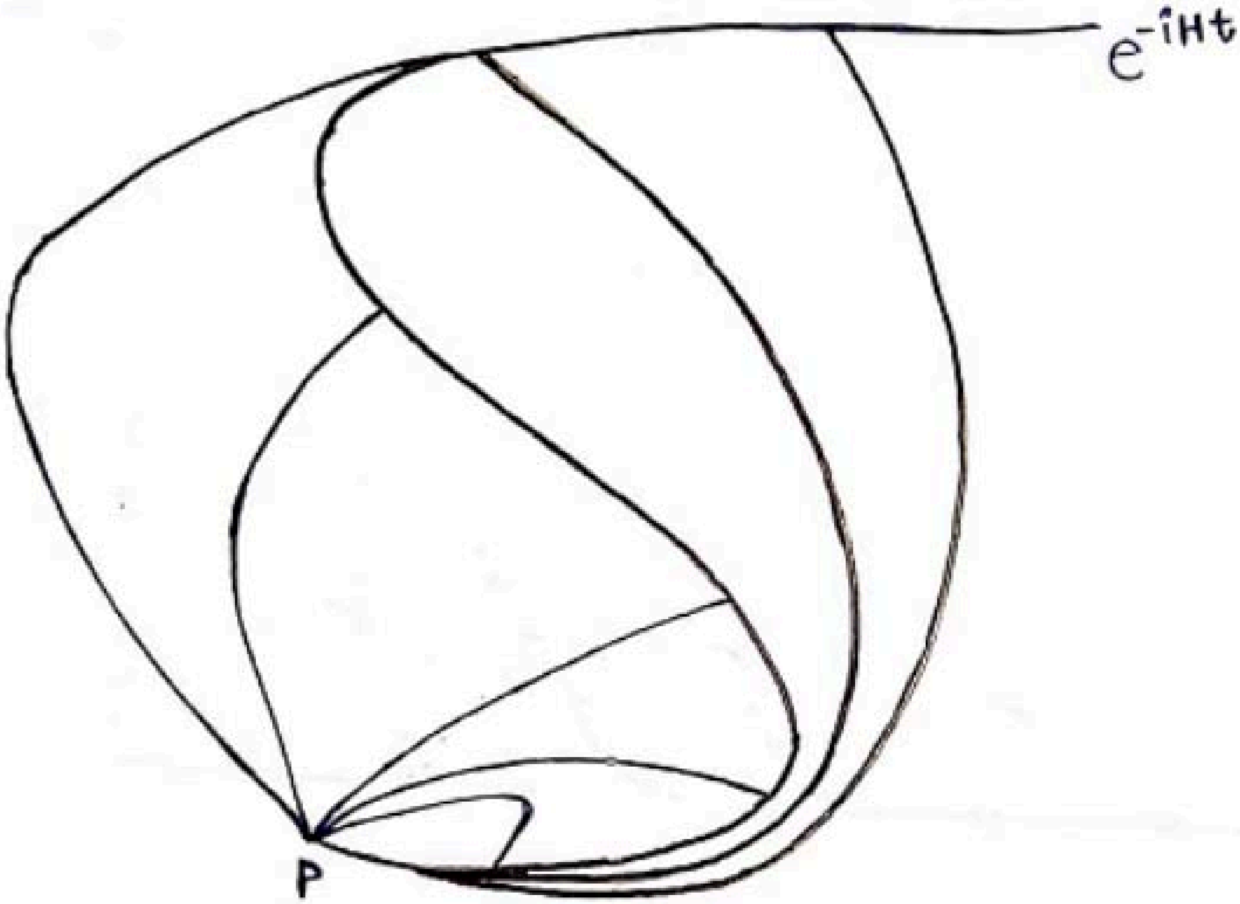

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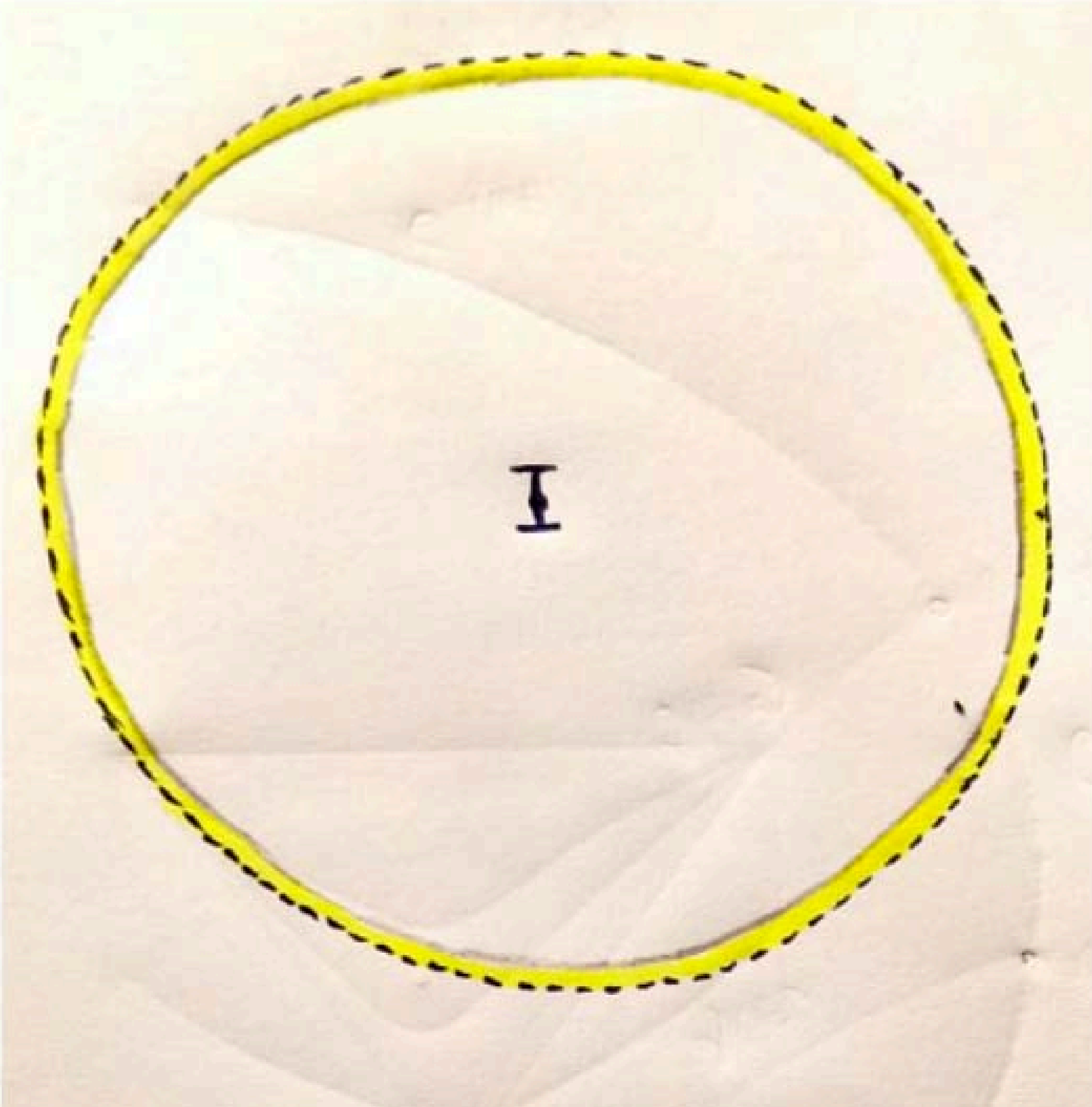

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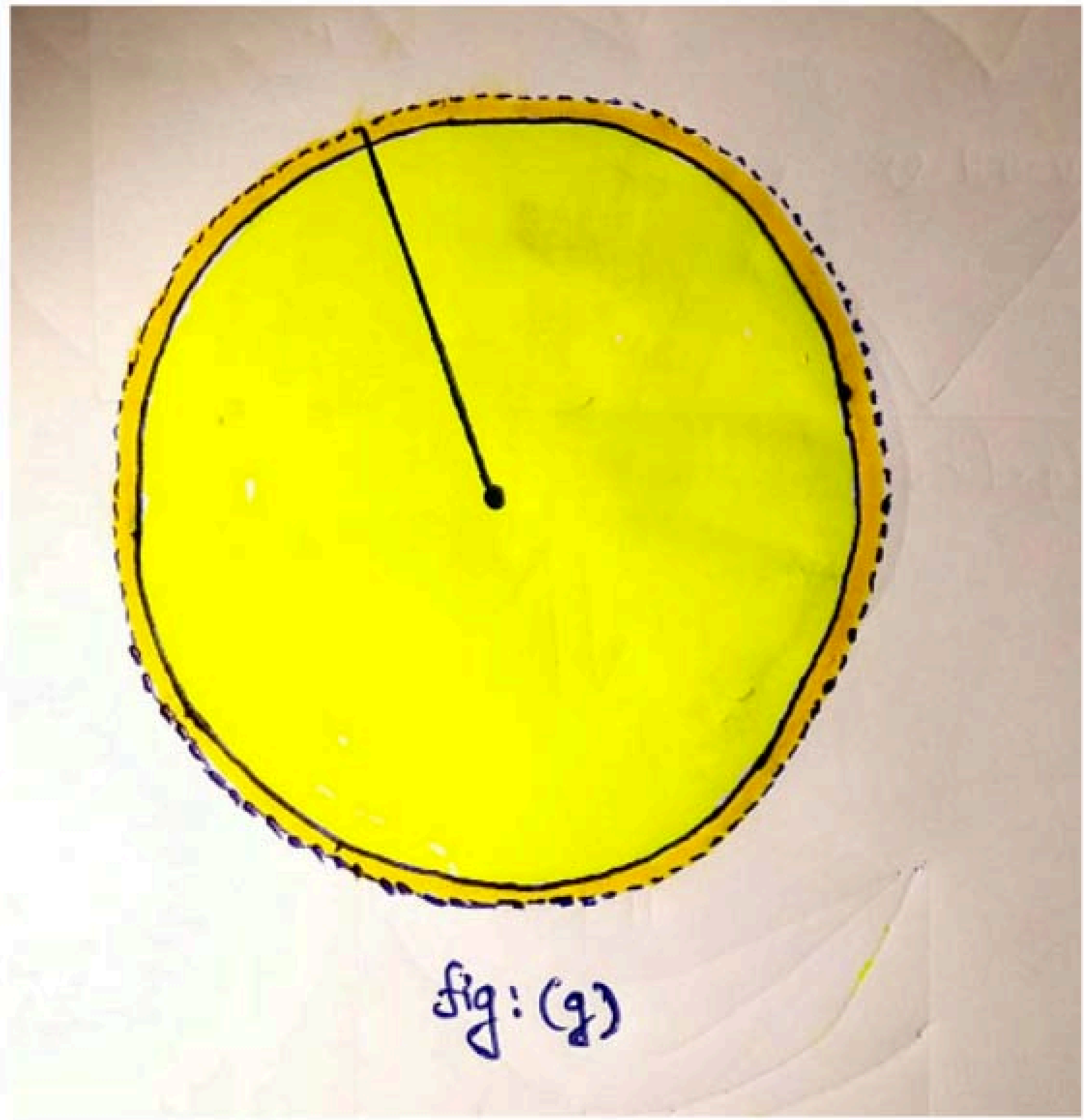


fig: (g)

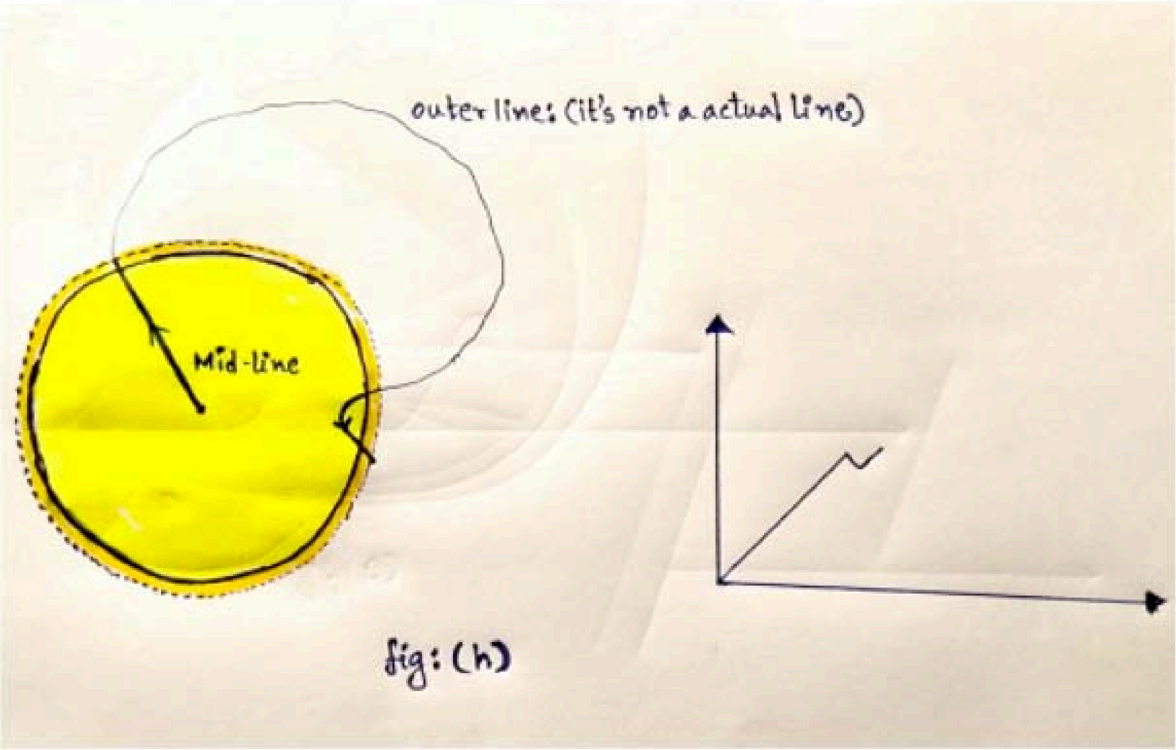

outer line: (it's not a actual line)
Mid-line

fig: (h)

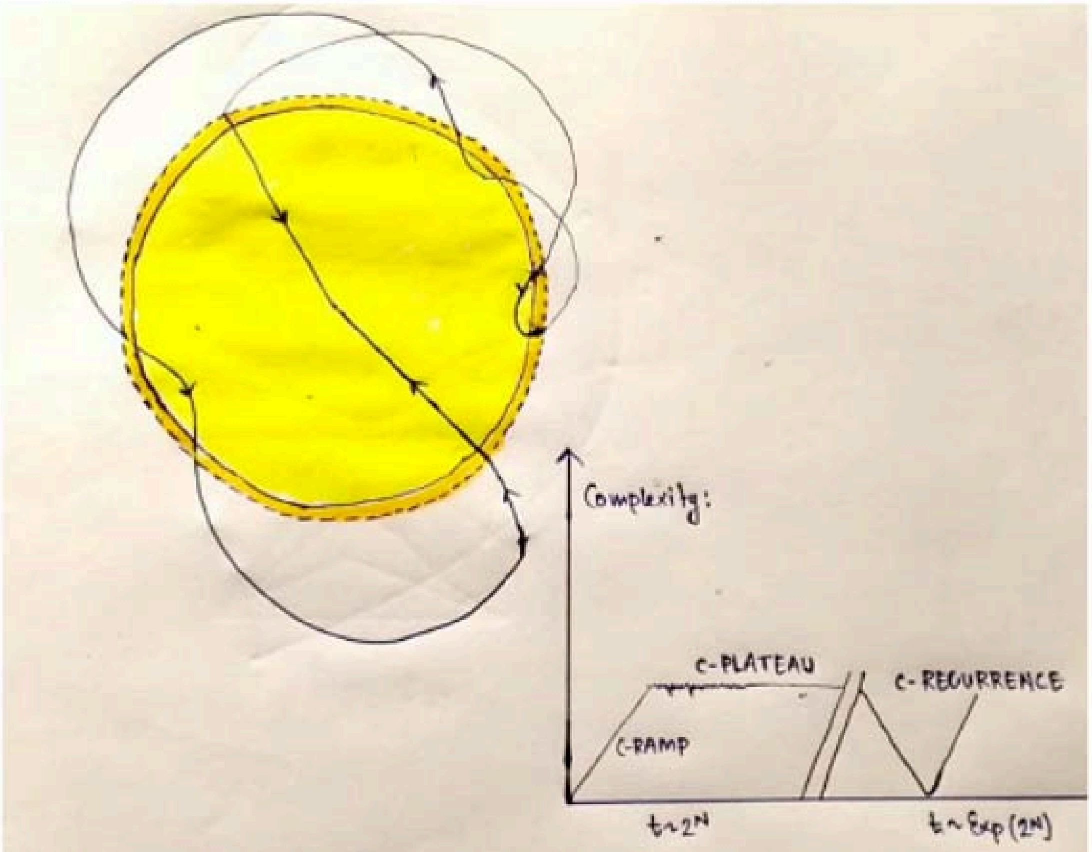

Complexity:
C-PLATEAU
C-RECURRENCE
C-RAMP
t~2^N
t~Exp(2^N)

fig: (i)

