



IMAGE: A MAP OF THE STARS OF THE ORION CONSTELLATION

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# JournalPreview

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## London Journal of Research in Science: Natural & Formal

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# 3 General Concerns and 12 Problems in Einstein's Paper and Book on Special Relativity

*Sean Yuxiang Wu & Lü Wu*

## ABSTRACT

The theory of relativity has been at the pinnacle of human scientific thought for 120 years. However, whether it should continue to lead is a question for us. Voices questioning the theory of relativity have been continuing, but since they are mostly from a mathematical or experimental point of view, they have not yielded convincing results. We have pioneered a new way of analyzing mathematical models from the perspective of reviewing the rationality of physical models, thus we have seen many problems with relativistic models from different perspectives, which we have grouped into three general concerns and 12 obvious or easy to prove problems. Through these intuitive discussions, we believe that the theory of relativity should no longer lead the scientific and technological thinking of mankind. This article focuses on special relativity. We will continue to discuss general relativity in future articles.

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# 3 General Concerns and 12 Problems in Einstein's Paper and Book on Special Relativity

Sean Yuxiang Wu<sup>a</sup> & Lü Wu<sup>a</sup>

## ABSTRACT

*The theory of relativity has been at the pinnacle of human scientific thought for 120 years. However, whether it should continue to lead is a question for us. Voices questioning the theory of relativity have been continuing, but since they are mostly from a mathematical or experimental point of view, they have not yielded convincing results. We have pioneered a new way of analyzing mathematical models from the perspective of reviewing the rationality of physical models, thus we have seen many problems with relativistic models from different perspectives, which we have grouped into three general concerns and 12 obvious or easy to prove problems. Through these intuitive discussions, we believe that the theory of relativity should no longer lead the scientific and technological thinking of mankind. This article focuses on special relativity. We will continue to discuss general relativity in future articles.*

**Keywords:** relativity, einstein, special theory of relativity.

## I. INTRODUCTION

The theory of relativity put Einstein on the altar of science. For 120 years, people also have been questioning the theory of relativity, but to no avail. The reason is that people always question it by mathematics or experiments, which is of no much effect.

Pioneering from the perspective of an application engineer, we saw that Einstein did not correctly and strictly define those physical models like design a precise engineering project, and thus could not use those models correctly according to his own intentions. Therefore, when he used those models, there were already full of loopholes in his applications.

This made us believe that the theory of relativity should no longer lead the scientific thinking of mankind.

This article is a critique of Einstein's special theory of relativity. Therefore, it first describes three key points that need to be concerned in the systems of special theory of relativity. Then, 12 problems of the special theory of relativity were analyzed.

### 1.1 Three General Concerns

In general, in Einstein's theory of relativity, he did not clearly point out the application restrictions between two reference bodies. This led to many general application problems. Three major concerns are listed below, because of their absence the relative system makes the relativistic application lack a scientific basis, especially when the three are used in combination.

#### First concern

Theory of relativity does not specify the distance between the two reference bodies. If a clock on the airplane can be relative to the ground clock, then the American flag pole on the moon should be able to be relative to a light beam on the Earth. If the distance between the two reference bodies is more than one light year, the length change or clock slow down would still happen?

#### Second concern

The theory of relativity does not specify the quality of a reference body. If a moving alloy rod has a diameter of 1000 meters, a ray flies back and forth over the moving rod, will the rod become shorter?

#### Third concern

The model of the light ray and the rigid rod does not explain how the two reference bodies are

bound to each other? How do they form a relative system?

Assuming there are 100 rigid rods, how could Einstein make his ray form a relative system with the No.4 rigid rod he wanted to be relative to? Is there any way to bind his ray and No.4 rod to each other so that they can be relative without disturbed by other rods?

For another example, in the experiment of a clock on the airplane and a ground navy clock, how does the airplane clock know it should be relative to the navy clock? Does the airplane clock also be relative to the clocks on space shuttles, on trains, or on cars.....?

This relative situation in which the objects join the relative system automatically without knowing by the experimenter's mind is called Passive Relative. Due to the unscientific system design of Einstein's physical model, the Passive Relative is unavoidable.

### B. 12 Problems

We listed 12 problems about the special theory of relativity from Einstein's paper [1] and book [3] below. It will follow the following format: In each problem, Einstein's quotation corresponding to the discussing problem is extracted from Einstein's paper [1] or book [3] into the problem. Then, "what is wrong" is discussed and analyzed through this problem, and the key concern in the problem is pointed out.

#### Problem 1

There are several problems associated with Einstein's Quotation-1 from Section VII of [3]: "Let us suppose our old friend the railway carriage to be travelling along the rails with a constant velocity  $v$ , and that a man traverses the length of the carriage in the direction of travel with a velocity  $w$ . How quickly or, in other words, with what velocity  $W$  does the man advance relative to the embankment during the process?"

<https://www.gutenberg.org/files/5001/5001-h/5001-h.htm#ch6>

Using the "man-carriage-embankment" system in above Quotation-1, together with following

Quotation-2, we prove that the synchronous transmission rule given by Einstein is an obvious error, and is not a "universally valid" rule.

From "§1. Definition of simultaneity" of [1], Einstein's Quotation-2 is such:

*"Suppose a ray of light leaves from A toward B at "A time"  $t_A$ , is reflected from B toward A at "B-time"  $t_B$ , and arrives back at A at "A-time"  $t'_A$ . The two clocks are synchronous by definition if*

$$t_B - t_A = t'_A - t_B.$$

*We assume that it is possible for this definition of synchronism to be free from contradictions, and to be so for arbitrarily many points, and that the following relations are therefore generally valid:*

1. *If the clock in B is synchronous with the clock in A, then the clock in A is synchronous.*
2. *If the clock in A is synchronous with the clock in B as well as with the clock in C, then the clocks in B and C are also synchronous relative to each other.*

<https://einsteinpapers.press.princeton.edu/vol2-trans/156>

To easily refer to, we label Einstein's above formula as (1) shown below, since formula (1) will be used from beginning to the end in this paper.

$$t_B - t_A = t'_A - t_B. \tag{1}$$

Now, we take the carriage as A, the man as B, and the embankment as C. Then, according to (1): A and B are synchronous, that is, the carriage and the man are synchronous.

A and C are also synchronous, that is, the carriage and the embankment are synchronous.

But B and C are not synchronous. That is, the man and the embankment are not synchronous. Because the speed of the man relative to the embankment is (the speed of the man + the speed of the carriage), the back-and-forth speed is not equal. The back and forth times are different, which violates (1).

This is to say item 2 in Quotation-2 is wrong.

### Problem 2

The following is an exactly same error as Problem 1. We take Footnote of Section VIII of [3] as Einstein's Quotation-3:

*"We suppose further that, when three events A, B and C take place in different places in such a manner that, if A is simultaneous with B, and B is simultaneous with C (simultaneous in the sense of the above definition), then the criterion for the simultaneity of the pair of events A, C is also satisfied. This assumption is a physical hypothesis about the law of propagation of light; it must certainly be fulfilled if we are to maintain the law of the constancy of the velocity of light in vacuo."*<https://www.gutenberg.org/files/5001/5001-h/5001-h.htm#ch8>

In Quotation-3, Einstein defined a *simultaneity* transmission rule. The meaning of two events simultaneously is that they satisfy (1). Using the same method and steps as the proof in *Problem 1*, we can prove that the simultaneity transmission rule defined by Einstein is also wrong.

In simple terms, let the moving carriage in Einstein's Quotation-1 be B, the moving man in the carriage be A, and the embankment be C. Then, A and B meet (1) so they are simultaneous, and B is simultaneous with C; but A is not simultaneous with C.

### Problem 3

The last sentence of Quotation-3 also leads to a serious problem. Einstein said that if this simultaneity transfer assumption is not fulfilled, then the law of the constancy of the velocity of light in vacuo is not held. Now according to our proof in above Problem-2, the simultaneity transfer assumption is really not fulfilled. How should we handle Einstein's law of the constancy of the velocity of light in vacuo?

### Problem 4

One of the problems in Quotation-1 is: Since the composite of the man-and-carriage moves relative to the embankment, then, let's assume that there are two points A and B on the embankment that are far apart. According to (1), we need to calculate the time required for the composite

speed of the man-and- carriage to move back and forth between A and B. Assume that the train is heading from A to B. Then, when the man moves from B to A in the opposite direction of the carriage,  $W = w - v$ . Since the man's speed  $w$  is much smaller than the running speed  $v$  of the carriage, so  $w - v < 0$ , means the man will only move further and further away from point A and will never reach point A. Therefore, the result of the calculation using (1) is  $t'_A - t_B = -\text{infinity}$ . Or we can say that the man-and-carriage complex only moves in the direction of the train's movement, and has no movement against the direction of the train's movement. This is a situation not handled within Einstein's theory. It can be seen from this that the physical model of man-and-carriage relative to the embankment given by Einstein in Quotation-1 is not a model of relativity, and should not be used to discuss relativity at all.