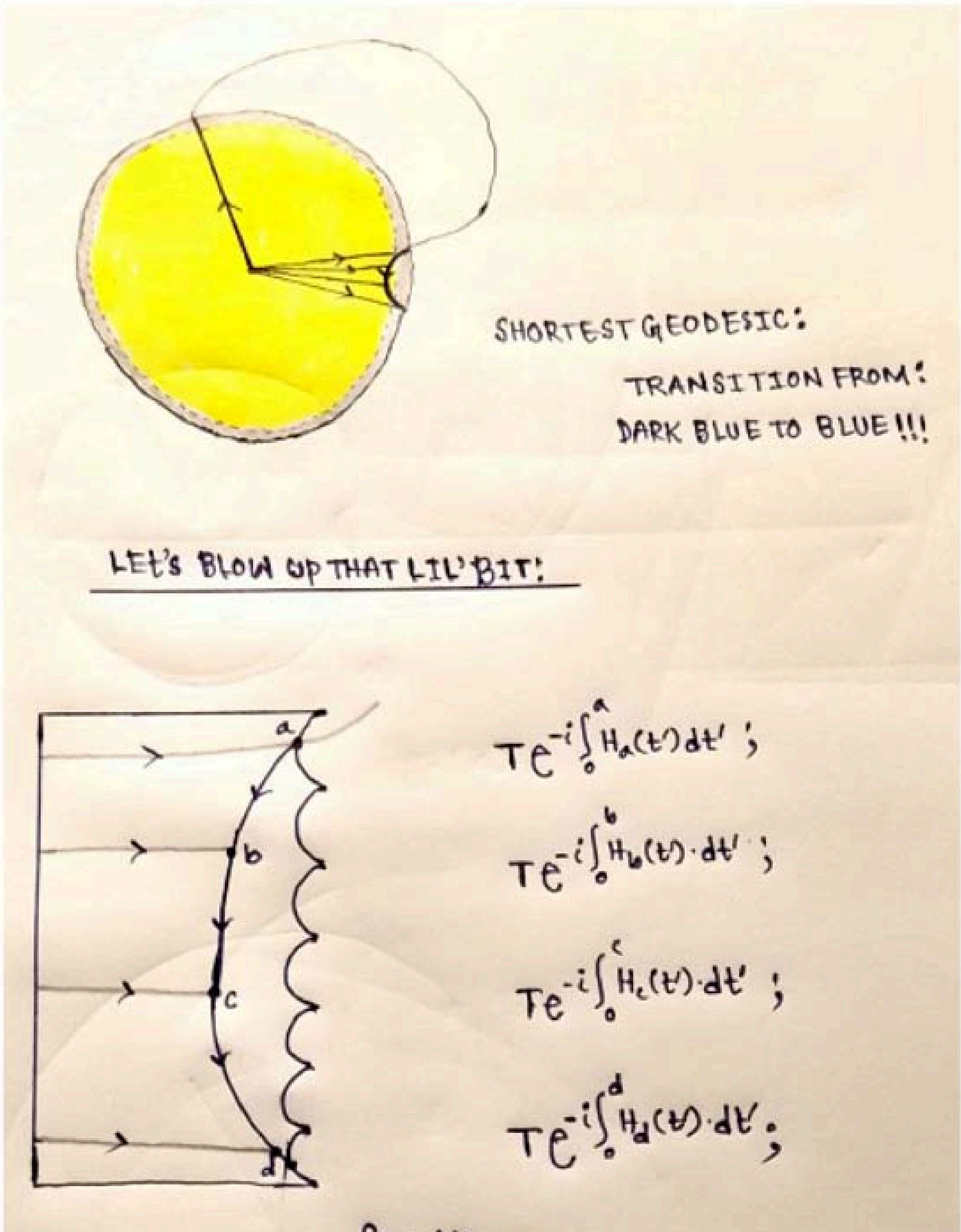
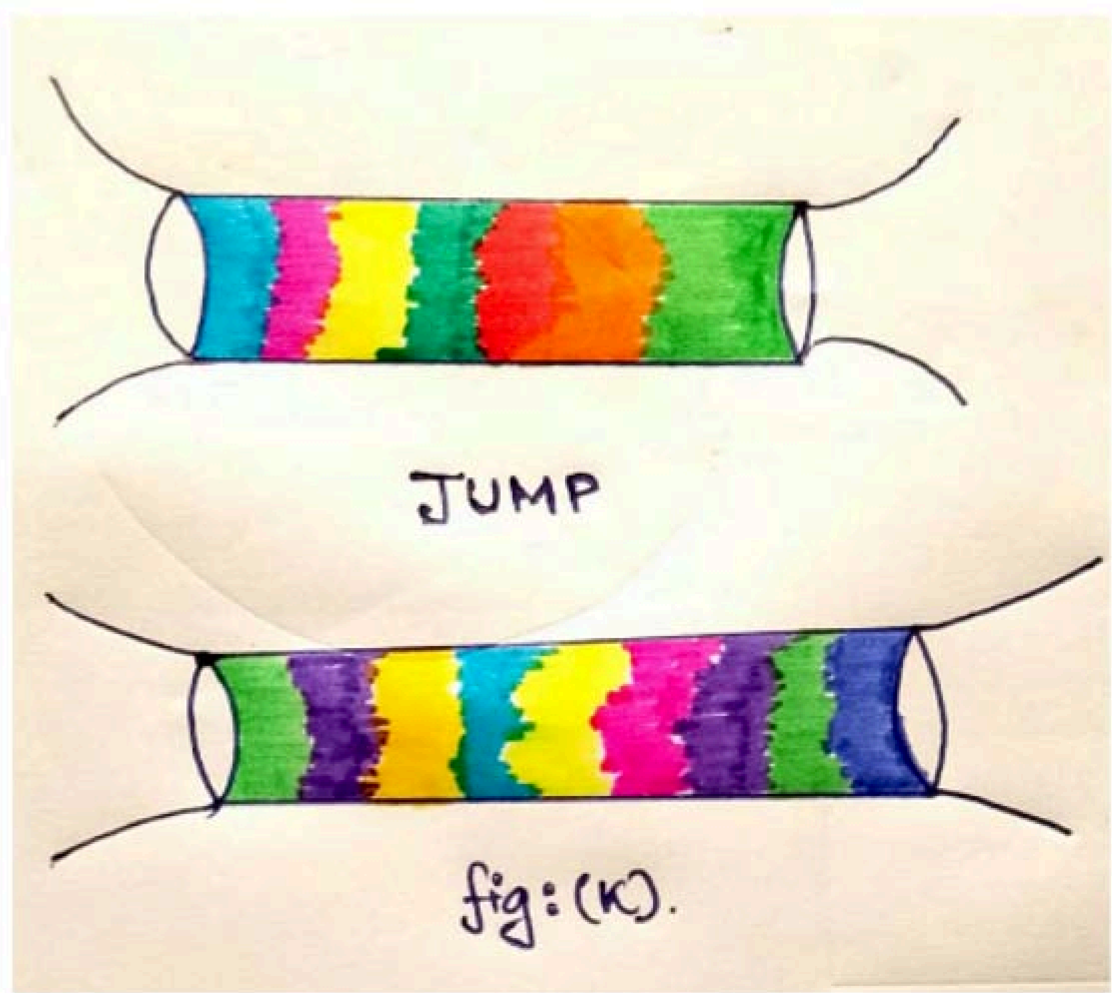

SHORTEST GEODESIC:
TRANSITION FROM:
DARK BLUE TO BLUE!!!
LET'S BLOW UP THAT LIL' BIT!
a
b
c
d
$Te^{-i\int_0^a H_a(t')dt'}$;
$Te^{-i\int_0^b H_b(t')\cdot dt'}$;
$Te^{-i\int_0^c H_c(t')\cdot dt'}$;
$Te^{-i\int_0^d H_d(t')\cdot dt'}$;

fig: (j)


JUMP
fig:(K).

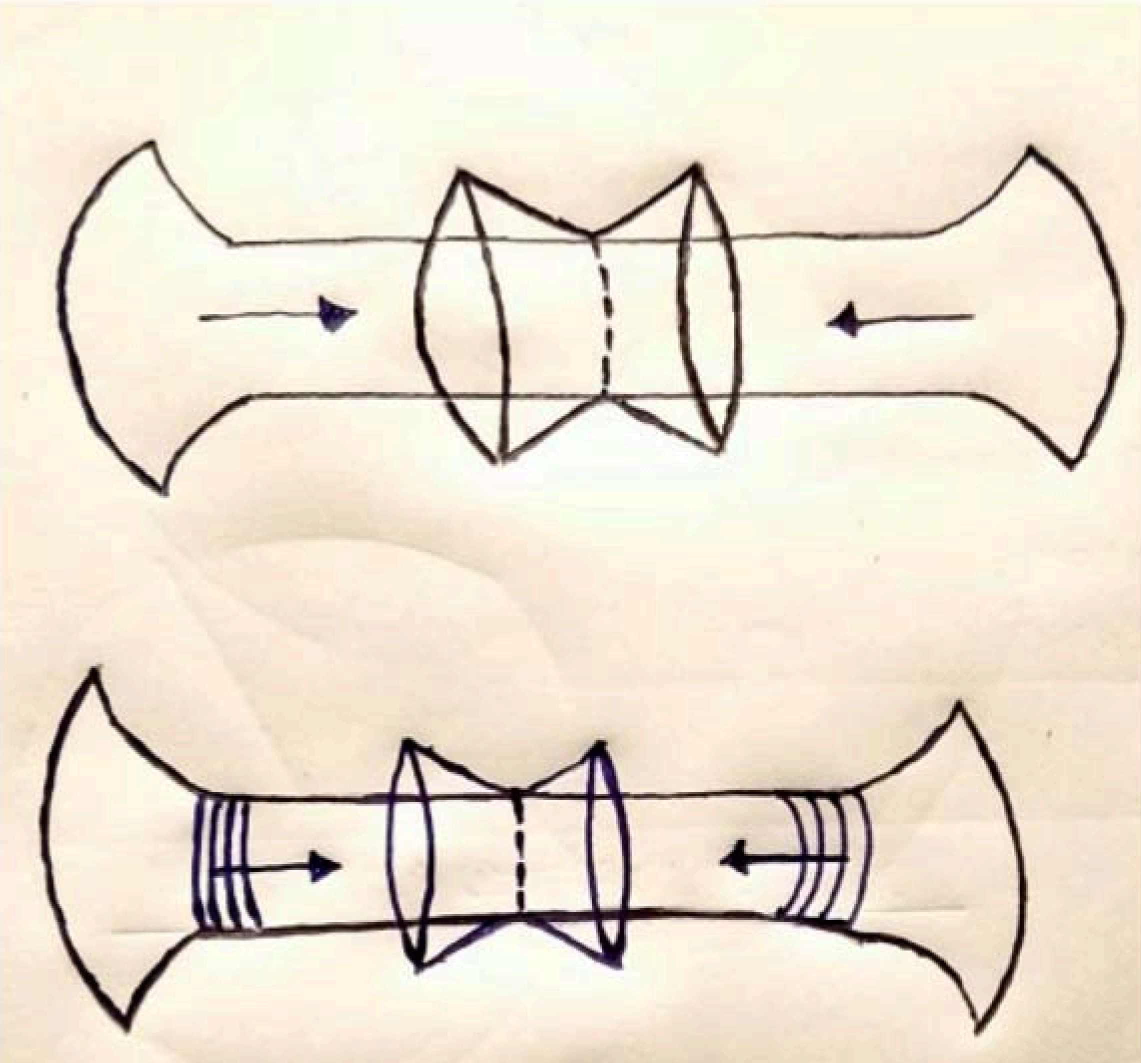
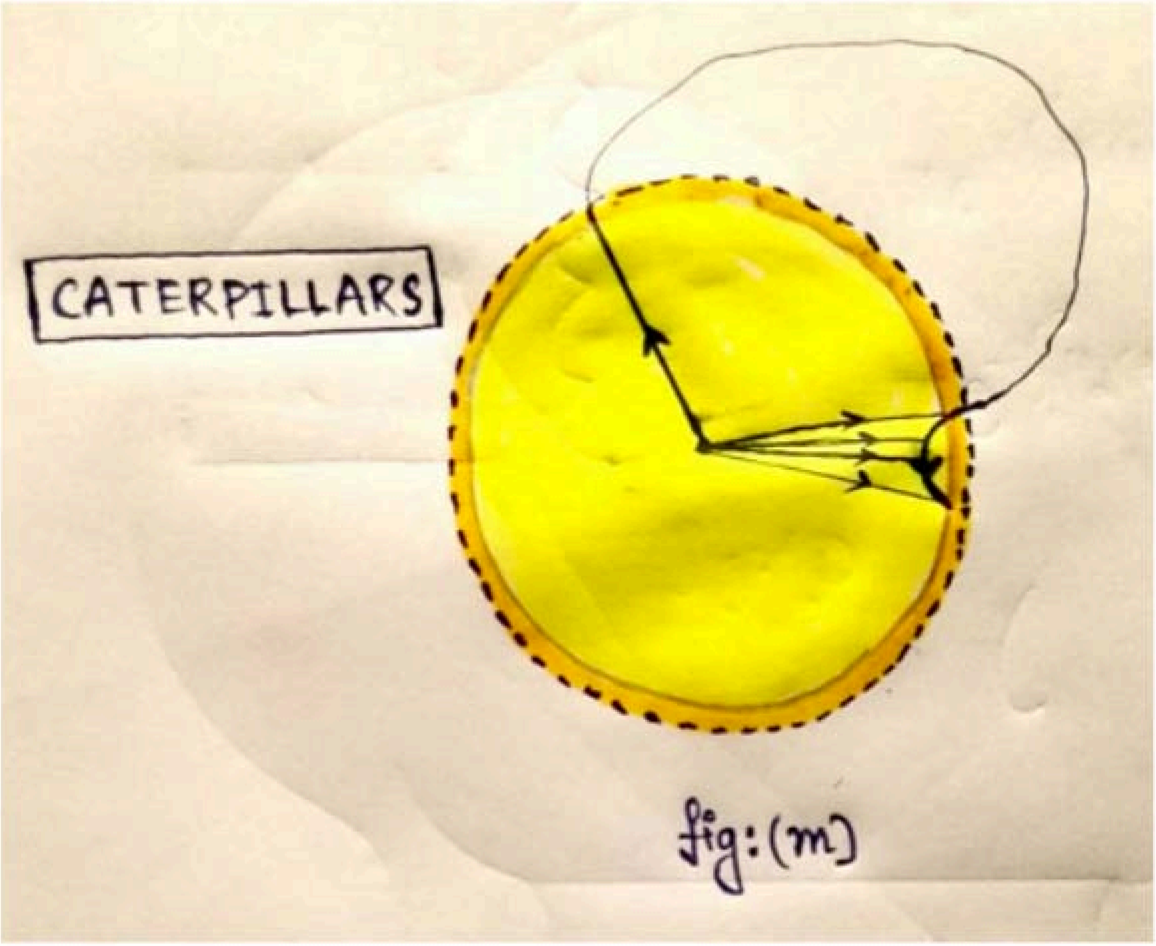


fig:(1)


CATERPILLARS
fig:(m)

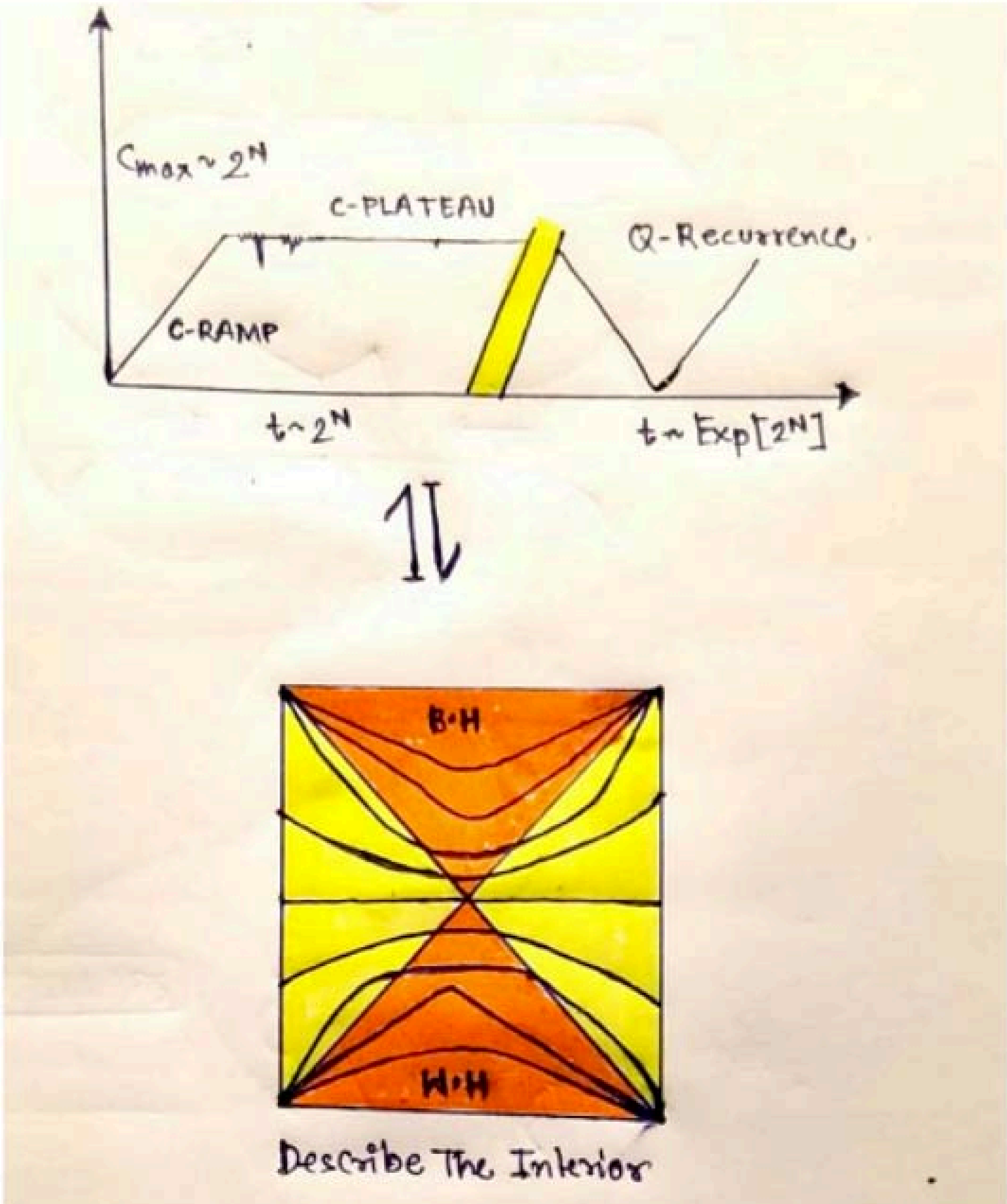

Cmax ~ 2^N
C-PLATEAU
Q-Recurrence
C-RAMP
t~2^N
t~Exp[2^N]
B·H
W·H
Describe The Interior

fig: (n)

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Optimizing Watermelon (Citrullus Lanatus) Production and Profitability on Sandy Ferralsol in Kenge through Integrated Organic and Mineral Fertilization

Adrien Ndonda, Jérémie Mukanza, Blanchard Tebo, Henri-Désiré Kinkani & Denis Ngwala

University of Kwango

ABSTRACT

This study evaluated the agronomic performance and economic viability of watermelon (Citrullus lanatus) in Kenge, Kwango Province (DRC), using a 2 × 5 factorial split-plot design with NPK as the main factor and five organic amendments as subplots. Growth and yield traits were measured across 12-plant subplots. Integrated nutrient management significantly improved plant performance, with Neptune's Harvest™ Fish Fertilizer + NPK producing the highest yield (24.4 t ha^{-1}). Organic amendments enhanced soil physico-chemical properties, increasing NPK retention and nutrient-use efficiency. Marketable yields ranged from 20.3 to 11.6 t ha^{-1} depending on NPK supplementation. Economic analysis showed strong profitability for combined treatments, with Marginal Rate of Return values of 1.83 for Neptune's Harvest™ Fish Fertilizer + NPK, 1.76 for biochar + NPK, and 1.70 for Tithonia + NPK, whereas treatments with MRR < 0.5 were not economically viable. Results demonstrate that watermelon production in Kenge is feasible and profitable under integrated nutrient management.

Keywords: watermelon, organic fertilizers, NPK, biochar, sandy ferralsols, soil amendment, crop yield, horticulture diversification, market viability, nutrient management.