### Problem 5

Another problem in Quotation-1 is that the physical model in this passage is that a man is walking in a moving carriage, but the man is required to be a reference body relative to the embankment, while the embankment is another reference body. The problem is that the speed  $W$  of the man moving relative to the embankment is the combination of the man's speed  $w$  and the carriage's speed  $v$ . The man and the carriage are together formed a reference body in the relative system, and the embankment is another reference body. Then,  $W$  is the speed of the reference body composed of the man-and-carriage. The back-and-forth speeds of  $W$  are different (coming:  $W = w + v$ ; going:  $W = w - v$ ), which violates the regulations of the qualified reference body that must have uniform speed stipulated by Einstein. Therefore, the composite of the man-and- carriage cannot be used as a reference body in Einstein's theory of relativity. In other words, Einstein widely used this unqualified composite object as a reference body in his book [3] to discuss his theory of relativity.

Someone may ask: Einstein uses such a pattern in many places. For example, in "§2. On the relativity of lengths and times" of [1], Quotation-4 says:

Let a ray of light depart from A at the time  $t_A$ , let it be reflected at B at the time  $t_B$ , and reach A again at the time  $t'_A$ . Taking into consideration the principle of the constancy of the velocity of light we find that

$$t_B - t_A = \frac{r_{AB}}{c - v} \text{ and } t'_A - t_B = \frac{r_{AB}}{c + v}$$

where  $r_{AB}$  denotes the length of the moving rod—measured in the stationary system. Observers moving with the moving rod would thus find that the two clocks were not synchronous, while observers in the stationary system would declare the clocks to be synchronous.

<https://einsteinpapers.press.princeton.edu/vol2-trans/159>

The time calculation formula used is labeled as (2) as following for later reference:

$$t_B - t_A = \frac{r_{AB}}{c - v} \text{ and } t'_A - t_B = \frac{r_{AB}}{c + v} \quad (2)$$

Then, doesn't formula (2) also mean that the speed of light does not meet the requirement that the reference body must move at a uniform speed? Because the speeds go back and forth in (2) look like  $(c - v)$  and  $(c + v)$ , they are different, just like  $(w - v)$  and  $(w + v)$ . Many of such patterns discussed by Einstein, like lightning striking from both ends of the carriage, or the raven flying over the carriage, are all the same pattern. Are they all wrong?

The answer is: None of them are wrong. In all of the works of Einstein that we know of, only this system model of man-and-carriage, and embankment that Einstein used extensively in [3] is wrong!

Regarding this question, please see the detailed discussion about the speed of light below. In Problem 7 we will give an answer to it. This is a difficult question. Try first to see if you can answer it.

#### Problem 6

Einstein emphasized the constancy speed of light in [4]. But in his work, there are contradictory statements.

In the second paragraph at the beginning of [1], Einstein emphasized the principle of the constancy of the speed of light. In the following §1, he said four times that the speed of light in a vacuum is constant and has nothing to do with the motion state of the observer or the light source.

However, in [1] and [3], his writing repeatedly violated this principle set by himself.

Quotation-5 is from “§4. The physical meaning of the equations obtained concerning moving rigid bodies and moving clocks” of [1]:

*An analogous consideration—applied to the axes of Y and Z—it being borne in mind that light is always propagated along these axes, when viewed from the stationary system, with the velocity  $\sqrt{(c^2 - v^2)}$  gives us*

$$\partial\tau/\partial y = 0, \partial\tau/\partial z = 0.$$

<https://einsteinpapers.press.princeton.edu/vol2-trans/166>

Here Einstein calculated that “light is always propagated along these axes, when viewed from the stationary system, with the velocity  $\sqrt{(c^2 - v^2)}$  .....”

#### Problem 7

Quotation-6 is From Section VII of [3]:

*“w is the required velocity of light with respect to the carriage, and we have  $w = c - v$ . The velocity of propagation of a ray of light relative to the carriage thus comes out smaller than c.”*

The above description obviously violates "the constancy of the velocity of light in vacuo." At the end of §2 in [1], a similar problem with the speed of light also appears.

This problem is not a big deal and can be corrected by the following calculation. Since the speed of light is constant, we cannot say that  $w$  is the speed of light relative to the carriage  $w = c - v$ . The speed of light is constant and has nothing to do with the motion state of the observer or the light source. The correct statement should be follows:

The speed of light is completely independent and will not be affected by anything. Suppose the time required for light to travel the length  $L$  of the carriage in the stationary system is  $T$ . When light at the speed of light travels from point B through the length of the moving carriage  $L$ , and arrives the original position of point A, but point A has already moved forward a certain distance at the speed  $v$ . The light needs more time to catch up to A. So, the total used time is bigger than  $T$ . Similarly, the total time required for light to reach B from A is slightly less than  $T$ .

Thus, the back-and-forth times the light used to travel moving distance  $L$  in different directions are not the same, and do not satisfy (1).

In this way, we have proved that (1) does not hold in this model, as Einstein wanted to prove; and we have also correctly explained the problem that the speed of light in (2) has not changed. In our proof, the speed of light is always constant, and it is the movement of another reference body - the carriage, that makes (1) not hold. Moreover, the speed  $v$  of the reference body (here is carriage) is always uniform, and the carriage can be used as another reference body in the relative system. Therefore, this is a qualified relative system.

This also answers the question that the reader was asked to think about at the end of Problem-5 before reading this section.

In the relative systems composed of light, lightning, flying raven, etc., which Einstein often used, each of them is completely independent to another reference body. In Einstein's theory of relativity, each of them is not affected by another reference body in the system and exists independently with a uniform speed.

#### Problem 8

Einstein's physical models often fail to take into account the application conditions, leading to various errors. Here is an example.

In Section V of [3] Einstein gave us a new protagonist raven in Quotation-7 below:

*Let us imagine a raven flying through the air in such a manner that its motion, as observed from*

*the embankment, is uniform and in a straight line. If we were to observe the flying raven from the moving railway carriage, we would find that the motion of the raven would be one of different velocity and direction, but that it would still be uniform and in a straight line.*

<https://www.gutenberg.org/files/5001/5001-h/5001-h.htm#ch6>

But there are obvious problems with the physical mode. The raven is different from the light beam in the relative system because their speeds are very different.

When two moving reference bodies are independent of each other, the relative system composed of them cannot maintain synchronization. In addition, it has certain requirements for the reference bodies. The raven flying over the carriage is independent of the carriage. Because the raven's speed is smaller than the speed of the carriage, the raven can never catch up to the other end of the carriage, and there is no way of using it as a reference body to form a relative system. The mathematical model abstracted from this physical model is completely invalid since it cannot use formula (1).

#### Problem 9

Using Einstein's theory to wipe out any enemy.

First, Einstein said in following Quotation-8 from section XVIII of [3]:

*If we formulate the general laws of nature as they are obtained from experience, by making use of*

- (a) *the embankment as reference-body,*
- (b) *the railway carriage as reference-body,*

*then these general laws of nature (e.g. the laws of mechanics or the law of the propagation of light in vacuo) have exactly the same form in both cases.....*

*As long as it is moving uniformly, the occupant of the carriage is not sensible of its motion, and it is for this reason that he can unreluctantly interpret the facts of the case as indicating that the carriage is at rest, but the embankment in motion. Moreover, according to the special*

*principle of relativity, this interpretation is quite justified also from a physical point of view.*

<https://www.gutenberg.org/files/5001/5001-h/5001-h.htm#ch18>

Quotation-8 describes the relative meaning between two reference bodies in a relative system. Everything in Quotation-8 seems perfect. But if we replace the protagonists with “a light beam” and “an enemy,” who is staying in a position or moving at a uniform speed, and replace the carriage with a light beam, and replace the embankment with the enemy. Now let the light and the enemy form a relative system.

What will happen after the replacement?

Using the model of above Quotation-8, the light is not moving, instead the enemy is moving with the speed of light.

In the case of the matter (enemy) as  $m$  moving at the speed of the light, according to Einstein's theory  $E = mc^2$ , the enemy is converted into energy  $E$ . He is wiped out, and exists as energy.

Einstein said in Quotation-9: *"It is clear that the same results hold good for bodies at rest in the "stationary" system, viewed from a system in uniform motion.*

So, using a light beam to be one reference-body, the enemy be another reference-body; by applying Quotation-8, we can easily and remotely wipe out any enemy.

If we can't wipe out the enemy, it means somehow the theory is wrong.

#### Problem 10

Quotation-9 is from § 4. Physical Meaning of the Equations Obtained in Respect to Moving Rigid Bodies and Moving Clocks of [1]: *the X dimension appears shortened in the ratio  $1: \sqrt{1 - v^2/c^2}$ , i.e. the greater the value of  $v$ , the greater the shortening. For  $v = c$  all moving objects—viewed from the "stationary" system—shrivel up into plane figures...*

*It is clear that the same results hold good for bodies at rest in the "stationary" system, viewed from a system in uniform motion.*

*...the travel clock on its arrival at A will be  $1/2 t(v/c)^2$  second slow," and "a balance-clock at the equator must go more slowly..."*

<https://einsteinpapers.press.princeton.edu/vol2-tans/166>

In Quotation-9 Einstein discussed the physical meaning of the equations obtained for a moving rigid body and a moving clock.