Classification: LCC Code: SB347.W3, SB191.5.F5, S605.5

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ABSTRACT

This study evaluated the agronomic performance and economic viability of watermelon (Citrullus lanatus) in Kenge, Kwango Province (DRC), using a 2 × 5 factorial split-plot design with NPK as the main factor and five organic amendments as subplots. Growth and yield traits were measured across 12-plant subplots. Integrated nutrient management significantly improved plant performance, with Neptune's Harvest™ Fish Fertilizer + NPK producing the highest yield (24.4 t ha^{-1}). Organic amendments enhanced soil physico-chemical properties, increasing NPK retention and nutrient-use efficiency. Marketable yields ranged from 20.3 to 11.6 t ha^{-1} depending on NPK supplementation. Economic analysis showed strong profitability for combined treatments, with Marginal Rate of Return values of 1.83 for Neptune's Harvest™ Fish Fertilizer + NPK, 1.76 for biochar + NPK, and 1.70 for Tithonia + NPK, whereas treatments with MRR < 0.5 were not economically viable. Results demonstrate that watermelon production in Kenge is feasible and profitable under integrated nutrient management.

Keywords: watermelon, organic fertilizers, NPK, biochar, sandy ferralsols, soil amendment, crop yield, horticulture diversification, market viability, nutrient management.

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I. INTRODUCTION

Kenge, located in Kwango Province, Democratic Republic of Congo, possesses considerable agricultural potential (Kisangani, 2021). However, local farming is predominantly focused on subsistence crops such as cassava, maize, and basic vegetables (Luyeye, 2020). Introducing crops like watermelon could diversify agriculture and enhance economic development (Mmbere, 2022).

Watermelon cultivation provides income in regions where it is practiced, yet in Kenge, it remains largely unfamiliar. Soils in the area are predominantly sandy, acidic, and low in organic matter, whereas watermelon thrives in sandy soils enriched with organic matter (Chambre Régionale d'Agriculture, 2017). Nearby regions with similar edaphic and climatic conditions, such as the Bateke Plateau, have achieved moderate yields, although fruit size and commercial value remain low. This highlights the agronomic challenge addressed in this study: optimizing watermelon production under local conditions.

The study aims to improve soil fertility and establish efficient production strategies. Developing sound agronomic practices in peri-urban Kenge could increase farmers' incomes and mitigate chronic food insecurity (Johnson and Lee, 2018). With favorable climate, appropriate rainfall, and temperatures (Ngoma, 2019; Mbuyi, 2021; Kamina, 2022), Kenge is suitable for watermelon cultivation, provided rational fertilization and technical support are applied (Kekulé, 2021; Mujinga and Kambale, 2022).

Watermelon, valued for its nutritional and commercial potential, can meet growing local and regional market demand, enhance farmer income, and contribute to crop diversification (Tshilombo, 2019; Kabongo, 2021; Tshibangu, 2020; Smith, 2020). Environmental sustainability and rational water management are essential to prevent soil degradation and ensure long-term productivity.

This study analyzes these factors to demonstrate how watermelon cultivation could serve as a strategic driver of economic growth in Kenge and to strengthen farmer capacity in addressing contemporary agricultural challenges (Mwamba, 2023).

II. MATERIALS AND METHODS

2.1 Study Area

The study was conducted at the experimental field of Congo Vert/Mangoy (figure 1) in Kenge (4° 46 '15.9'' S, 16° 59' 04.8'' E; 570 m above sea level). Kenge has a sub-equatorial (AW4, Köppen) climate with four alternating seasons: major rainy (15 Sep–15 Jan), short dry (15 Jan–15 Mar), short rainy (15 Mar–15 May), and major dry (15 May–15 Sep). Mean annual temperature is ~25°C, and annual rainfall averages 1,507 mm.

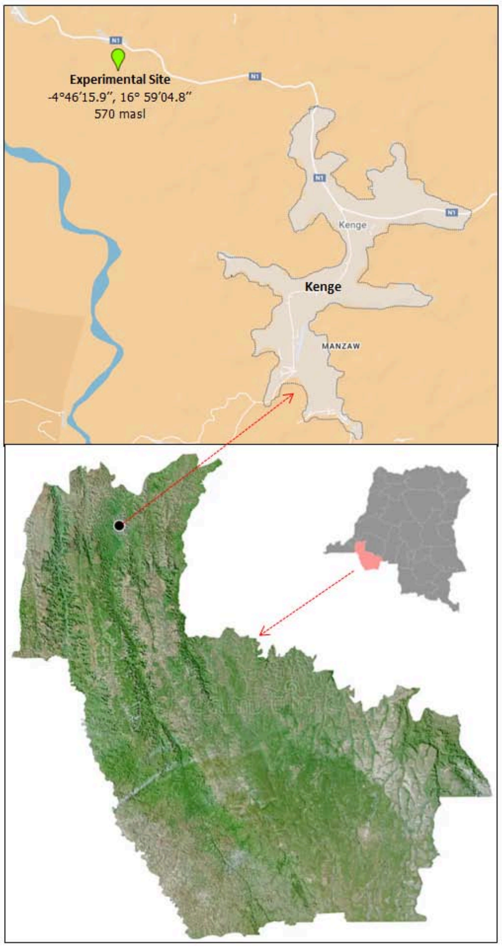

Figure 1: Experimental site, Congo Vert (Mangoy), Kenge, DRC.

Soils are predominantly sandy (sandy loam to sandy clay) with <20% clay. They are light, weakly cohesive, with unstable aggregates, high porosity, and good infiltration but low water retention. Surface horizons are leached despite soil depth of several meters. Soils are acidic to highly acidic (pH 4.5–5.5), typical ferralsols with low cation exchange capacity (CEC ≈ 10 meq/100 g) and poor chemical fertility. Organic matter is low (≤2%) due to rapid mineralization under hot, humid conditions. The soils are deficient in N, P, and K, but rich in Fe and Al, which may reach toxic levels. Calcium (Ca), magnesium (Mg), and exchangeable bases are low, with base saturation <30% (Ndonda *et al.*, 2025).

The physico-chemical properties of the soil prior to the establishment of the experimental trial are presented in Table 1.

Table 1: Physico-chemical properties of the experimental site soil before the start of the trial (Ndonda *et al.* 2025)

Parameter	pH (H_2O)	Cation Exchange Capacity (CEC) cmol(+) kg^{-1}	% Total Nitrogen (N)	Available Phosphorus (P) - ppm	Exchangeable Potassium (K) cmol(+) kg^{-1}	Exchangeable Calcium (Ca) cmol(+) kg^{-1}	Exchangeable Magnesium (Mg) cmol(+) kg^{-1}	Exchangeable Sodium (Na) cmol(+) kg^{-1}	Clay	Silt	Sand
Value	5	15	0.1	3.5	0.17	3.2	1.5	0.05	20	20	60

Source: Soil analyses conducted using the Agritech Insight digital kit in Kinshasa, 2025.

The dominant vegetation at the experimental site is herbaceous, mainly grasses, with perennial species such as *Hyparrhenia diplandra*, *Imperata cylindrica*, and *Chromolaena odorata*. Soils are generally sandy, with sandy-clay patches in Kolokoso and Dinga sectors. The experiment was conducted on relatively poor sandy soil covered by *Hyparrhenia*-type grasses with low vegetative vigor.

2.2 Materials

The study used watermelon (*Citrullus lanatus*, var. Congo watermelon; Cucurbitaceae). Soil amendment treatments included poultry manure, biochar, *Tithonia diversifolia* leaf biomass, and the commercial biofertilizer Neptune's Harvest™ Fish Fertilizer. Inferential statistical analyses were conducted in GenStat, multivariate procedures in PaST, and additional graphics in Microsoft Excel. Climatic parameters were estimated using New_LocClim Estimator, and daily temperatures were monitored with an integrated digital thermometer. Geographic coordinates were obtained via the "Mes coordonnées" mobile application, and geospatial visualization was performed using Google MyMaps.

2.3 Organic Fertilizers

(a) Neptune's Harvest™ Fish Fertilizer (figure 2) : The Fish Fertilizer formulation was used in this study (Ndonda *et al.*, 2024).

Source photo: https://mysmithland.com/neptunes-harvest-Fish-fertilizer-2-4-1-gallon/

Figure 2: Neptune's Harvest Fish Fertilizer

Neptune's Harvest™ Fish Fertilizer is an organic liquid fertilizer derived from fresh North Atlantic fish via enzymatic hydrolysis, which preserves vitamins, amino acids, enzymes, and growth hormones. It contains both macro- and micro-nutrients naturally present in fish tissues, with chelated nitrogen and other nutrients immediately available for plant uptake. Unlike conventional fish emulsions, it retains fish proteins and oils without producing an unpleasant odor. The fertilizer enhances plant vigor, improves resilience to environmental stresses such as frost and heat, and promotes flowering and fruiting. In this experiment, 70 ml of Neptune's Harvest™ Fish Fertilizer diluted in 1 L of water was applied when plants reached approximately 10 cm in height.

(b) Tithonia diversifolia

Tithonia diversifolia (Mexican sunflower) is a fast-growing perennial shrub of the Asteraceae family (figure 3). Under favorable conditions, it reaches 1.5–4 m in height and exhibits a bushy habit with upright, robust, often hollow stems. Leaves are alternate, simple, lobed (3–7 lobes), large (15–35 cm), with serrated margins and long, sturdy petioles. The plant is rich in nutrients (N, P, K, Ca, and Mg) and is widely used as green manure or compost accelerator. Its fresh biomass yield is high, ranging from 50 to 80 $t \cdot ha^{-1} \cdot yr^{-1}$.

Figure 3: Photograph of a stand of *Tithonia diversifolia*

The nutrient content of *Tithonia diversifolia* leaves varies with plant maturity, growth conditions, and plant part analyzed. Published studies report approximate mean ranges (dry weight basis): Nitrogen (N) 2.5–4.0%, Phosphorus (P) 0.2–0.5%, and Potassium (K) 2.0–4.0%, highlighting its suitability as a nitrogen- and potassium-rich green manure (Ropa *et al.*, 2020). Other minerals include Calcium (Ca) 1.0–2.0%, Magnesium (Mg) 0.3–0.6%, and organic matter >80% of dry mass. Decomposition of this biomass improves soil structure, water-holding capacity, and microbial activity (Ndonda, 2025), making it a valuable organic fertilizer.

(c) Poultry Manure

Poultry manure consists of solid and liquid excreta from chickens, rich in N, P, and K. The solid fraction is brown, while the urates are chalky white. Typical composition includes 1.5–3.5% N, 1.5–3.0% P_2O_5, 1–2% K_2O, 2–5% Ca, and 60–80% organic matter, with an alkaline pH of 7.5–8.5. Its chemical characteristics make it a concentrated, nutrient-rich organic fertilizer.

(d) Biochar

Biochar is a carbon-rich soil amendment produced by pyrolysis of plant biomass in low-oxygen conditions. In this study, 24 kg of charcoal was crushed, sieved, and layered with kitchen waste in a 70 cm pit. The mixture was turned biweekly and watered daily during the first week; after one month, it was applied at 2 $kg \cdot m^{-2}$ (20 $t \cdot ha^{-1}$). Biochar's microporous structure (50–90% stable carbon) enhances water retention, cation exchange capacity, porosity, nutrient availability, microbial activity, and reduces nutrient leaching. It also increases soil pH, sequesters carbon, and improves long-term fertility, particularly in acidic, coarse soils such as those in Kenge (Moango *et al.*, 2023).

2.4 Experimental Design

A 2 × 5 factorial split-plot design with four replications was implemented. The main factor was NPK 17-17-17 application (10 g per plant), applied in furrows 10 cm from each plant when plants reached 15–20 cm height. The secondary factor consisted of five organic treatments:

1. No organic fertilizer (T0)
2. Neptune's Harvest™ Fish Fertilizer at 270 ml per plant (T1) Goat manure at 20 $t \cdot ha^{-1}$ (T2)
3. Poultry manure at 15 $t \cdot ha^{-1}$ (T3)
4. Leaf biomass of *Tithonia diversifolia* at 20 $t \cdot ha^{-1}$ (T4)

III. DATA COLLECTION

Measured parameters included sowing dates, 50% flowering, vine growth (length and stem diameter), number of marketable fruits per plant, weight of marketable fruits, and marketable fruit yield.

IV. RESULTS AND DISCUSSION

4.1 Results

Table 2: Results of the observed data from the experiment.

Primary Factor (Mineral Fertilizer)	Secondary Factor (Organic Fertilizers)	Surviving Plants per Plot/12	Stem Diameter at 3 DAS (cm)	Vine Length at 3 DAS (m)	Average Number of Marketable Fruits per Plant	Marketable Fruit Weight per Plot (kg)	Average Fruit Weight (kg)	Marketable Fruit Yield ($t \cdot ha^{-1}$)
Without NPK	Control	9	0.38^{a}	0.74^{a}	6.8^{a}	5.1^{a}	3.8^{ab}	8.49^{a}
	Biochar	11	0.53^{a}	1.43^{c}	7.7^{a}	7.3^{a}	2.9^{a}	12.23^{a}
	Tithonia	10	0.53^{a}	1.31^{c}	8.9^{b}	6.6^{a}	4^{ab}	10.99^{a}
	Neptune's Harvest	10	0.58^{a}	1.29^{c}	9.2^{b}	8.4^{a}	4^{ab}	13.93^{a}
	Poultry Manure	10	0.57^{a}	1.26^{bc}	8.8^{b}	7.3^{a}	5^{b}	12.11^{a}
Mean without NPK		10	0.50^{a}	1.2^{ns}	8.3^{ns}	6.9^{ns}	4.06^{ns}	11.6^{a}
With NPK	Sole NPK	9	0.65^{b}	0.92^{a}	6.4^{a}	8.4^{a}	4.1^{ab}	13.98^{a}
	Biochar	11	0.80^{b}	1.37^{c}	9.2^{b}	12.6^{ab}	3.2^{a}	21.04^{b}
	Tithonia	11	0.75^{b}	1.40^{c}	9.2^{b}	13.4^{ab}	3.5^{a}	22.26^{b}
	Neptune's Harvest	10	0.86^{b}	1.45^{c}	9.7^{b}	14.6^{b}	4.2^{ab}	24.39^{bc}
	Poultry Manure	10	0.79^{b}	1.49^{c}	9.7^{b}	11.8^{ab}	4.7^{b}	19.60^{ab}
Mean with NPK		10.3	0.80^{b}	1.3^{ns}	8.8^{ns}	12.2^{ns}	3.94^{ns}	20.3^{b}
Overall Mean		10	0.63	1.26	8.5	9.3	4	15.5
LSD.05								
Mineral Fertilizer (A)			0.95^{ns}	0.26^{ns}	1.1^{ns}	7.6^{ns}	1.68^{ns}	8.6*

Organic Fertilizers (B)			0.23*	0.29*	1.17*	5.2*	1.72*	8.8**
A × B			0.39*	0.64*	5.99^{ns}	12.0^{ns}	2.8^{ns}	14.38*
CV%								
Mineral Fertilizer (A)			12.9	36.3	14.0	13.2	20.5	10.2
Organic Fertilizers (B)			13.6	22.4	15.4	21.0	31.2	21.0
A × B			5.7	9.4	6.6	9.2	15.1	9.2

4.2 Seedling Emergence

The number of emerged seedlings per plot was not significantly affected by either organic or mineral fertilizers. On average, 10 out of 12 sown seeds germinated across all treatments, and neither NPK 17–17–17 nor organic amendments influenced initial seedling establishment. No interaction effect between organic and mineral fertilizers was observed at this stage.

4.3 Stem Diameter at Collar – 3 Months after Sowing

Stem diameter varied significantly with fertilization. Organic fertilizers alone produced a mean diameter of 0.50 cm, while combinations with NPK increased it to 0.80 cm (LSD.0$_5$ = 0.30 cm; CV = 50.9%). Non-NPK treatments ranged from 0.38 cm in the control to 0.58 cm with Neptune's Harvest™ Fish Fertilizer, whereas NPK-supplemented treatments ranged from 0.65 to 0.86 cm, all significantly higher than their non-NPK counterparts

4.4 Vine Length – 30 Days after Sowing

Significant differences were observed among organic treatments. The shortest vines were in the control (0.75 m without NPK; 0.92 m with NPK). Biochar increased vine length to 1.43 m without NPK and 1.37 m with NPK, while *Tithonia diversifolia* reached 1.31 m alone and 1.40 m with NPK. Neptune's Harvest™ Fish Fertilizer and poultry manure also improved vine length, particularly when combined with NPK ($p < 0.05$).

4.5 Number of Marketable Fruits per Plant

NPK alone did not significantly affect fruit number, whereas organic fertilizers increased it. Marketable fruits per plant ranged from 6 in control with NPK to 10 in Neptune's Harvest™ Fish Fertilizer + NPK and poultry manure + NPK (LSD.0$_5$ = 1.17; CV = 15.4). This highlights the positive effect of organic amendments, further enhanced by mineral fertilization.

4.5 Weight of Marketable Fruits per Plot

Commercial fruit weight differed significantly among treatments. Combined NPK + organic fertilizers produced the highest yields: Neptune's Harvest™ Fish Fertilizer + NPK (14.6 kg/plot), Tithonia + NPK (13.4 kg), biochar + NPK (12.6 kg), and poultry manure + NPK (11.8 kg). Organic amendments without NPK produced lower yields (5–8 kg/plot). These results indicate that integrated nutrient management, particularly Neptune's Harvest™ Fish Fertilizer + NPK, is the most effective strategy to optimize watermelon productivity under Kenge's agro-ecological conditions.

4.6 Marketable Fruit Yield Response to Organic Fertilizers and NPK Supplementation

Significant differences in marketable fruit yield were detected among nutrient management strategies ($LSD_{.05}$ = 8.6, $p < 0.05$). Application of organic fertilizers alone resulted in a mean yield of 11.6 t ha^{-1}, whereas the integration of NPK with biofertilizers increased yield by 75%**,** reaching 20.3 t ha^{-1}.

Yield responses varied markedly among the organic fertilizer sources. (i) Biochar: Yield increased from 12.23 t ha^{-1} (without NPK) to 21.04 t ha^{-1} (with NPK), representing a 72% improvement. (ii) *Tithonia diversifolia*: Yield doubled from 10.99 t ha^{-1} to 22.26 t ha^{-1} when supplemented with NPK. (iii) Neptune's Harvest™ fish fertilizer: Produced the highest yield under NPK integration (24.39 t ha^{-1}), compared with 13.93 t ha^{-1} without NPK. (iv) Poultry manure: Increased from 12.11 t ha^{-1} alone to 19.6 t ha^{-1} with NPK. Overall, the results demonstrate a consistent and substantial synergistic effect of combining NPK with organic fertilizers, with improvements ranging from 55% to 102% depending on the organic source. The analysis reveals a significant interdependence between the variables 'marketable fruit weight per plot' and overall yield, as well as between average fruit weight and yield (figure 4).