In whole §4 of [1], there is no other physical or matter content except coordinate motion and transformation. That is a pure mathematics section. But playing the pure mathematic, using the motion of the reference bodies, Einstein concluded that the moving length be shortened in the ratio  $1: \sqrt{1 - v^2/c^2}$ , and *the travelled clock will be  $1/2 t (v/c)^2$  second slow," and "a balance-clock at the equator must go more slowly."*

If we take two beams of light with the same conditions but moving in completely opposite directions, and make them be relative to the same one rigid rod at the same time, what will be the result?

Can a moving diamond rod become shorter by relative motion?

We cannot prevent moving objects in the world passively relative to Einstein's moving rod. We also cannot prevent moving clocks in the airplanes, in the running trains..., be passively relative to Einstein's clock at any location.

We want to ask: Does the §4 of [1] mean that as long as the mathematics is beautiful, the application can be arbitrary? Will the material world be changed according to pure mathematical inference or calculation?

Lorentz transformation is a theory about electromagnetic fields. Can it be extended to rigid rods, carriages, and other matters at will?

#### Problem 11

The Quotation-10 in Section VII of [3] Einstein says

*"since the ray of light plays the part of the man walking along relatively to the carriage. The*

velocity  $W$  of the man relative to the embankment is here replaced by the velocity of light relative to the embankment." This sentence is incorrect. The "light relative to the embankment " and "the man relative to the embankment " are two completely different modes.

The light and the carriage are independent, so their speeds cannot be superimposed! The light relative to the embankment is also good to form a static relative system.

But for the man walking in a moving carriage, his speed and the carriage's speed must be superimposed. (1) is hold in the relative system they composed. But the motion of the man can't form a relative system with the embankment, which we discussed in Problem-4 and Problem-5. More importantly, it damaged Einstein's conclusion that "absolute simultaneity does not exist." We continue discussing this below.

#### Problem 12

Einstein emphasized the *relativity of simultaneity* and rejected absolute simultaneity in the theory of relativity. It seems that if there is absolute simultaneity, the whole relative system will crash. Quotation-11 below are the stories about definition of simultaneity.

- a) From [5]: *That is why the theory of relativity rejects the concept of absolute simultaneity, absolute speed, absolute acceleration, etc., they can have no unequivocal link with experiences.*
- b) From [1], after eighteen years, Two key words ["simultaneous," or] were added into this paragraph: *Thus with the help of certain imaginary physical experiments we have settled what is to be understood by synchronous stationary clocks located at different places, and have evidently obtained a definition of "simultaneous," or "synchronous" and of "time."*  
[https://www.physics.umd.edu/courses/Phys606/spring\\_2011/einstein\\_electrodynamics\\_of\\_moving\\_bodies.pdf](https://www.physics.umd.edu/courses/Phys606/spring_2011/einstein_electrodynamics_of_moving_bodies.pdf)
- c) From [2]: (This paragraph missing two key words "simultaneous," or) *With the help of some physical (thought) experiments, we have thus laid down what is to be understood by*

*synchronous clocks at rest that are situated at different places, and have obviously obtained thereby a definition of "synchronous" and of "time."*

<https://einsteinpapers.press.princeton.edu/vol2-trans/157>

Then, what is simultaneity? How can a system be judged as a relativistic system with absolute simultaneity?

The title of §1 in [2] is "§1 Definition of simultaneity". However, it is strange that in this about 1,000-word paragraph of §1, readers cannot find the definition of simultaneity. There is only one sentence related to the definition of "simultaneity" in §1 of the paper (Quotation-2), but still readers have no way to figure out the precise meaning of simultaneity.

Instead, there is a formula defined as synchronous. In §1, "simultaneous" appears 5 times and "synchronous" appears 7 times. The precise definition of synchronous was given by the formula which we referred to as (2) in Problem 1. But people still don't have a clear definition of "simultaneous."

If we don't have a precise definition of anything, how can we comment on this thing?

So, Einstein set a trap for readers in §1: the title is "§1 Definition of Simultaneity", but he did not give a clear definition of it, instead he gave us a precise mathematical definition for "synchronous". Generally speaking, "simultaneity" is not equal to "synchronous".