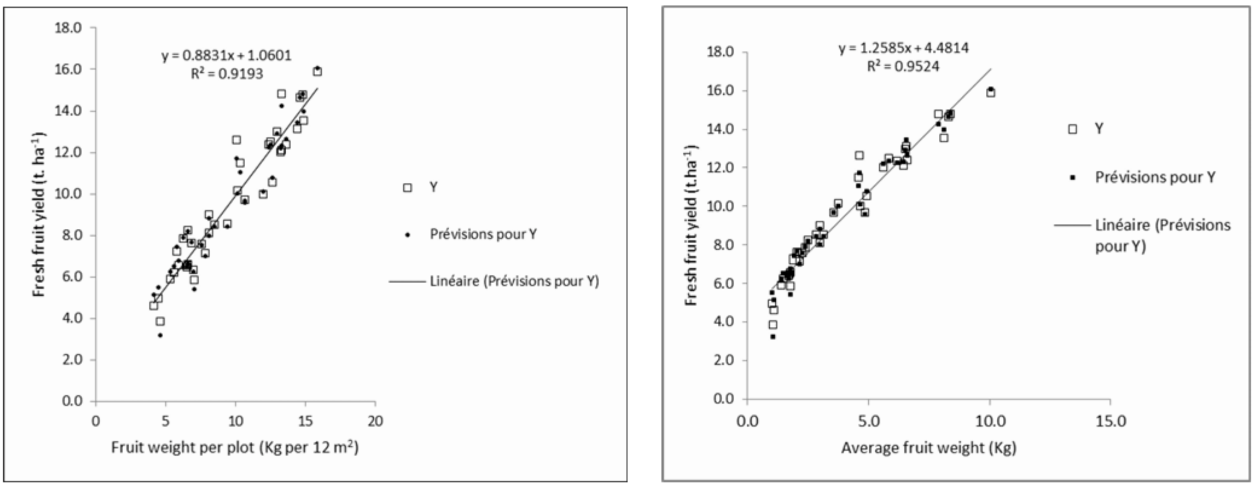

Figure SEQ Figure * ARABIC 4: Regression curve between fruit weight per plot and yield (left) and between average fruit weight and yield (right)

4.7 Economic Profitability of the Applied Practices

Low adoption of agricultural innovations is often linked to insufficient consideration of the agro-economic context and the practical relevance of tested technologies. Meaningful recommendations require a thorough understanding of farmers' production conditions, obtained through field diagnostics and farmer interviews. Such assessments help identify major constraints to productivity and determine feasible technological improvements.

Data from these evaluations inform on-farm experimental programs conducted under real farming conditions. Trials on representative farms generate essential information for planning future research and refining technologies in subsequent cropping seasons (CIMMYT, 1989).

4.8 Partial Budget Analysis

Partial budgeting organizes experimental data to clearly quantify the costs and benefits of different treatments. In this study, it was used to assess agronomic practices for improving the fertility of Kenge's poor soils for watermelon cultivation—a crop increasingly promoted as a cash crop in a region characterized by low purchasing power, traditional non-resilient farming, and speculative mono-cropping and poor-quality seeds.

Table 3: Partial budget analysis of fertilizers applied to watermelon crop

	Without NPK					With NPK				
	Control	Biochar	Neptune's Harvest™	Poultry manure	*Tithonia diversifolia*	Sole NPK	Biochar	Neptune's Harvest™	Poultry manure	*Tithonia diversifolia*
Mean yield per hectare (kg ha^{-1})	8488	12233	10988	13933	12113	13983	21042	22263	24388	19596
Adjusted mean yield (−20%) (kg ha^{-1})	6790	9787	8790	11147	9690	11187	16833	17810	19510	15677
Gross field revenue (USD ha^{-1})	2425	3495	3139	3981	3461	3995	6012	6361	6968	5599
Seed expenditure (USD ha^{-1})	40	40	40	40	40	40	40	40	40	40
Expenditure for *Tithonia diversifolia* leaf biomass (USD ha^{-1})	0	0	0	0	400	0	0	0	0	400
Expenditure for mineral fertilizer and other soil amendments (USD ha^{-1})	0	510	640	1200	0	600	1150	1240	1800	600
General labor expenditure (USD ha^{-1})	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
Labor expenditure for the application of mineral and organic fertilizers (USD ha^{-1})	0	100	100	100	0	100	150	150	150	50
Labor expenditure for the application of *Tithonia diversifolia* leaf biomass (USD ha^{-1})	0	0	0	0	125	0	0	0	0	125
Total variable expenditures (USD ha^{-1})	1040	1650	1780	2340	1565	1740	2340	2430	2990	2215
Net economic returns (USD ha^{-1})	1385	1845	1359	1641	1896	2255	3672	3931	3978	3384

Partial budget analysis revealed that combining organic fertilizers with NPK produced the highest net benefits: Neptune's Harvest™ Fish Fertilizer ($3978 ha^{-1}), poultry manure ($3931 ha^{-1}), biochar ($3672 ha^{-1}), and *Tithonia diversifolia* ($3384 ha^{-1}). When applied alone, net benefits ranged from $1350 to $2250 ha^{-1}, with the control yielding the lowest ($1385 ha^{-1}).

4.9 Marginal Rate of Return (MRR) and Cost-Benefit Ratio (CBR)

The marginal rate of return indicates the average gain a farmer can expect from investing in a new practice or set of practices compared to the control (CIMMYT, 1989).

Table 4: Marginal Rate Return (MRR) and Cost-Benefit Ratio (CBR) analysis

	Without NPK					With NPK				
	Control	Biochar	Neptune's Harvest™	Poultry manure	*Tithonia*	Sole NPK	Biochar	Neptune's Harvest™	Poultry manure	*Tithonia*
Marginal cost ($ ha⁻¹)	0	610	740	1300	525	700	1300	1390	1950	1175
Marginal net benefit ($ ha⁻¹)	0	460	-26	256	511	870	2287	2546	2593	1999
Marginal Rate of Return (MRR)	0	0.75	-0.03	0.20	0.97	1.24	1.76	1.83	1.33	1.70
Cost-Benefit Ratio (CBR)	1.33	1.12	0.76	0.70	1.21	1.30	1.57	1.62	1.33	1.53

MRR quantifies the average gain from adopting a new practice relative to the control (CIMMYT, 1989). Treatments with MRR ≥ 0.5 and CBR > 1 are considered economically viable. In this study, biochar alone (MRR = 0.75), *Tithonia diversifolia* alone (MRR = 0.97), and all organic + NPK combinations (MRR = 1.24–1.83) met this criterion. Single applications of Neptune's Harvest™ Fish Fertilizer or poultry manure were not recommended due to low MRR (<0.5) and CBR (<1).

V. DISCUSSION

This study evaluated the effects of organic and mineral fertilizers on watermelon production in Kenge's sandy, acidic soils. Among the treatments, poultry manure, Neptune's Harvest™, *Tithonia diversifolia*, and biochar combined with NPK consistently produced the best outcomes. Beyond high seedling survival rates (>90%), these treatments significantly improved fruit development and yield: average fruit weights ranged from 1.0 kg to 4.6 kg, fresh fruit yields from 4.6 t·ha⁻¹ to 12.6 t·ha⁻¹, and net benefits reached up to $3,978 ha⁻¹ for Neptune's Harvest™ combined with NPK. These results indicate tangible improvements in both horticultural performance and economic returns.

The superior performance of Neptune's Harvest™ and biochar on acidic ferralsol can be attributed to multiple mechanisms. Neptune's Harvest™, rich in proteins, amino acids, and micronutrients, enhanced nutrient assimilation, early vigor, and microbial activity, while biochar contributed to soil pH buffering, water retention, and aggregate stabilization. Its high cation exchange capacity improved NPK retention, reducing nutrient losses and promoting sustained growth. Integrating organic and mineral fertilizers optimized vegetative and reproductive development, as evidenced by larger fruits, improved fresh yield, and enhanced marketable fruit quality.

These findings align with regional studies in nutrient-constrained horticultural systems, where integrated soil fertility management increased plant growth, fruit bulking, and soluble solids (°Brix) under acidic, sandy conditions (International Institute of Tropical Agriculture, 2022; FAO, 2021). Observed flowering windows and fruit mass further support the physiological benefits of combined organic–mineral fertilization, highlighting practical implications for farmers seeking both yield and quality improvements.

By synthesizing literature into thematic clusters—early vigor, nutrient retention, and reproductive yield—this study moves beyond confirmatory results, providing mechanistic insights into how fertilizer choice mediates watermelon performance in acidic, nutrient-poor soils. These findings offer actionable guidance for the design of fertilization strategies and cooperative-based horticultural practices in Kenge and similar agroecological zones.

VI. CONCLUSION

Watermelon cultivation in Kenge is both agronomically feasible and economically profitable when supported by appropriate fertilization. Poultry manure, Neptune's Harvest™ Fish fertilizer, *Tithonia diversifolia*, and biochar produced the best results, with average fruit weights of 4.6 kg, 1.1 kg, 1.8 kg, and 1.0 kg per fruit, respectively, and fresh fruit yields of 12.6, 4.6, 6.6, and 5.0 $t \cdot ha^{-1}$. Corresponding net benefits were highest with Neptune's Harvest™Fish Fertilizer combined with NPK, reaching \$3,978 ha^{-1}, compared to \$1,385 ha^{-1} for the unfertilized control. From a socio-economic perspective, watermelon production provides opportunities for agricultural diversification, increased household incomes, and enhanced food security. Widespread adoption will depend on the formation of farmer cooperatives, site-specific fertilization strategies, improved market access, and the implementation of agroecological practices such as soil conservation and sustainable water management.

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Analysis and Control of Malaria Dynamic Models

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ABSTRACT

Malaria is one of the infectious and life-threatening vector-borne diseases that causes life-threatening complications. Effective and efficient strategies must be implemented to minimize the damage caused by malaria, and to do this, we must understand the dynamics of the measles transmission and implement control methods that are beneficial and cost-effective.

In this work, bifurcation analysis and multiobjective nonlinear model predictive control is performed on two dynamic models involving malaria transmission. Bifurcation analysis is a powerful mathematical tool used to deal with the nonlinear dynamics of any process. Several factors must be considered, and multiple objectives must be met simultaneously. The MATLAB program MATCONT was used to perform the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the state-of-the-art global optimization solvers IPOPT and BARON. The bifurcation analysis revealed the existence of branch points in both models. The MNLMPC calculations converged to the Utopia solution in both models. The branch points (which cause multiple steady-state solutions from a singular point) are very beneficial because they enable the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in both models.

Keywords: bifurcation, optimization, control, measles.

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Keywords: bifurcation, optimization, control, measles.

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I. BACKGROUND

Romero-Leiton, et al(2019)[1] conducted stability analysis and discussed optimal control intervention strategies of a malaria mathematical model. Ibrahim et al (2020)[2] investigated the impact of awareness on controlling malaria disease using a mathematical modelling approach. Abioye Adesoye Idowu, et al (2020)[3] performed optimal control on a mathematical model of malaria. Kobe (2020)[4], developed a mathematical model of controlling the Spread of Malaria Disease Using Intervention Strategies. Ndii et al (2021)[5] studied the effects of individual awareness and vector controls on malaria transmission dynamics using multiple optimal control. Adepoju et al (2021)[6] discussed the stability and optimal control of a disease model with vertical transmission and saturated incidence. Keno et al (2021)[7] discussed optimal control and cost effectiveness analysis of SIRS malaria disease model with temperature variability factor. Aldila Dipo et al (2021)[8] studied an optimal control problem and backward bifurcation on malaria transmission with vector bias. Tasman, et al (2021)[9] researched an optimal control problem of a malaria model with seasonality effect using real data. Al Basir et al (2021)[10] explored the effects of awareness and time delay in controlling malaria disease propagation. Sinan Muhammad et al (2022)[11], developed a Fractional mathematical model of malaria disease with treatment & insecticides. Al Basir et al (2023)[12] published an article on mathematical modelling and optimal control of malaria using awareness-based interventions. Olaniyi Samson et al (2023)[13], performed an optimal control analysis of a mathematical model for recurrent malaria dynamics. Wako et al (2025)[14] analysed and performed optimal control calculations of a model describing malaria and its associated complications.

This work aims to perform bifurcation analysis and multiobjective nonlinear control (MNLMPC) studies in two measles transmission models, which are discussed in Al Basir et al (2023)(model 1)[13] and Wako et al (2025) [14](model 2). The paper is organized as follows. First, the model equations are presented, followed by a discussion of the numerical techniques involving bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC). The results and discussion are then presented, followed by the conclusions.

II. MODEL EQUATIONS (MODEL 1)AL BASIR ET AL (2023)

In model 1, hu, ha, and hi represent the susceptible unaware, susceptible, aware, and infected human populations. vs and vi represent the susceptible and infected vector populations, while mv represents the level of awareness. The model equations are

$$\frac{d(hu)}{dt} = \pi h - \alpha hu(mv) - \frac{\lambda 1(hu)vi}{nv} - dh(hu) + \frac{g(ha)}{1+mv}$$
$$\frac{d(ha)}{dt} = \alpha hu(mv) - dh(ha) + c1r(hi)mv - \frac{\lambda 2(ha)vi}{nv} - \frac{g(ha)}{1+mv}$$
$$\frac{d(hi)}{dt} = \frac{\lambda 1(hu)vi}{nv} - (dh + delta)hi - c1(r)(hi)mv + \frac{\lambda 2(ha)vi}{nv}$$
$$\frac{d(vs)}{dt} = \pi v - \frac{\beta(hi)vs}{nv} - \mu vs - c2\gamma(vs)mv$$
$$\frac{d(vi)}{dt} = \frac{\beta(hi)vs}{nv} - \mu vi - c2(\gamma)vi(mv)$$
$$\frac{d(mv)}{dt} = c3\omega + \sigma hi - \theta mv$$

The base parameter values are

$$\lambda 1 = 0.02; \alpha = 0.001; \lambda 2 = 0.002; \beta = 0.25; \pi h = 400; \pi v = 10000;$$
$$\mu = 0.12; r = 0.001; dh = 0.002; \delta = 0.01; \gamma = 0.003; \theta = 0.01;$$
$$c1 = 0.2; c2 = 0.2; c3 = 0.2; g = 0.01; \ \omega = 0.05; \ \sigma = 0.05;$$

c1, c2 and c3 are the control parameters. More details can be found in Al Basir et al (2023).

III. MODEL EQUATIONS (MODEL 2)WAKO ET AL (2025)

Model 2 is a scaled model where sh, im, ic, rh, are the scaled variables representing the susceptible humans, malaria-infected humans, those with induced complications, and recovered human beings. The scaled susceptible and infected mosquito populations are represented by sv and iv.

The model equations are

$$gfac = \frac{\gamma 1}{1+\varepsilon ic}$$

$$\frac{d(sh)}{dt} = \mu 1 - \alpha 1 \sigma q(iv)sh - \mu 1 sh$$

$$\frac{d(im)}{dt} = \alpha 1 \sigma q(iv)sh - (\omega 1 + \mu 1)im - \gamma 0(u0)im$$

$$\frac{d(ic)}{dt} = \omega 1(im) - \mu 1(ic) - u1(gfac)ic - \delta 0(ic)$$

$$\frac{d(rh)}{dt} = \gamma 0(u0)im + u1(gfac)ic - \mu 1(rh)$$

$$\frac{d(sv)}{dt} = \mu 2 - \alpha 2 \sigma (im)sv - \mu 2(sv)$$

$$\frac{d(iv)}{dt} = \alpha 2(\sigma)im(sv) - \mu 2(iv)$$

The base parameter values are

$$\alpha 1 = 0.75; \alpha 2 = 0.2; \sigma = 0.3; \mu 1 = 3.9139e-05;\ \mu 2 = 0.12;\ \omega 1 = 0.2185;\ \gamma 0 = 0.0056;$$
$$\gamma 1 = 0.0056;\ \delta 0 = 0.0244; \varepsilon = 0.007; q = 2;\ u0 = 0.2; u1 = 0.2;$$

More details can be found in Wako et al (2025).

IV. BIFURCATION ANALYSIS

The MATLAB software MATCONT is used to perform the bifurcation calculations. Bifurcation analysis deals with multiple steady-states and limit cycles. Multiple steady states occur because of the existence of branch and limit points. Hopf bifurcation points cause limit cycles . A commonly used MATLAB program that locates limit points, branch points, and Hopf bifurcation points is MATCONT(Dhooge Govearts, and Kuznetsov, 2003[15]; Dhooge Govearts, Kuznetsov, Mestrom and Riet, 2004[16]). This program detects Limit points(LP), branch points(BP), and Hopf bifurcation points(H) for an ODE system

$$\frac{dx}{dt} = f(x, \alpha)$$

$x \in R^n$ Let the bifurcation parameter be α. Since the gradient is orthogonal to the tangent vector,

The tangent plane at any point $w = [w_1, w_2, w_3, w_4, ...w_{n+1}]$ must satisfy

$$Aw = 0$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha]$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the Jacobian matrix $J = [\partial f / \partial x]$ must be singular.

For a limit point, there is only one tangent at the point of singularity. At this singular point, there is a single non-zero vector, y, where Jy=0. This vector is of dimension n. Since there is only one tangent the vector

$y = (y_1, y_2, y_3, y_4, ... y_n)$ must align with $\hat{w} = (w_1, w_2, w_3, w_4, ... w_n)$. Since

$$J\hat{w} = Aw = 0$$

the n+1 th component of the tangent vector w_{n+1} = 0 at a limit point (LP).

For a branch point, there must exist two tangents at the singularity. Let the two tangents be z and w. This implies that

$$Az = 0$$
$$Aw = 0$$

Consider a vector v that is orthogonal to one of the tangents (say w). v can be expressed as a linear combination of z and w ($v = \alpha z + \beta w$). Since $Az = Aw = 0$; $Av = 0$ and since w and v are orthogonal,

$w^T v = 0$. Hence $Bv = \begin{bmatrix} A \\ w^T \end{bmatrix} v = 0$ which implies that B is singular.

Hence, for a branch point (BP) the matrix $B = \begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular.

At a Hopf bifurcation point,

$$\det(2f_x(x,\alpha) @ I_n) = 0$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov (1998[17]; 2009[18]) and Govaerts [2000] [19].

V. MULTIOBJECTIVE NONLINEAR MODEL PREDICTIVE CONTROL(MNLMPC)

The rigorous multiobjective nonlinear model predictive control (MNLMPC) method developed by Flores Tlacuahuaz et al (2012)[20] was used.

Consider a problem where the variables $\sum_{t_{i=0}}^{t_i = t_f} q_j(t_i)$ (j=1, 2..n) have to be optimized simultaneously for a dynamic problem

$$\frac{dx}{dt} = F(x,u)$$

t_f being the final time value, and n the total number of objective variables and u the control parameter. The single objective optimal control problem is solved individually optimizing each of the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ The optimization of $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ will lead to the values q^*_j. Then, the multiobjective optimal control (MOOC) problem that will be solved is

$$\min(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)-q_j^*))^2$$

$$subject\ to\ \frac{dx}{dt}=F(x,u);$$

This will provide the values of u at various times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where ($\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)=q_j^*$ for all j) is obtained.

Pyomo (Hart et al, 2017)[21] is used for these calculations. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method The NLP is solved using IPOPT (Wächter And Biegler, 2006)[22]and confirmed as a global solution with BARON (Tawarmalani, M. and N. V. Sahinidis 2005)[23].

The steps of the algorithm are as follows

1. Optimize $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ and obtain q_j^*.
2. Minimize $(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)-q_j^*))^2$ and get the control values at various times.
3. Implement the first obtained control values
4. Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when

   $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)=q_j^*$ for all j.

Sridhar (2024)[24] demonstrated that when the bifurcation analysis revealed the presence of limit and branch points tthe MNLMPC calculations to converge to the Utopia solution . For this, the singularity condition, caused by the presence of the limit or branch points was imposed on the co-state equation (Upreti, 2013)[25]. If the minimization of q_1 lead to the value q_1^* and the minimization of q_2 lead to the value q_2^* The MNLPMC calculations will minimize the function $(q_1-q_1^*)^2+(q_2-q_2^*)^2$. The multiobjective optimal control problem is $\min \quad (q_1-q_1^*)^2+(q_2-q_2^*)^2 \quad subject\ to \ \frac{dx}{dt}=F(x,u)$

Differentiating the objective function results in

$$\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)=2(q_1-q_1^*)\frac{d}{dx_i}(q_1-q_1^*)+2(q_2-q_2^*)\frac{d}{dx_i}(q_2-q_2^*)$$

The Utopia point requires that both $(q_1-q_1^*)$ and $(q_2-q_2^*)$ are zero. Hence

$$\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)=0$$

The optimal control co-state equation (Upreti; 2013)[25] is

$$\frac{d}{dt}(\lambda_i)=-\frac{d}{dx_i}((q_1-q_1^*)^2+(q_2-q_2^*)^2)-f_x\lambda_i; \quad \lambda_i(t_f)=0$$

λ_i is the Lagrangian multiplier. t_f is the final time. The first term in this equation is 0 and hence

$$\frac{d}{dt}(\lambda_i)=-f_x\lambda_i;\lambda_i(t_f)=0$$

At a limit or a branch point, for the set of ODE $\frac{dx}{dt}=f(x,u)$ f_x is singular. Hence there are two different vectors-values for $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i)>0$ and $\frac{d}{dt}(\lambda_i)<0$. In between there is a vector $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i)=0$. This coupled with the boundary condition $\lambda_i(t_f)=0$ will lead to $[\lambda_i]=0$ This makes the problem an unconstrained optimization problem, and the optimal solution is the Utopia solution.

VI. RESULTS AND DISCUSSION

The bifurcation analysis for model 1; with c1 as the bifurcation parameter, revealed a branch point at (hu; ha; hi; vs; vi; mv;c1) values of (175000; 25000; 0; 82918.739635; 0; 1; 3.255054) (Fig. 1a).

For theMNLMPC calculations, hu (0) is set to 100000; $\sum_{t_{i=0}}^{t_i=t_f} ha(t_i)$ were maximized and resulted in a value of 200000 and $\sum_{t_{i=0}}^{t_i=t_f} hi(t_i)$ was minimized and resulted in a value of 0. The overall optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} ha(t_i)-200000)^2+(\sum_{t_{i=0}}^{t_i=t_f} hi(t_i)-0)^2$ was minimized subject to the equations governing the model. This led to a value of zero (the Utopia point).

The MNLMPC values of the control variables, c1, c2, and c3 were 0.5004, 0.5072, and, 0.6403. The various MNMPC figures are shown in figures 1b-1i. The control profiles c1, c2, c3 (Fig. 1h) exhibited noise, and this was remedied using the Savitzky-Golay filter to produce the smooth control profiles c1sg, c2sg, and c3sg (Fig. 1i). It is seen that the presence of the a branch point is beneficial because it allows the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar(2024)[24].

The bifurcation analysis for model 2 with u0 as the bifurcation parameter revealed a branch point at (sh; im; ic; rh; sv; iv;u0) values of (1 0 0 0 1 0 1.153725)(Fig. 2a).

For theMNLMPC calculations, sh(0)is set to 0.7 ; $\sum_{t_{i=0}}^{t_i=t_f} im(t_i), \sum_{t_{i=0}}^{t_i=t_f} ic(t_i), \sum_{t_{i=0}}^{t_i=t_f} iv(t_i)$ were minimized individually and each resulted in a value of 0. The overall optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} im(t_i)-0)^2+(\sum_{t_{i=0}}^{t_i=t_f} ic(t_i)-0)^2+(\sum_{t_{i=0}}^{t_i=t_f} iv(t_i)-0)^2$

was minimized subject to the equations governing the model. This led to a value of zero (the Utopia point).

The MNLMPC values of the control variables, u0, u1 were 0.3778 and 0.3899. The various MNMPC figures are shown in figures 2b-2i. The control profiles u0, u1 (Fig. 2h) exhibited noise, which was remedied using the Savitzky-Golay filter to produce the smooth control profiles u0sg, u1sg. (Fig. 2i). It is seen that the presence of the a branch point is beneficial because it allows the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar(2024)[24].

In both models, the presence of the a branch point is beneficial because it allows the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar(2024)[24].

VII. CONCLUSIONS

Bifurcation analysis and multiobjective nonlinear control (MNLMPC) studies in two malaria transmission models. The bifurcation analysis revealed the existence of branch points in both models. The branch points (which cause multiple steady-state solutions from a singular point) are very beneficial because they enable the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in the models. A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control (MNLMPC) for malaria transmission models is the main contribution of this paper.

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Data Availability Statement

All data used is presented in the paper

Conflict of interest

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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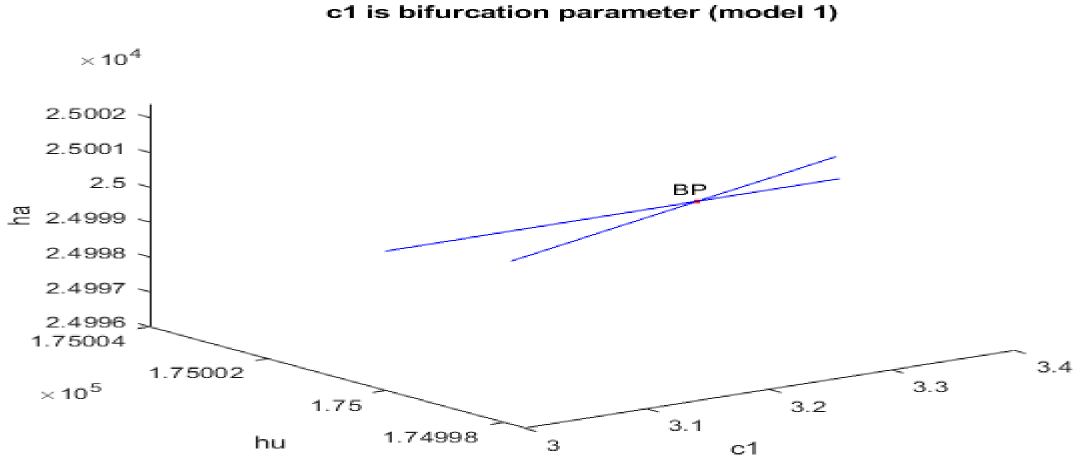

Fig. 1a: Bifurcation analysis of model 1

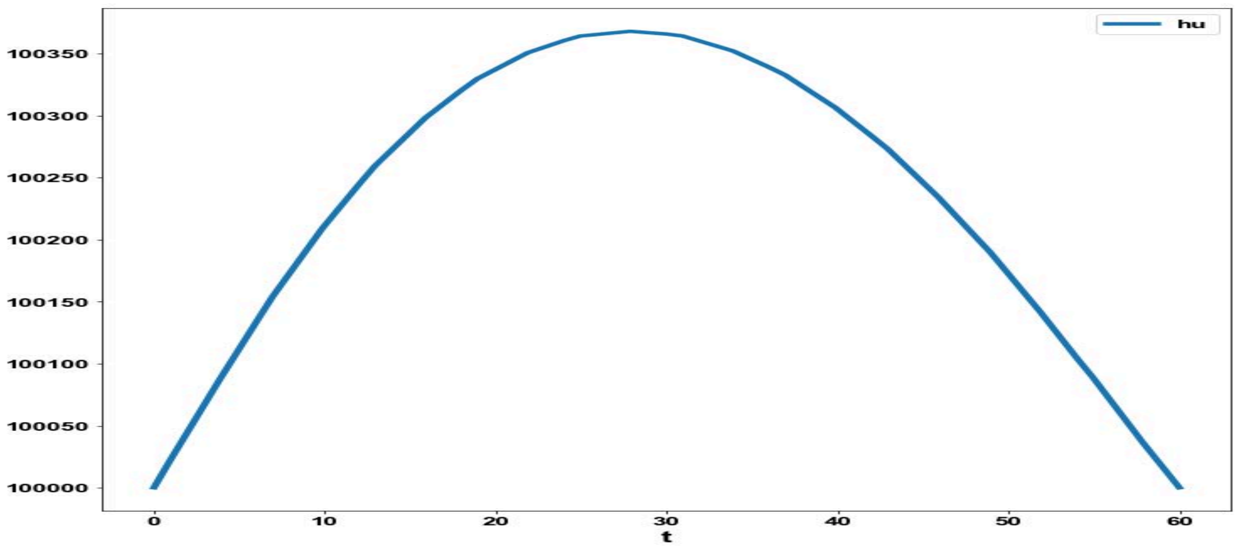

Fig. 1b: MNLMPC model 1(hu vs t)

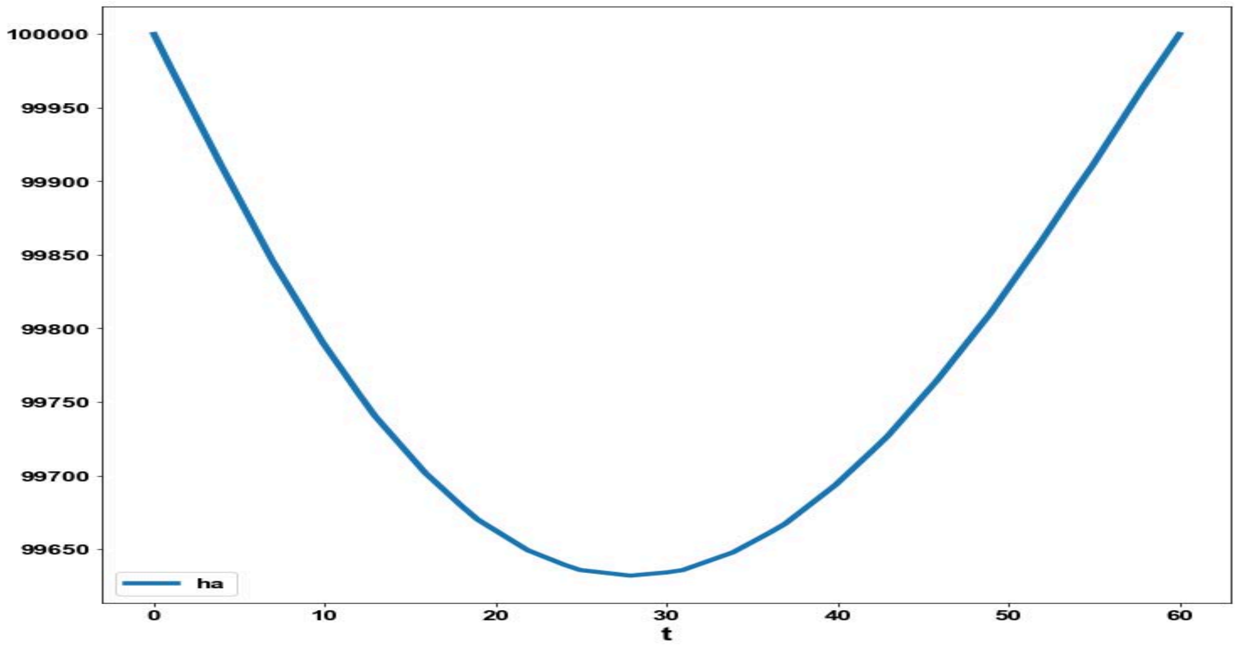

Fig. 1c: MNLMPC model 1(ha vs t)

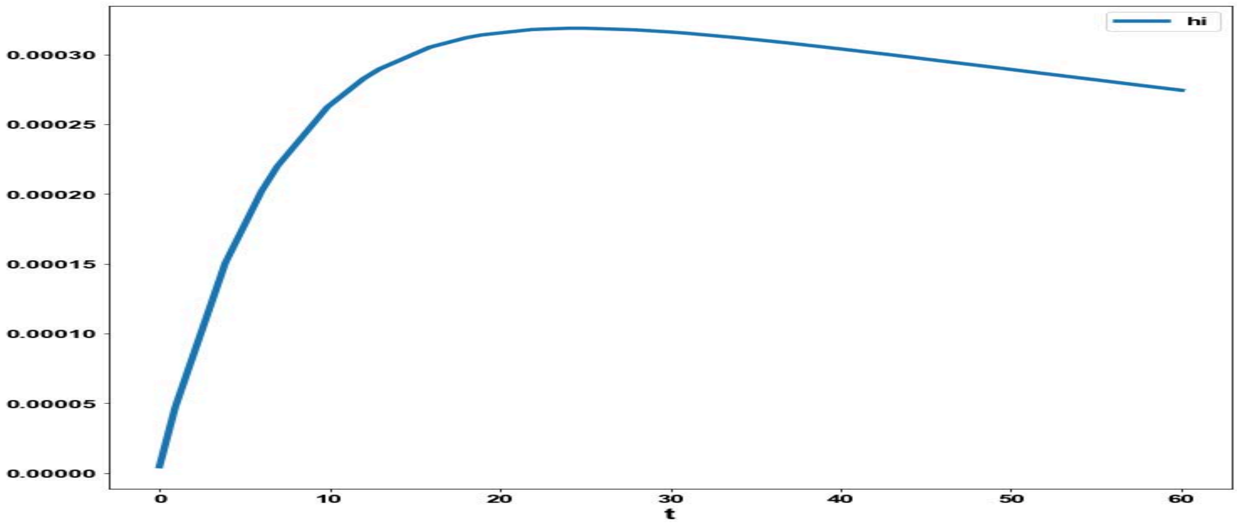

Fig. 1d: MNLMPC model 1(hi vs t)

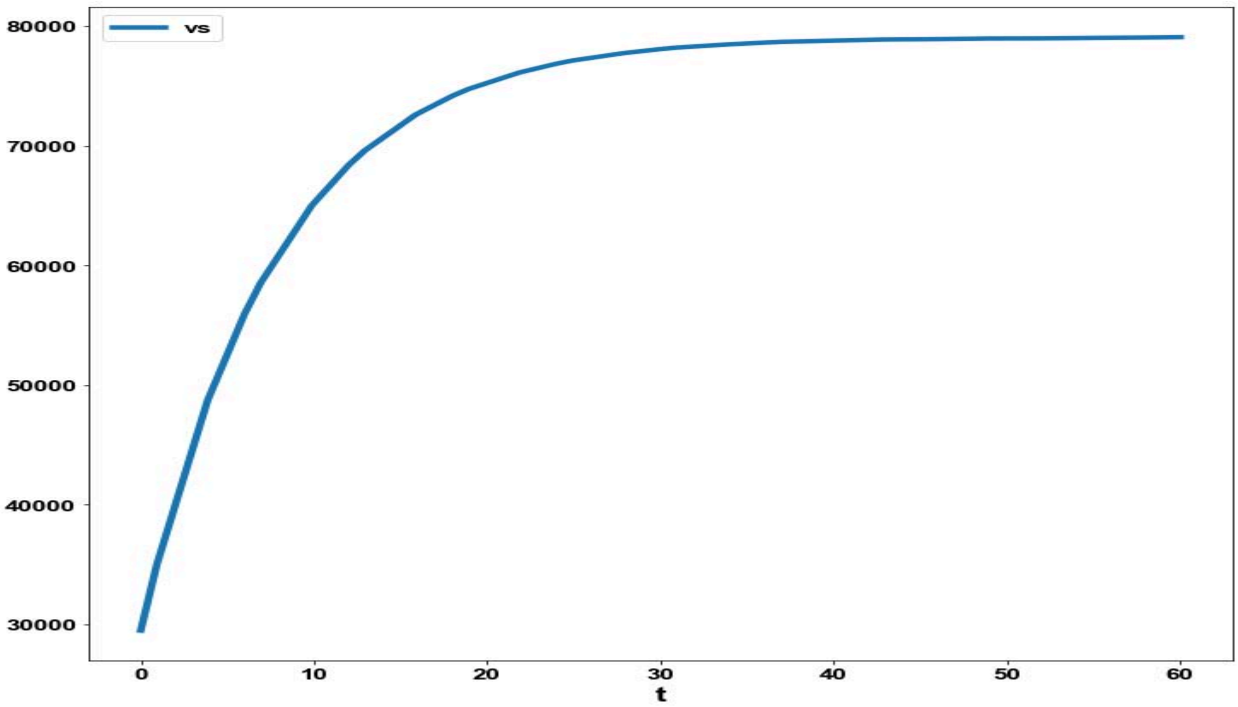

Fig. 1e: MNLMPC model 1(vs vs t)

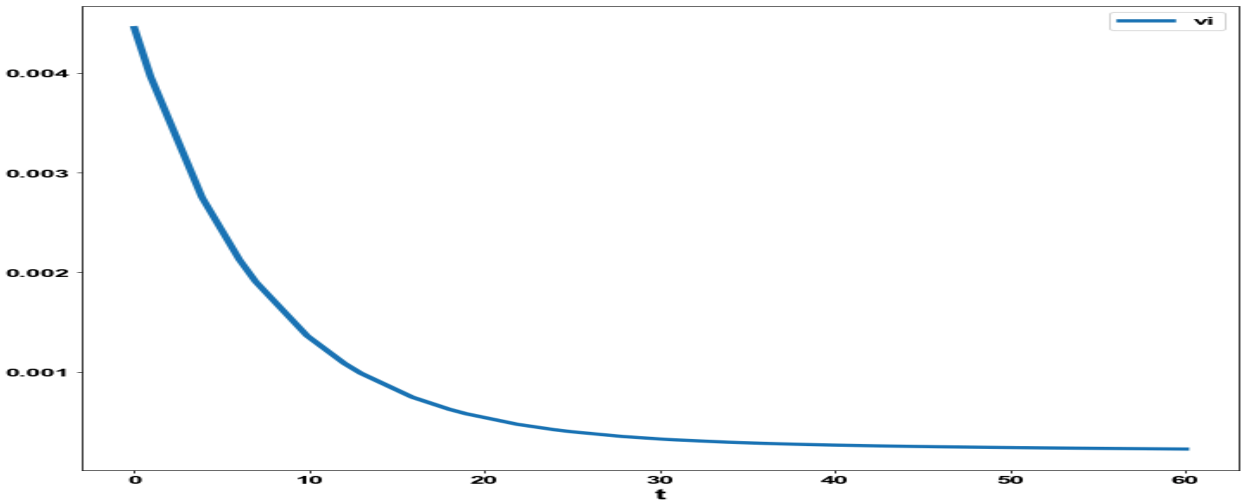

Fig. 1f: MNLMPC model 1(vi vs t)