This has trapped many people, and it is certain that many so-called "masters of theory of relativity" do not truly understand the theory of relativity.

This trap also protected Einstein's theory of relativity. Because people could not accurately understand the key concepts of relativity, they had to follow the so-called masters who "understood" relativity to support relativity. 18 years later, relativity theory had established its unshakable position, few people would still be interested in what Einstein's simultaneity was.

After eighteen years, the two key words were quietly added into his paper "On the Electrodynamics of Moving Bodies" by Einstein. This modified paper was included in the book "The Principle of Relativity" [1] which Einstein personally arranged to reprint in 1923. He quietly inserted two key words into this English version of the paper 18 years later. Thus, the definition of simultaneity became Quotation-11 b), made it clear that "simultaneity" and "synchronous" in Einstein's paper are the same!

However, in other languages besides English [6-8], the articles still do not contain these two keywords. Einstein's secret revision of the key points of his paper 18 years later is not a decent behavior.

Now we know simultaneity = synchronous, and synchronous has a precise definition by (1). Then we can judge if a relative system is an absolute simultaneity system or not. The judging rule should be:

If a relativistic system always meets (1), then it is an absolute simultaneity relativistic system.

We believe that relative systems satisfying absolute simultaneity exist according to equation (1). An obvious example is the system composed of railway embankment, train carriage, and a man, an old friend in Quotation-1 of [3] discussed above. Continue Problem 11, taking out the man and the train carriage to build up a relativistic system; taking out the man and the embankment to build up another relativistic system. Both relativistic systems maintain absolute simultaneity. Because even if observed from the moon, the systems composed of the man and the train carriage always satisfies the equation (1) – the back-and-forth time will always be the same.

According to Einstein's definitions on absolute and relative synchronous or simultaneity of a relative system, we can find that there are a large number of relative systems that maintain absolute simultaneity in reality, such as the raven walking back and forth on a running train, stewardess walking on flying plane..., their movement always satisfies equation (1), and they

are all relative systems that maintain absolute simultaneity. And that's why a sprinter doesn't need to consider running in the same or different direction of the Earth's rotation.

The existence of the absolute simultaneity relative system damaged and negated Einstein's theory of relativity.

### III. RESULTS

Since Einstein did not attach importance to the physical model used to abstract out the mathematical model, from the perspective of the rationality of the physical model, the special theory of relativity has various theoretical defects; due to the lack of rigor in Einstein's theory and writing, there are many self-contradictory and unjustifiable statements in his paper and monograph as we listed.

### IV. DISCUSSION

Over the past century, under the influence of Einstein, the academic world has been filled with an atmosphere of mathematical supremacy, and Einstein's theory of relativity seems to be.

Starting from analyzing the rationality of the physical model and then discussing the mathematical model derived from it can ensure the rationality of the mathematical model, especially in analyzing the relativistic model that is closely integrated with the application. This also applies to analyzing the mathematical model of general relativity.

### V. CONCLUSIONS

Summarizing all problems above, we would like to ask: is the theory of special relativity worth to be the top scientific holy object to continue leading the scientific thinking for another 120 years and more? The answer is negative.

### REFERENCE

1. LORENTZ, A. & Einstein, A, et al, (1952), *THE PRINCIPLE OF RELATIVITY*, Translated by W. PERRETT and G. B. JEFFERY, DOVER