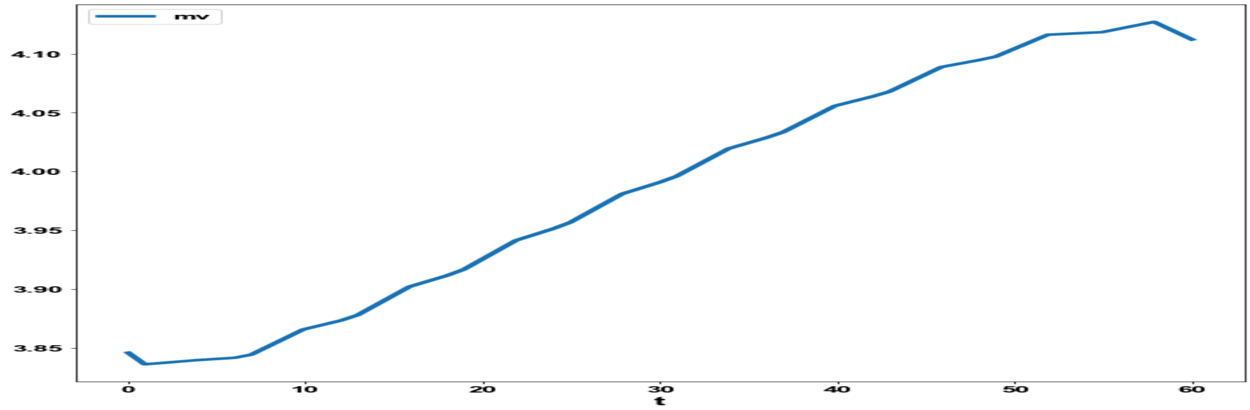

Fig. 1g: MNLMPC model 1(mv vs t)

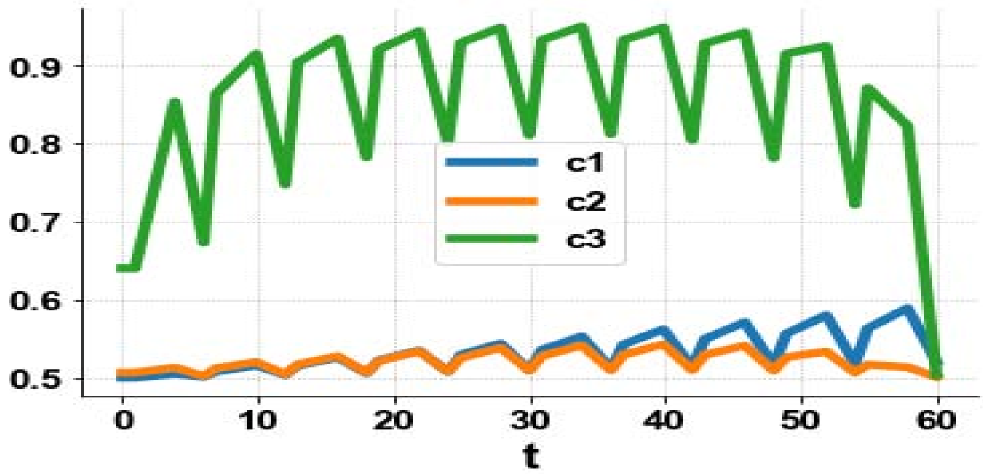

Fig. 1h: MNLMPC model 1(u1,u2,u3 vs t)

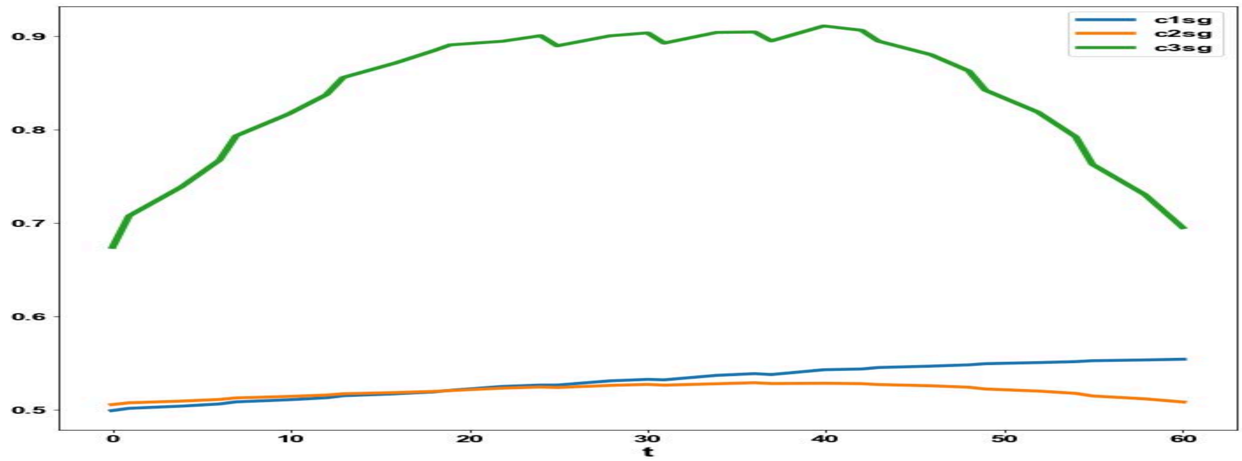

Fig. 1i: MNLMPC model 1(u1sg,u2sg,u3sg vs t)

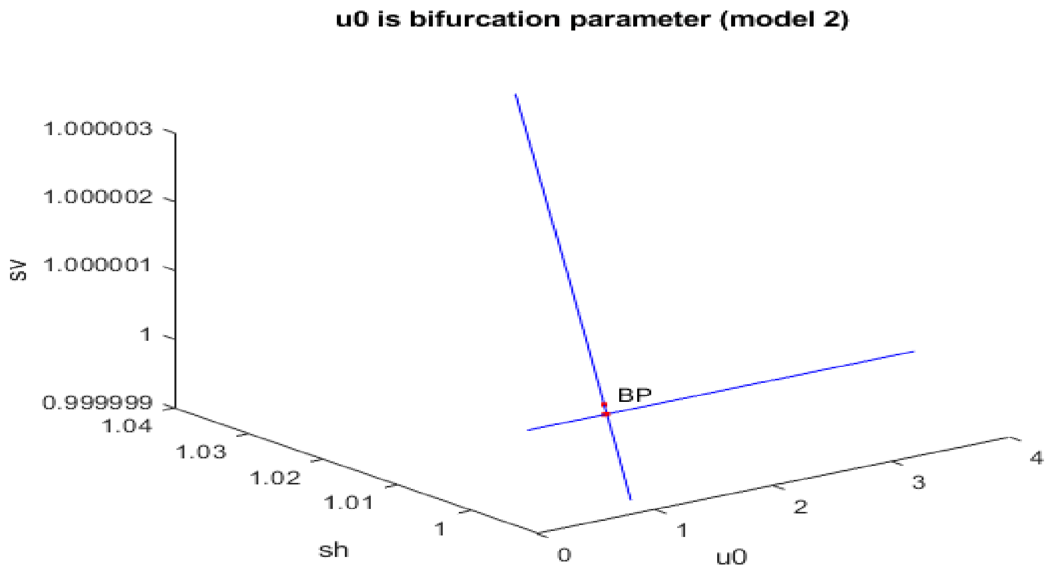

Fig. 2a: Bifurcation analysis of model 2

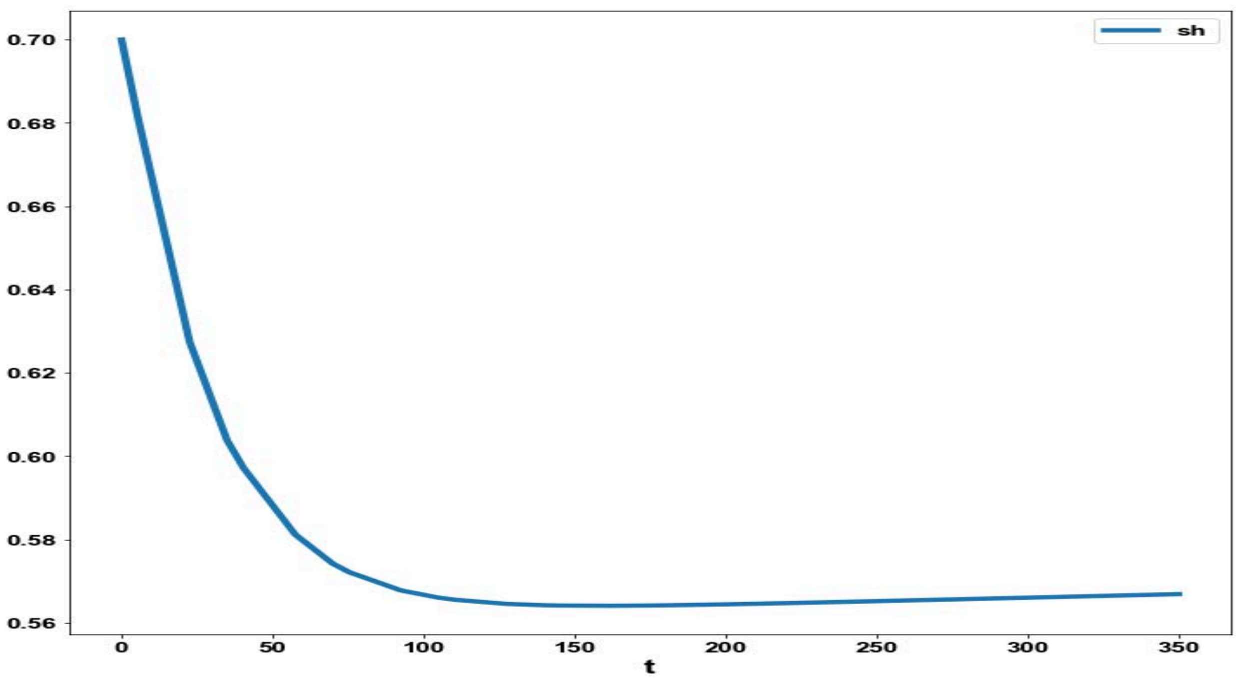

Fig. 2b: MNLMPC model 2(sh vs t)

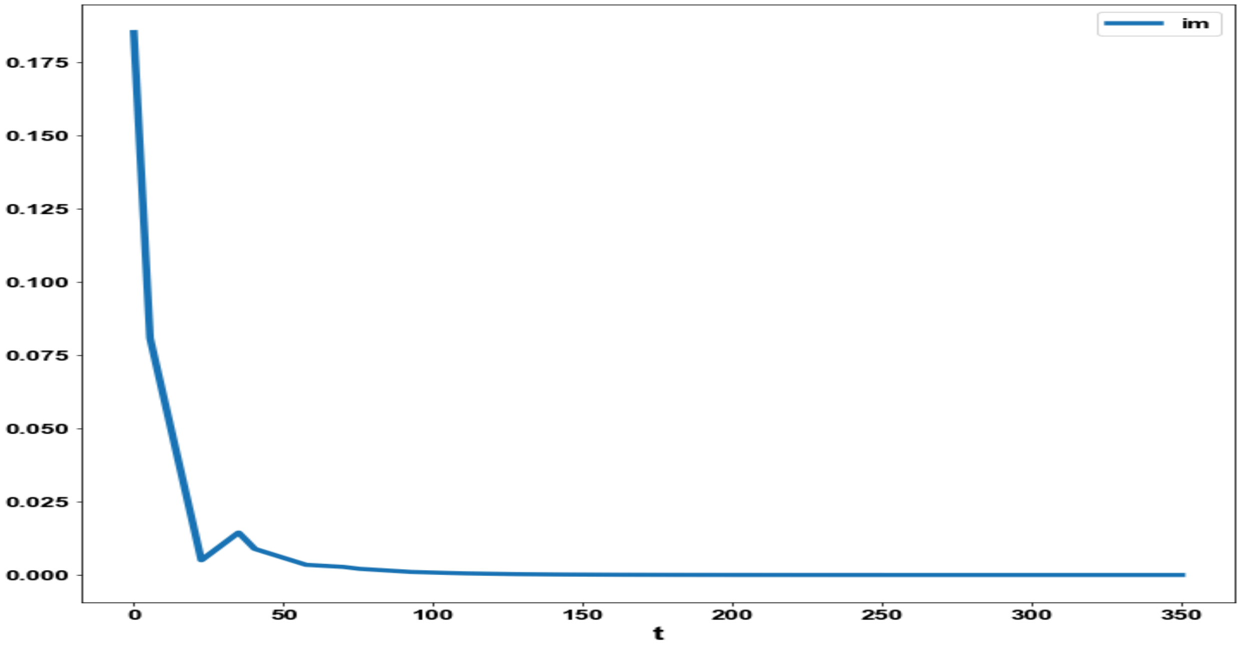

Fig. 2c: MNLMPC model 2 (im vs t)

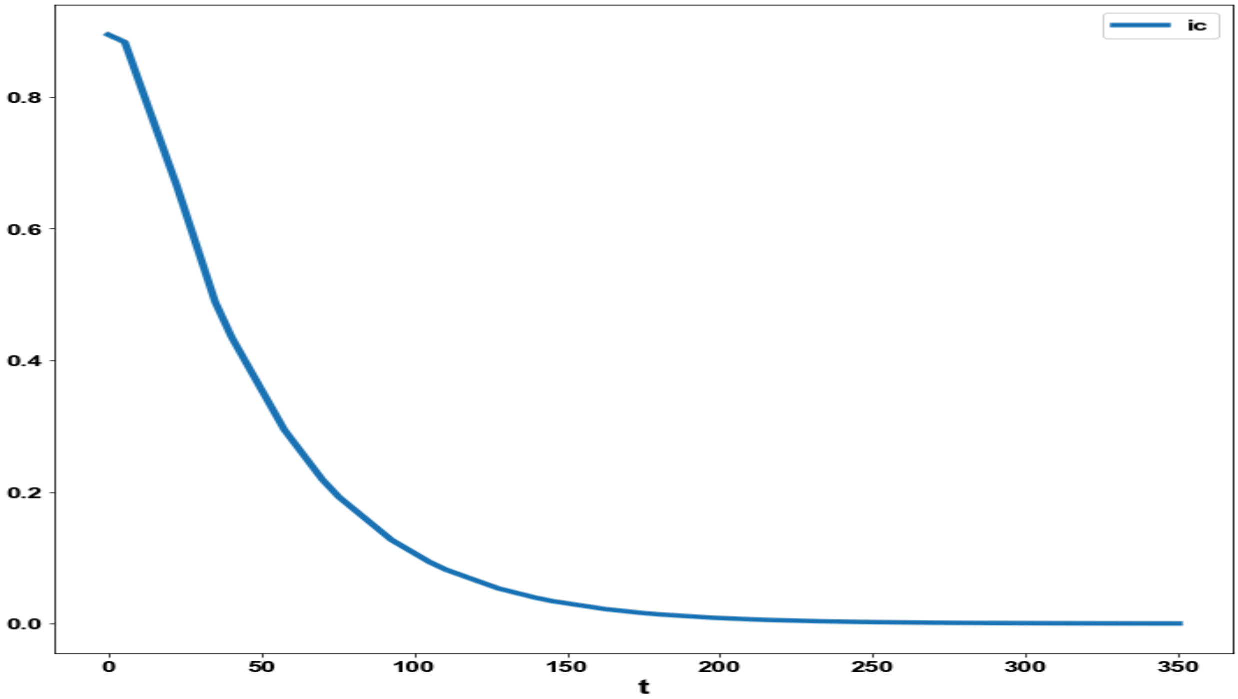

Fig. 2d: MNLMPC model 2 (ic vs t)

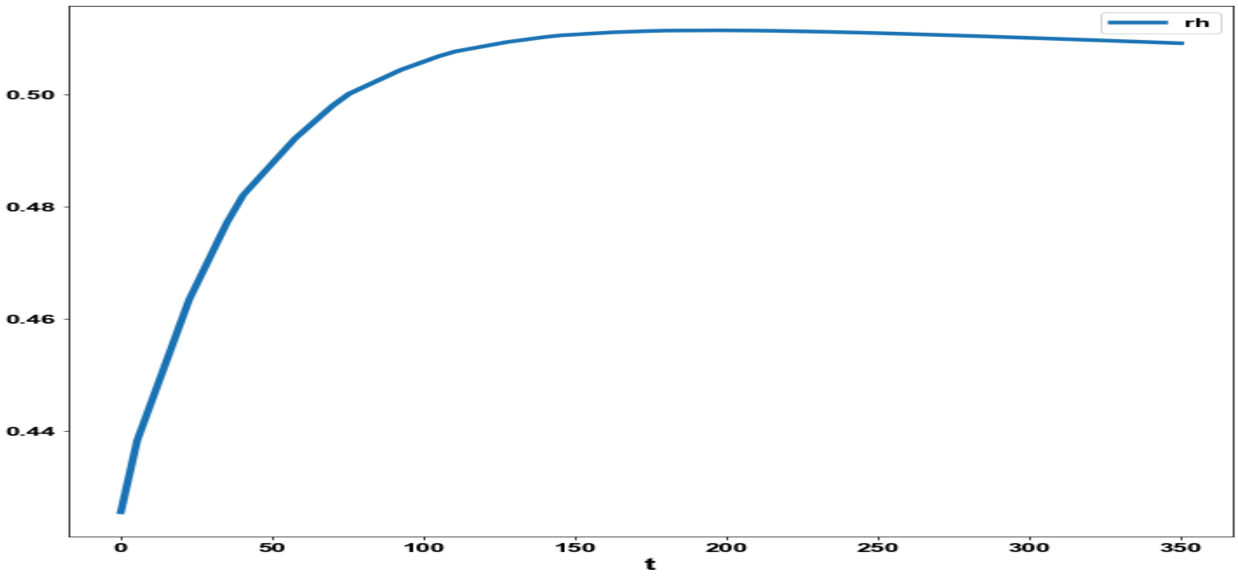

Fig. 2e: MNLMPC model 2 (rh vs t)

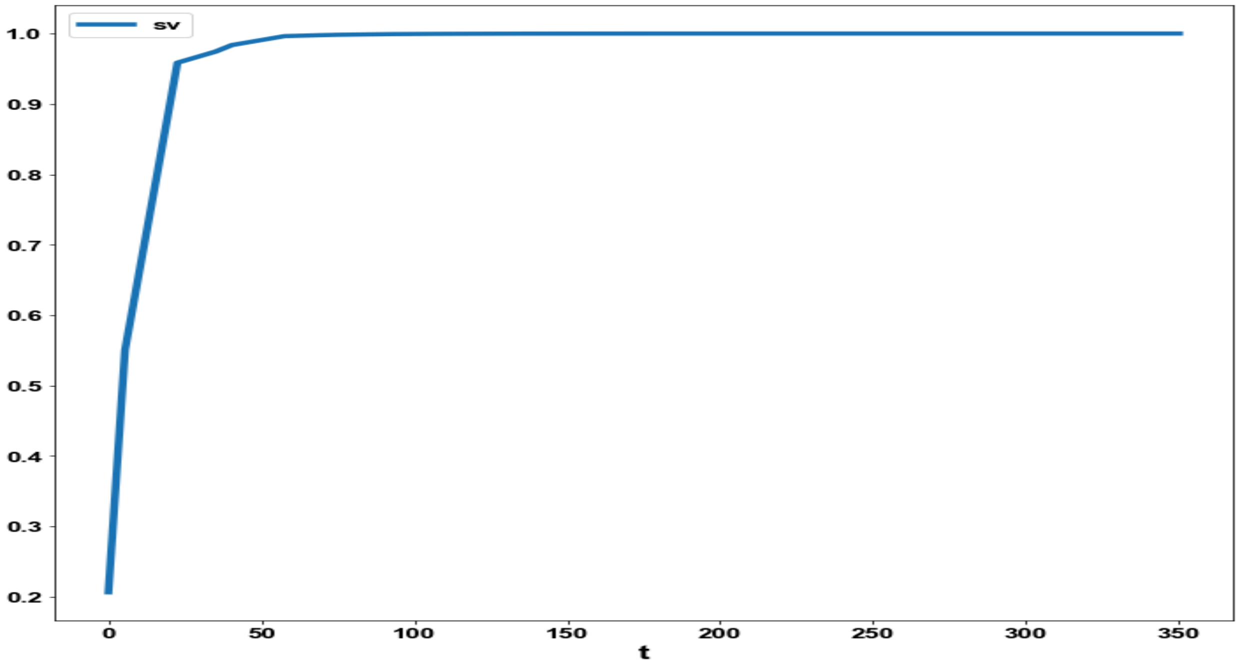

Fig. 2f: MNLMPC model 2 (sv vs t)

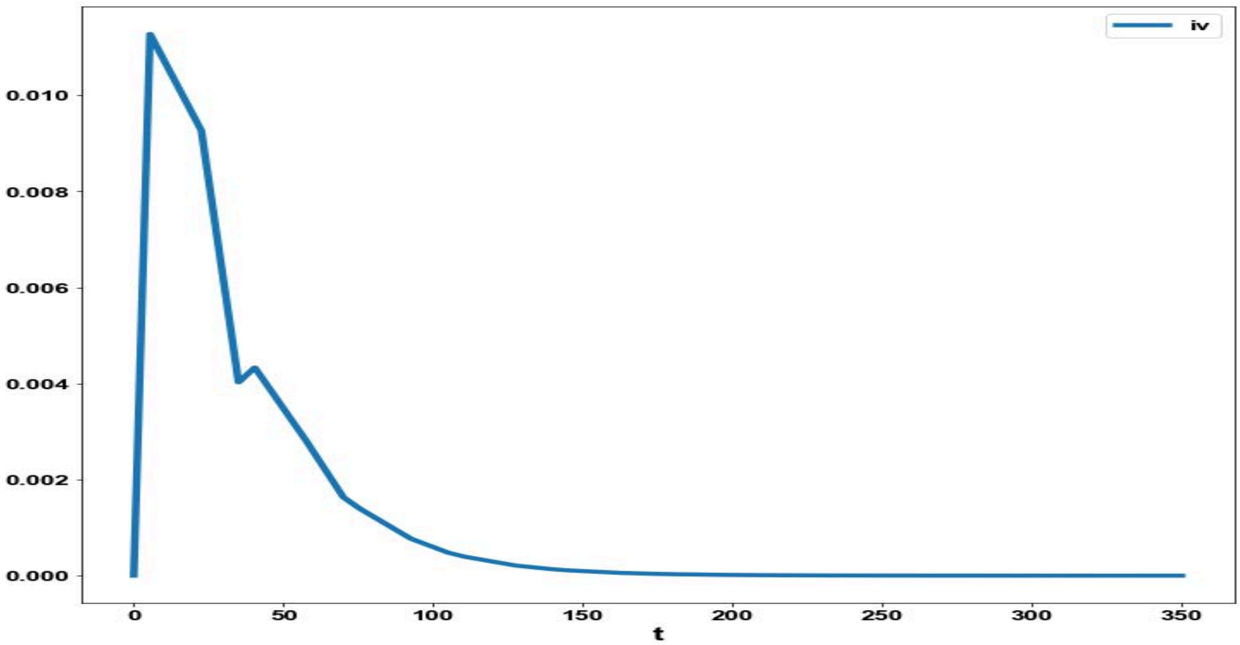

Fig. 2g: MNLMPC model 2 (iv vs t)

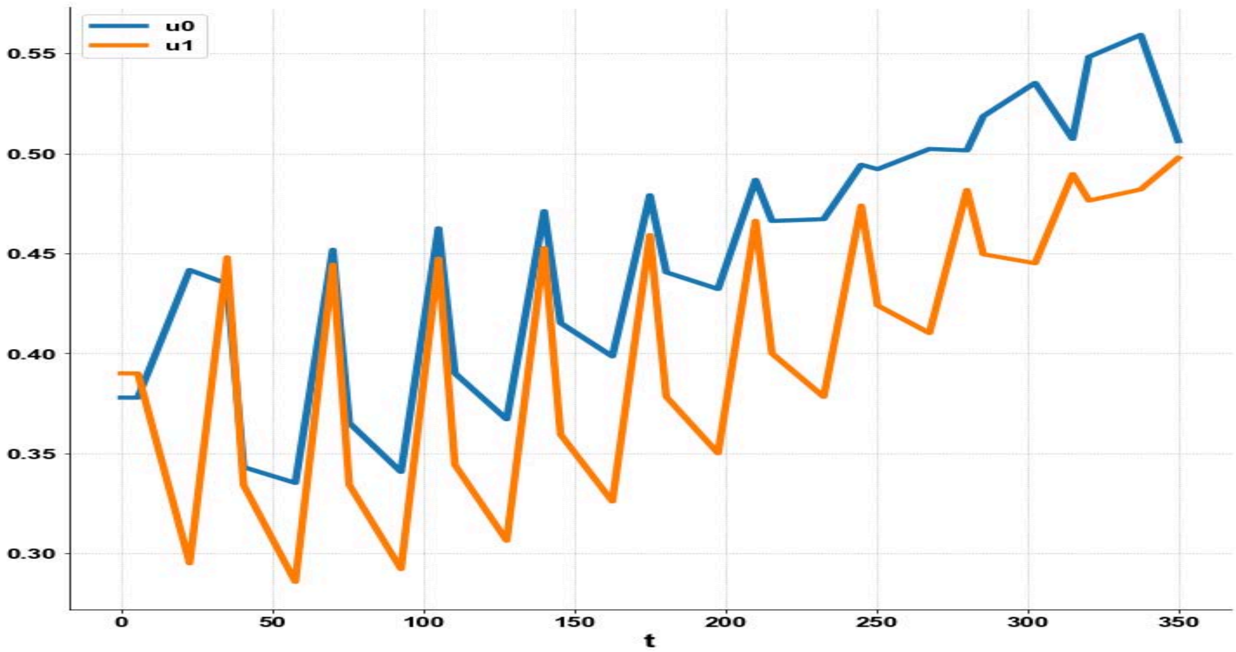

Fig. 2h: MNLMPC model 2 (control profiles noise exhibited)

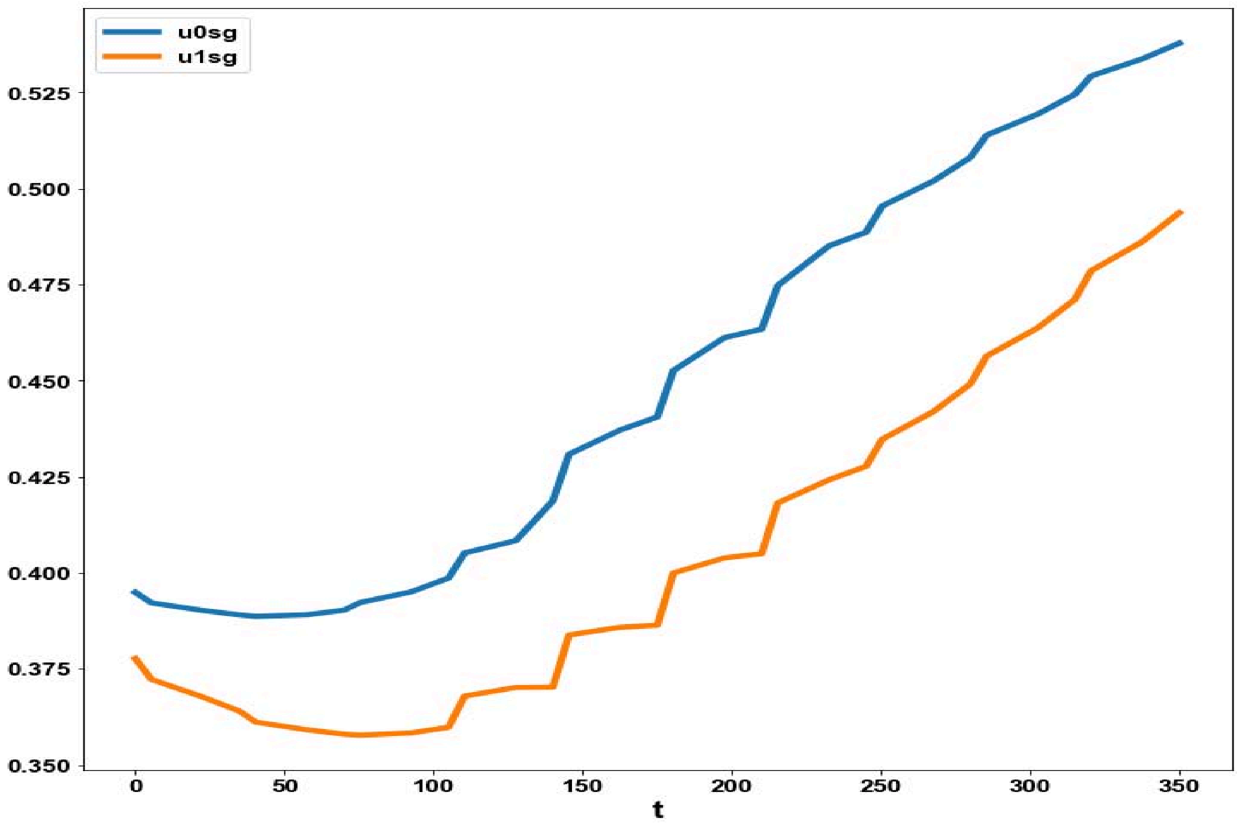

Fig. 2i: MNLMPC model 2 (control profiles noise eliminated)

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A Paper on "String-Theory, AdS/cFT, and Sarkar Equation of Holography

Najmuj Sahadat Sarkar

ABSTRACT

This paper analyzes how UV/IR relations in AdS/CFT holography determine the behavior of degrees of freedom, thermal fluctuations, and entropy in the AdS bulk and its conformal field theory boundary. First, using a UV regulator δ on the AdS boundary, the expectation values of coordinate fluctuations behave as $\langle z\rangle^2 \sim \delta^{-2}$ and $\langle y\rangle^2 \sim \delta^2$. This demonstrates that the SYM UV cut off on the boundary maps to the IR cut-off inside the bulk. Considering a boundary patch of area 1, with $1/\delta^3$ cutoff cells and N^2 degrees of freedom per cell (from the adjoint of U(N)), the total degrees of freedom per unit area becomes $N\_dof \approx N^2/\delta^3$. The regulated 8-dimensional area scales as R^8/δ^3, giving $N\_dof/A \sim N^2/R^8$. Using $1/G \sim N^2/R^8$, this matches the holographic principle. Next, thermal fluctuations of the SYM field Φ satisfy $\langle \Phi^2 \rangle = T^2\_SYM$, replacing earlier UV fluctuation expressions. Thermal fluctuations become significant at $z \sim T\_SYM$, corresponding to $(1-r) \sim 1/T\_SYM$, similar to the AdS black hole horizon relation. Then i introduces the Sarkar Approximation, assuming the boundary cut-off δ is very small (close to Planck length), creating a quantum domain where degrees of freedom satisfy a "position–momentum" uncertainty relation $r \cdot p \sim \hbar$. Using $r \approx 1 - 1/T\_SYM$ and $E\_SYM = cN^2T^4\_SYM$, one obtains.

Keywords: AdS, boundary, bulk, UV/IR, Super String, AdS/cFT, SYM, D3-brane, holography, sarkar approximation, sarkar equation of holography.

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$(T^4\_SYM - T^3\_SYM)\ c\ N\_dof \approx 1$. Solving this differential relation yields: $N\_dof = e^i \times (1 - T\_SYM) / T^3\_SYM$, where e^i is a dimensionless factor determined by physical limits. At $T\_SYM \rightarrow \infty$ (boundary), $N\_dof \rightarrow 0$ unless $e^i \rightarrow \infty$.At $T\_SYM \rightarrow 0$ (bulk), $N\_dof \rightarrow \infty$ unless $e^i \rightarrow 0$. Thus N_dof remains fixed in both regimes. Using $S = |\ln(N\_dof)|$, the entropy of the AdS boundary and the bulk are equal. This equality expresses a holographic limit, and equation (40) is termed the "Sarkar Equation of Holography."

Keywords: AdS, boundary, bulk, UV/IR, Super String, AdS/cFT, SYM, D3-brane, holography, sarkar approximation, sarkar equation of holography.

I. INTRODUCTION

AdS space has certain specific properties that make it a natural representative for a Holographic Hamiltonian description. In this paper, I will discuss a remarkable form or structure of AdS holography, which originates from the mathematics of string theory.

This remarkable form comes from the higher-order symmetry of the theory, which is actually a powerful version of supersymmetry. However, we will ignore the mathematical aspects of the theory and emphasize the simple physical principles. The particular space I am interested in working with is not five-dimensional AdS, but rather AdS(5)⊗S(5). This is actually a 10-dimensional product space constructed from two factors: first, a 5-dimensional AdS, and second, the 5-sphere or S(5). But why S(5)? Because higher-order supersymmetry operates in superstring theory. Typically, the supergravity theories that arise from string theory do not have a cosmological constant. But by bending some of the spatial directions into a compact manifold, we can obtain a lower-dimensional "Kaluza-Klein" theory, where a cosmological constant can be generated in a certain manner. Conceptually speaking, the internal 5-sphere is not important, but from a mathematical standpoint, it is quite important if we want to construct a precise statement. I want to begin my paper by giving a detailed idea of AdS geometry. For this purpose, we assume that the 5-dimensional AdS space is actually a 4-dimensional solid ball multiplied by an infinite 1- dimensional time axis. This geometry can be described in terms of some dimensionless coordinates such as t, r, Ω. Here t is time, r is a radial coordinate whose value varies as 0, or between 0 and 1, meaning $0 \le r < 1$, and Ω parameterizes the unit 3-sphere.

This geometry has uniform curvature R^{-2}, where R is the curvature radius. Under these circumstances, the metric becomes:

$$d\tau^2=[R^2/(1-r^2)^2]\{(1+r^2)^2dt^2-4dr^2-4r^2d\Omega^2\} \tag{1}$$

This metric can also be written in another way:

$$d\tau^2=(R^2/y^2)\{dt^2-dx^idx^i-dy^2\} \tag{2}$$

where i=1,2,3. Equation (2) can be related to equation (1) in two different ways. First, at the boundary r=1, equation (2) works as an approximation of equation (1). The 3-sphere can be replaced by a flat tangential plane (parameterized by x^i), and the radial coordinate can be replaced by y, where y=(1−r). The second way in which equation (1) and equation (2) can be related is that equation (2) actually works as an exact metric for an incomplete patch of AdS space. A time-like geodesic reaches $y=\infty$ in finite proper time; therefore, the space in equation (2) is geodesically incomplete. Its horizon lies at $y=\infty$, and thus the time coordinate in equation (1) is not equal to the time coordinate in equation (2). We can also write the metric in equation (2) in the form z=1/y, giving:

$$d\tau^2=R^2\{z^2(dt^2-dx^idx^i)-(1/z^2)dz^2\} \tag{3}$$

According to this form, we can say that at z=0 we find a horizon because the time-time component of the metric vanishes there, and at $z=\infty$ we find the boundary. However, to construct $AdS(5)\otimes S(5)$ space, we need to introduce 5 additional coordinates ω_5 describing the unit 5-sphere, whose metric is:

$$ds_5{}^2=R^2d\omega_5{}^2 \tag{4}$$

Although the AdS boundary is at infinite proper distance from any point inside the ball, light can travel to that boundary and return within a finite time. For example, it takes a total time $t=\pi$ to go from the origin r=0 to the boundary r=1 and return!

For all practical purposes we may say that AdS space is a finite cavity with perfectly reflecting walls. The size of the cavity is denoted by R. And R is much larger than any microscopic scale such as the Planck scale or the string scale.

We shall call $AdS(5)\otimes S(5)$ the bulk space, and the 4-dimensional boundary the AdS boundary, where y=0.

According to the holographic principle, everything inside the bulk can be described by a theory whose degrees of freedom can be identified at the boundary y=0.

The holographic principle claims something more: it states that the boundary theory should not contain more than one degree of freedom per Planck area.

To understand this, let us determine the area of the boundary. From equation (1) it is clear that the metric diverges near the boundary. Later we will regulate this divergence by moving slightly away from the boundary y=0. But for now, we assume that the number of degrees of freedom per unit coordinate area is infinite.

This indicates that the boundary theory will be a quantum field theory.

Another important aspect relates to the symmetry of AdS. Look again at the metric of equation (2)! It is obvious that the geometry is invariant under ordinary 4-dimensional Minkowski Poincaré transformations of coordinates t and x^i. Additionally, there is a dilational symmetry:

$$t \rightarrow \lambda t$$
$$x^i \rightarrow \lambda x^i$$
$$Y \rightarrow \lambda y \tag{5}$$

On the other hand, if we consider only the representation of AdS in equation (1), we find additional symmetry. For example, all the rotations of the sphere Ω fall into the symmetry group. The full symmetry group of AdS(5) is O(4|2). Additionally, there is an O(6) symmetry corresponding to the rotations of the internal 5-sphere.

Since our goal is to argue that the holographic description of physics inside the bulk space-time is important, we must show how the symmetries of AdS act on the boundary. The 4-dimensional Poincaré symmetry acting on the boundary is straightforward. The dilational symmetry acts easily on the coordinates t and x.

These transformations acting on the boundary are known as conformal transformations, which preserve light-like directions on the boundary.

Indeed, the full AdS symmetry group, when acting on the boundary y=0, becomes the conformal group of 4-dimensional Minkowski space.

The implication of this boundary symmetry is that the holographic boundary theory must be invariant under conformal group transformations.

From this perspective (along with the infinite coordinate density of the boundary), the holographic theory is actually a CFT or conformal quantum field theory.

As previously mentioned, AdS(5)⊗S(5) is a solution of 10-dimensional supergravity, which is the low-energy limit of superstring theory.

It is true that additional symmetries may exist beyond the conformal group, such as the O(6) symmetry related to the internal 5-sphere. These extra symmetries correspond to N=4 supersymmetry.

This symmetry is easily understood in the holographic theory.