- PUBLICATIONS, INC. Standard book number: 486-60081-5, 1952.
2. Inside the book collected following renewed paper: “*On the Electrodynamics of Moving Bodies*,” Einstein, A. (1905), Page 35-65. But actually, it is not (1905) but (1923) version.
  3. Einstein, A (1905), The (1905) version is as follows: “*On the Electrodynamics of Moving Bodies*,” Einstein, A (1905). The collected papers of Albert Einstein, Volume 2: The Swiss years, Doc. 23, Annalen der Physik 17 (1905): 891-921, <https://einsteinpapers.press.princeton.edu/vol2-trans/157>
  4. Einstein, A (1916), *Relativity: The Special and General Theory*, republished by A Digireads.com Book, Digireads.com Publishing, Translated by Robert W. Lawson, ISBN10: 1-4200-4633-1, ISBN 13-978-1-4209-4633-8, 2012 <https://www.Gutenberg.org/files/5001/5001-h/5001-h.htm>
  5. Einstein, A (1919), “What Is the Theory of Relativity?” *The London Times*, November 28, 1919. Source of English translation: Albert Einstein, *Einstein, Ideas and Opinions*, translated by Sonja Bargmann. New York: Crown, 1952, pp. 100-05. Source of original German text: Albert Einstein, *Mein Weltbild*. Amsterdam: Querido Verlag, 1934, pp. 220-28.
  6. Einstein, A., (1920), *Letters to Solovine, 1906-1955*, Page 21, Philosophical Library, ISBN: 978-1-4532-0488-7, 1934.
  7. Einstein, A., (1905), *Zur Elektrodynamik bewegter Körper (On the Electrodynamics of Moving Bodies)*, German language version, there is no the two critical words in this German version. <https://einsteinpapers.press.princeton.edu/vol2-doc/315>
  8. Einstein, A., (2018), *On the Electrodynamics of Moving Bodies*, Chinese language version, 爱因斯坦文集第二卷, Page 96, ISBN 978-7-100-07166-6, 商务印书馆, 2018. There is no the two critical words in this Chinese version.
  9. <http://archive.keyllo.com/F-%E5%8F%91%E7%8E%Bo/%E8%AE%BA%E5%8A%A8%E4%BD%93%E7%9A%84%E7%94%B5%E5%8A%A8%E5%8A%9B%E5%AD%A6%EF%BC%88%E7%88%B1%E5%9B%A0%E6%96%AF%E5%9D%A6%EF%BC%89.pdf>
  10. Einstein, A., (2005), *On the Electrodynamics of Moving Bodies*, Spanish language version, there are no two critical words in this Spanish version. <http://webs.ftmc.uam.es/juancarlos.cuevas/Teaching/articulo-original.pdf>
  11. Wu, Sean Yuxiang (2024), *Five Errors that Have Never Been Discussed Over the Theory of Special Relativity*, *Applied Physics Research*; Vol. 16, No. 1. <https://ccsenet.org/journal/index.php/apr/article/view/0/49839>

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# Influence of LEDs on Light Compensation Point of *L. Solanum Tuberosum* Aeroponic Plantlets

*Martha E. Dueñas Jaco, Martha A. Rodríguez Mendiola, Carlos Arias Castro, Juan José Villalobos & Laura Isabel Arias Rodríguez*

## ABSTRACT

Aeroponic systems have demonstrated their efficacy for seed potato production in greenhouses using sunlight or artificial light, achieving yields comparable or superior to traditional methods. However, the optimal light requirements remain understudied, particularly with LED lights in aeroponic farming. This study aimed to determine the light compensation point (Ic) required for potato growth under LED lighting.

The light compensation point is the irradiance where photosynthesis equals respiration, varying by species and environment. Selecting an appropriate light intensity is crucial for crop productivity and cost-efficiency. Limited research exists on potato cultivation under LED lighting, especially concerning plant photosynthetic rates. This study evaluated the effects of LEDs on potato plants, focusing on CO<sub>2</sub> exchange and biomass production

Results indicated that blue light was the most efficient for CO<sub>2</sub> absorption, with an Ic of ~50 μmol m<sup>-2</sup> s<sup>-1</sup>, while red light required 67 μmol m<sup>-2</sup> s<sup>-1</sup>. White and mixed lights exhibited higher Ic values (84.6 and 108.5 μmol m<sup>-2</sup> s<sup>-1</sup>, respectively). Plants exposed to blue light developed robust structures but had lower biomass compared to those under red light treatments, underscoring the need for further optimization of LED spectra in aeroponic farming systems.

**Keywords:** aeroponic, potatoes, LED, light compensation point, net photosynthesis.

**Classification:** LCC Code: 0706, 0703, 0607

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# Influence of LEDs on Light Compensation Point of *L. Solanum Tuberosum* Aeroponic Plantlets

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Juan José Villalobos<sup>co</sup> & Laura Isabel Arias Rodríguez<sup>δ</sup>

## ABSTRACT

*Aeroponic systems have demonstrated their efficacy for seed potato production in greenhouses using sunlight or artificial light, achieving yields comparable or superior to traditional methods. However, the optimal light requirements remain understudied, particularly with LED lights in aeroponic farming. This study aimed to determine the light compensation point (Ic) required for potato growth under LED lighting.*

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## I. INTRODUCTION

Artificial light source is a critical component in indoor farming since light quality and intensity are one of the most important environmental factors affecting plant growth and morphology (Rabara et al., 2017), but also to prevail economically within the limits of plant growth and cost reduction (Domurath et al., 2012). The light compensation point is the irradiance at which the photosynthetic activity of a plant is equal to its respiratory activity by CO<sub>2</sub> exchange (Nobel, 2009) and it depends from species and growing environments (Taiz & Zeiger, 2002). So, it provides a basis for the selection of light intensity suitable for growing of a certain crop. Moreover, when we graph the Net Photosynthesis vs irradiance the slope of the line at the light compensation point indicates quantum yield ( $\phi$ ), mol of CO<sub>2</sub> fixed by mol of photons irradiated (Long & Hällgren, 1993). It is also necessary to take into account that 9 mol photons are necessary to fix a mol of CO<sub>2</sub> (Osborne & Geider, 1987). So, these measurements help to understand better the irradiance effect on the crops.