Ultimately, we arrive at the conclusion that the quantum gravity inside AdS(5)⊗S(5) is actually equivalent to a superconformal Lorentz-invariant QFT on the AdS boundary.

To get a measure of the number of degrees of freedom in AdS(5)⊗S(5), we must construct a cutoff field theory inside the bulk.

A simple cutoff corresponds to a microscopic length equal to the 10-dimensional Planck length l(p). One way to do this is to introduce a sort of spatial lattice in the 9-dimensional space. It need not be a regular lattice; only its average spacing must be l(p).

Then we can construct a simple Hamiltonian lattice theory.

To count the degrees of freedom, we consider the area of AdS as infinite.

For that, we consider an L-surface at $r=1-\delta$.

The 9-dimensional spatial volume inside the L-surface can be obtained easily from equation (2), and it is critically divergent:

$$V(\delta) \sim R^9/\delta^3 \tag{6}$$

Thus, the number of lattice sites or degrees of freedom inside the bulk becomes:

$$V/l(p)^9 \sim (1/\delta^3) R^9/l(p)^9 \tag{7}$$

In such a theory, we can compute the maximum entropy. According to the holographic bound:

$$S\_max \sim A/l(p)^8 \tag{8}$$

where A is the area of the L-boundary. Therefore:

$$S\_max \sim (1/\delta^3) R^8/l(p)^8 \tag{9}$$

In other words, when R/l(p) is large, the holographic description requires a reduction factor of {l(p)/R} in the number of degrees of freedom per unit Planck volume.

Stated differently, the holographic principle can describe all microscopic physics inside a very large AdS space using {l(p)/R} degrees of freedom per Planck volume.

No matter how large R is, the theory remains able to describe microscopic bulk physics beautifully.

To obtain the holographic description of AdS(5)⊗S(5), we must understand the symmetries clearly. That is why I began with a conceptual description.

Even more importantly, only a particular class of known systems possesses N=4 supersymmetry, namely SU(N) supersymmetric Yang-Mills theories (SYM).

The connection between gravity, or the string-theoretic generalization of AdS(5)⊗S(5), and the super Yang-Mills theory on the boundary has deep conceptual aspects.

In this paper I will discuss only certain specific features.

The correspondence states that superstring theory in the bulk of AdS(5)⊗S(5) is equivalent to N=4, 3+1-dimensional SU(N) SYM theory on the AdS boundary.

In this paper SYM always refers to the specific supersymmetric gauge theory. Here N denotes the amount of supersymmetry, and N denotes the dimension of the Young-Mills gauge group.

It is clear that SYM is conformally invariant and therefore does not have any additional dimensional parameters. The theory is defined on the boundary using dimensionless coordinates t, Ω or t, x. Even the momenta on the boundary are dimensionless. Therefore it is appropriate to say that all SYM quantities are dimensionless.

On the other hand, bulk gravity variables such as mass, length, temperature carry their usual dimensions.

To convert SYM variables to bulk variables, we can use a conversion factor R. Thus if E_SYM and M represent the energies of SYM and the bulk theory respectively, then:

$$E_{s\gamma m} = M \times R$$

Similarly, a bulk time interval equals the t-interval multiplied by R.

But one question may seem confusing: since AdS(5)⊗S(5) is a 10-dimensional spacetime, one might think that the boundary is (8+1)-dimensional.
However, it is actually (3+1)-dimensional, and the reason is precise.

To see this, let us consider a Weyl rescaling. The metric [equation (1)] is rescaled by a factor $R^2/(1-r^2)^2$ so that the rescaled metric becomes finite on the boundary.

The new metric becomes:

$$ds^2=\{(1+r^2)^2dt^2-4dr^2-4r^2d\Omega^2\}+\{(1-r^2)^2d\omega_5{}^2\} \tag{10}$$

Observe that the size of the 5-sphere shrinks to zero as r→1 at the boundary. Thus, the boundary geometry is actually (3+1)-dimensional.

Let us return to the connection between bulk and boundary theories.

The 10-dimensional bulk theory has two dimensionless parameters:

i. The curvature radius of AdS measured in string units R/l(s). Alternatively, R can be measured in 10-dimensional Planck units, giving the relationship:

$$g^2 l(s)^8 = l(p)^8$$

ii. g is a dimensionless coupling constant. The string coupling constant and the string length scale are mathematically related to the 10-dimensional Planck length and the Newton constant:

$$l(p)^8 = g^2\, l(s)^8 = G \tag{11}$$

The gauge theory has only two constants:

(a) The rank of the gauge group N.
(b) The gauge coupling $g_{\gamma m}$.

Of course, the two bulk parameters R and g can be determined by N and $g_{\gamma m}$:

$$R/l(s) = (Ng_{\gamma m}{}^2)^{1/4}$$
$$g = g_{\gamma m}{}^2 \tag{12}$$

Thus, we can write the 10-dimensional Newton constant as:

$$G = R^8/N^2 \tag{13}$$

There are two different limits which are quite interesting.

The AdS/CFT connection is useful for understanding the behavior of gauge theory in the 't Hooft limit, defined as:

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$$g_{\gamma m} \to 0$$

$$N \to \infty$$

$$g^2_{\gamma m} N = \text{constant} \tag{14}$$

In the bulk string perspective:

$$g \to 0$$

$$R/l(s) = \text{constant} \tag{15}$$

Thus, the 't Hooft limit corresponds to the classical string theory limit inside a fixed and large AdS space.

This limit is classical supergravity.

But this remarkable limit is completely different from that of the holographic principle. We consider a situation where the AdS radius increases but the parameters describing microscopic bulk physics remain fixed:

$$g = \text{constant}$$

$$R/l(s) \to \infty \tag{16}$$

From the gauge theory viewpoint:

$$g_{\gamma m} = \text{constant}$$

$$N \to \infty \tag{17}$$

Our goal is to show that the number of quantum degrees of freedom in the gauge theory satisfies the holographic behavior of equation (8)!

In the metrics of equation (1) and equation (2), the proper area of any finite coordinate patch diverges as it approaches the AdS boundary. Therefore, the number of degrees of freedom associated with that patch also diverges. This is consistent with continuum QFT such as SYM, where any finite 3-dimensional region contains infinitely many modes.

For a refined counting, we must regulate both the area of the AdS boundary and the UV degrees of freedom in SYM.

These two regulators are actually two aspects of the same physical idea.

We have already considered an L-boundary inside $r=1-\delta$, where $y=\delta$ (which acts as an IR regulator!).

According to the IR/UV relation, the IR regulator in the bulk corresponds to a UV regulator in SYM. In many cases, this is similar to string behavior (when studied in short time intervals). To understand this IR/UV relation in AdS, we must understand the connection between D-branes and AdS(5)⊗S(5).

D-branes are objects that appear in superstring theory. They are stable "impurities" of various dimensionalities within the vacuum.

A Dp-brane is a p-dimensional object.

We are particularly interested in the D3-brane.

These objects fill 3 spatial coordinates x^i and a time coordinate.

The remaining 6 coordinates we denote by z^n, where $z=\sqrt{(z^n \cdot z^n)}$.

We now place N D3-branes stacked at z=0.

A single D3-brane has some local degrees of freedom—for example, its location in the z coordinates may fluctuate.

Thus, the z-location can be regarded as a scalar field living on the D3-brane. Additionally, there are modes on the brane associated with directions t, x, which can be described by vector fields, along with fermionic modes required for supersymmetry.

The point is that the action for the fluctuations z(x,t) can be obtained from string theory calculations, producing an ordinary 3+1-dimensional scalar field theory.

The mass of a stack of N D3-branes is M, and as N increases the charge of the D3-brane also increases.

This mass and charge act as sources for bulk fields such as the gravitational field. What makes the D3-brane stack interesting is that it resembles the geometry of $AdS(5)\otimes S(5)$. Indeed, the geometry defined by equations (3) and (4) is closely related to the geometry emerging from the D3-brane stack.

In particular, the geometry arising from D3-branes is a specific solution of supergravity equations:

$$ds^2=F(z)(dt^2-dx^2)-F(z)^{-1}dz\cdot dz \tag{18}$$

where:

$$F(z)=[1+cg_sN/z^4]^{-1/2} \tag{19}$$

Here c is a numerical constant.

If we consider the limit in which $(cg_sN/z^4)>>1$, then we can replace F(z) by:

$$F(z)\approx z^2/(cg_sN)^{1/2} \tag{20}$$

Then it becomes easy to see that the D-brane metric is essentially the same as equations (3) and (4).

Moreover, the fluctuation theory of the D-brane stack is the N=4 SYM theory. All fields in this theory live within a single supermultiplet in the adjoint (N×N) representation of SU(N).

In this paper, I will attempt to provide an argument for the IR/UV connection based on the quantum fluctuations of the D3-brane position.

This is localized near z=0 in equation (3).

The location of a point on a D3-brane is defined by six coordinates z,ω_5.

We assume that the six coordinates are Cartesian $(z^1,\ldots z^6)$.

The actual or original coordinate z is defined by:

$$z^2=(z^1)^2+\ldots\ldots+(z^6)^2 \tag{21}$$

As indicated, the coordinates z^n can be represented by six scalar fields on the brane's worldvolume in the SYM theory.

Thus, if the six scalar fields Φ^i are canonically normalized, the relation between z and Φ is:

$$z=(g_{\gamma m}\, l_s^2/R^2)\Phi \tag{22}$$

Strictly speaking, equation (22) does not have a literal meaning because Φ fields in SU(N) are N×N matrices. The geometry therefore becomes non-commutative, and only configurations in which the six matrix-valued fields commute have a definite classical interpretation. Furthermore, the radial coordinate $z=\sqrt{(z^n \cdot z^n)}$ is defined as:

$$z^2=(g_{\gamma m}\, l_s^2/R^2)\times(1/N)\ \mathrm{Tr}\ \Phi^2 \tag{23}$$

A common question is: where is the D3-brane located inside AdS space?

The usual answer is the horizon, where z=0.

But the correct answer is subtler. The localization of information depends on the frequency range in which it exists.

At high frequency or short time scales, the string appears stretched. At low frequencies it is well localized.

To provide an analogous answer for the D3-brane, we must study the quantum fluctuations of its position.

Of course, Φ is a scalar quantum field whose scaling dimension is $(\text{length})^{-1}$. In this case, Φ satisfies for any of its N^2 components:

$$\langle \Phi^2_{kl} \rangle \sim \delta^{-2} \tag{24}$$

Where, δ is the ultra-violet regulator of field theory. According to equation (20), the average value of z will be:

$$\langle z \rangle^2 \sim (g_{\gamma m} l^2_s/R^2)^2 (N/\delta^2) \tag{25}$$

Or, using equation (12):

$$\langle z \rangle^2 \sim \delta^{-2} \tag{26}$$

In terms of the y coordinate, which vanishes at the AdS-boundary, we can write:

$$\langle y \rangle^2 \sim \delta^2 \tag{27}$$

We know that just below the boundary, the ultra-violet cut-off is located. And any low-frequency detector at z=0 will be able to see the branes, but as z increases, all the way up to z=∞, we will see higher-frequency branes. Thus it becomes clear that the SYM UV-cutoff of the boundary acts as the bulk IR-cutoff, which is evident from equation (23).

Now let us compute how many degrees of freedom are required to describe the region y>δ. The UV/IR connection implies that the theory can be described or analyzed with an ultra-violet regulator having δ cut-off. Let us consider a patch of the boundary that has unit coordinate area! Within that patch, there are $1/\delta^3$ cutoff cells whose size is δ. For each cell, the fields in the cut-off theory remain constant. This means that in each cell we obtain N^2 degrees of freedom, which are associated with the N⊗N components of the adjoint representation of U(N). Thus the number of degrees of freedom per unit area becomes:

$$N\_dof \approx N^2/\delta^3 \quad (28)$$

On the other hand, the regulated patch will have 8-dimensional area:

$$A=(R^3/\delta^3)\times R^5=R^8/\delta^3 \quad (29)$$

Thus, per unit area, the number of degrees of freedom will be:

Thus, per unit area, the number of degrees of freedom will be:

$$N\_dof/A \sim N^2/R^8 \quad (30)$$

Finally, using equation (13):

$$N\_dof/A \sim 1/G \quad (31)$$

This result is quite remarkable, because it is precisely what the holographic principle demands.

The high-frequency quantum fluctuations of the D3-brane's location can never be seen by a low frequency detector. In short, this is ensured by the renormalization group of the SYM-description acting on the brane. The renormalization group is what ensures that our body or any macroscopic structure cannot be easily destroyed or damaged by constant high-frequency vacuum fluctuations. However, we cannot be protected in the same way from classical fluctuations. A significant example is the thermal fluctuation of fields at high temperature. Any detector, whether low-energy or high-energy, can sense the thermal fluctuations of a brane's location. Let us now return to equation (24), but instead of using equation (25), we will use the thermal field fluctuations of Φ. For each of the N^2 components, the thermal fluctuation takes the form:

$$\langle \Phi^2 \rangle = T^2_{sym} \quad (32)$$

And equations (26) and (27) are replaced by:

$$\langle z \rangle^2 T^{-2}_{sym} \quad (33)$$

That means the coordinate-distance $z=T_{sym}$ strongly feels the thermal fluctuation. But if we write the expression for r, then it becomes:

$$(1-r) \sim 1/T_{sym} \quad (34)$$

Interestingly, this relation is equivalent to the horizon location of the AdS-black hole.

Now let us come to the main idea, which I have termed the "Sarkar-Approximation". From the AdS boundary at δ cut-off, I obtain the UV-regulator. At $r=1-\delta$, I am making an approximation where $\delta<<1$ but not zero! And at the AdS-boundary, $\delta=0$, hence $r=1$. I am also assuming that this δ cut-off is very close to the "Planck length"! Anyway, within this δ cut-off, a very large number of degrees of freedom operate. I can think of them as particles or modes, as I have explained earlier! Moreover, since δ is close to the Planck length, it acts as a "quantum mechanical" domain. The degrees of freedom present inside it satisfy Heisenberg's "position-momentum" uncertainty principle. Since $r=1-\delta$ is close to 1, we will represent the position uncertainty of a specific degree of freedom by Δr(Ads:boundary) and the momentum uncertainty by Δp(Ads:boundary). Without going into such complicated terminology, we will simply replace them by Δr(Ads:boundary)~r and Δp(Ads:boundary)~p. Therefore:

Δr(Ads:boundary)•Δp(Ads:boundary)~ħ

$$R \bullet p \sim \hbar \quad (35)$$

Earlier, from equation (34) we obtained:

$$(1-r) \sim 1/T_{s\gamma m}$$

$$r \sim 1-1/T_{s\gamma m} \quad (36)$$

$T_{s\gamma m}$ is the temperature of the "supersymmetric Yang-Mills" theory effective on the boundary. In some sense, the three quantities $p \approx M_{s\gamma m} \approx E_{s\gamma m}$ become equal! Another important point is that ħ=c=G=1; we are writing these three natural constants in standard-unit notation. Therefore equation (35) becomes:

$$r \bullet p \sim \hbar$$

$$(1-1/T_{s\gamma m}) \bullet E_{s\gamma m} \approx 1 \quad (37)$$

The entity $E_{\square\gamma\square}$ in equation (37) has a special value, it equals:

$$E_{s\gamma m} = cN^2T^4_{s\gamma m} \quad (38)$$

c is a constant coming from the supersymmetric Yang-Mills theory. It is not the speed of light! Substituting the value of $E_{s\gamma m}$ from equation (38) into equation (37), we obtain:

$$(1-1/T_{s\gamma m}) \bullet cN^2T^4_{s\gamma m} \approx 1$$

$$(T^4_{s\gamma m}-T^3_{s\gamma m}) \bullet cN^2 \approx 1 \quad (39)$$

Another point is that $(N^2/R^8) \sim N^2 = N\_dof$ represents the number of degrees of freedom per unit area.

Thus equation (39) becomes:

$$(T^4_{s\gamma m}-T^3_{s\gamma m}) \bullet cN^2 \approx 1$$

$$(T^4_{s\gamma m}-T^3_{s\gamma m}) \bullet cN\_dof \approx 1$$

$$(4T^3_{s\gamma m}-3T^2_{s\gamma m}) \bullet cN\_dof \bullet dT_{s\gamma m} + (T^4_{s\gamma m}-T^3_{s\gamma m}) \bullet c \bullet dN\_dof \approx 0$$

$$T^2_{s\gamma m}(4T_{s\gamma m}-3) \bullet N\_dof \bullet dT_{s\gamma m} = T^3_{s\gamma m}(1-T_{s\gamma m}) \bullet dN\_dof$$

$$(4T_{s\gamma m}-3) \bullet N\_dof \bullet dT_{s\gamma m} = T_{s\gamma m}(1-T_{s\gamma m}) \bullet dN\_dof$$

$$\int dN\_dof/N\_dof = \int[(4T_{s\gamma m}-3)/T_{\square\gamma\square}(1-T_{s\gamma m})] \bullet dT_{s\gamma m}$$

$$\ln(N\_dof) = \int[dT_{s\gamma m}/(1-T_{s\gamma m})] - 3\int dT_{s\gamma m}/T_{s\gamma m}$$

$$\ln(N\_dof) = \ln(1-T_{s\gamma m}) - 3\ln(T_{s\gamma m}) + i$$

$$\ln(N\_dof) = \ln[(1-T_{s\gamma m})/T^3_{s\gamma m}] + i$$

$$N\_dof = e^i \times [(1-T_{s\gamma m})/T^3_{s\gamma m}] \quad (40)$$

From the mathematical structure of equation (40), we can analyze a special limit. In the case of the AdS-boundary, if $T_{sym}\to+\infty$, then N_dof→0 begins to occur. But to keep N_dof unchanged, $e^i\to+\infty$ or $i\to+\infty$ must hold. On the other hand, if we consider the bulk, then if $T_{sym}\to 0$ occurs there, N_dof→+∞ begins. But to keep N_dof unchanged, we must have $e^i\to 0$ or $i\to-\infty$. Thus in both cases, N_dof remains fixed or unchanged. Now, according to the formulation of entropy:

$$S=|\ln(N\_dof)|$$

$$N\_dof=e^s$$

Now, according to my defined criteria, N_dof remains unchanged in the bulk space and also remains unchanged in the AdS-boundary space. Therefore, the entropy S is equal in both cases. That means:

$$e^s(Ads:Boundary)=N\_dof=Fixed$$

$$S(Ads:Boundary)\sim\text{"Fixed"} \quad (41)$$

On the other hand:

$$e^s(Bulk:Space)=N\_dof=Fixed$$

$$S(Bulk:space)\sim\text{"Fixed"} \quad (42)$$

II. CONCLUSION

Therefore, according to equations (41) and (42), they are mutually equal or equivalent. From this approach of mine, AdS/CFT can also be stated. Moreover, since the entropy of the lower-bound region equals the entropy of the upper-bound region, we can consider this as the “Holographic Limit”. Equation (40) is called the “Sarkar Equation of Holography”. And e^i is a dimensionless factor whose behavior or nature is determined according to specific limits.

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