As an artificial illumination source, Light-emitting Diodes (LEDs) are the most popular for vertical farming or urban agriculture since they can emit a specific wavelength or a combination of them, they have a variety of designs, low heat emission, energy efficiency as well as their long lifetime (Janda et al., 2015; Rabara et al., 2017).

Potato is a highly productive crop and tubers constitute an excellent source of carbohydrate as well as a good source of proteins. Field farming is associated with several risks and uncertainties in vital and environmental stresses, such as high winds, floods, droughts, and pest attacks (Tunio et al., 2020), and also the poor efficiency in the use of natural resources such as soil producing degradation on it and water waste (Lakhiar et al., 2018). Aeroponic culture is soil less method that offer an innovative solution to ensure the environmental and economic sustainability of food supplies with high nutritional quality, non toxic food and labor (Lakhiar et al., 2018).

Potato (*Solanum tuberosum L*) is, due to its nutritional and energy value, a basic and necessary food in the diet of Mexicans and its cultivation has great economic and social importance for many mexican families (NOM-041-FITO-2002, 2003). One of the main challenges for the producers of this tuber is the quality of the plant propagation material, since the seed degenerates due to pathogens and diseases in the planting material continued in the cycles of vegetative propagation, affecting yields and quality of the crop. The *in vitro* production of potato seed is very convenient, since the seedlings are manipulated more easily to eradicate any pathogen present in the tissue (Tapia y Figueroa et al., n.d.). The pre-basic seed is obtained from *in vitro* plants that originate from the culture of meristems and these can be used directly for the production of minitubers (*ex vitro*) or for the formation of microtubers (*in vitro*) (NOM-041-FITO-2002, 2003; Tapia y Figueroa et al., n.d.).

As potato is traditionally grown under open-field conditions, fewer studies refer to cultivation in controlled environment illuminated with LED related to photosynthetic rates. No information seems to be available in scientific literature on the effect of monochromatic or dichromatic LED on *in vivo* potato plants and, more in general, on the influence of the different light spectrums on the process of tuberization in potato (Paradiso et al., 2019) or the light compensation point.

To have a profound base of the minimum light necessary to grow this culture, tests were carried out on photosynthetic rates on aeroponic potato culture under industry-relevant light intensities and various light qualities, measuring CO<sub>2</sub> leaf gas exchange. With these curves, light compensation points (LCP) were determined under the different experimental conditions to propose light intensity suitable for growing of *Solanum Tuberosum L.* aeroponic plantlets.

## II. MATERIALS AND METHODS

### 2.1 Plant material, growth chamber, environmental conditions and experimental design

Potato (*Solanum tuberosum L.*) variety Fianna plants *in vitro* were donated by Plant Biotechnology Laboratory of the Instituto Tecnológico de Tlajomulco and transplanted *in vivo* in an aeroponic chamber divided into four zones. Each zone was isolated from any external light source and was illuminated for 10 days with white LED light by Philips HUE multicolor smart bulbs. After transplantation, 10 plants were allocated to each of the four light treatments and kept them for the 8-week experimental period, Figure 1. The environmental conditions inside the growth chamber were set to 12/12 h photoperiod. The energy provided by the LEDs in the chamber was fixed at 10.65 J m<sup>-2</sup> s<sup>-1</sup>, which means an irradiance of 61, 42, 51 and 48 μmol m<sup>-2</sup> s<sup>-1</sup> for the colors red, blue, 50% blue:50% red (mix) and white (33% blue, 32% green, 8% yellow, 15% orange and 12% red) respectively, measured with the limited by the technology of the HUE lamps for the blue light. Planck-Einstein's equation (6) were used to convert from PPF to energy.

The Planck-Einstein equation relates the energy of a photon to its wavelength. It is expressed as:

$$E = h\left(\frac{c}{\lambda}\right) \quad (6)$$

Where:

- E is the energy of the photon (in joules).
- h is Planck's constant, approximately  $6.626 \times 10^{-34} \text{ J m}^2 \text{ s}^{-1}$
- c is the speed of light in a vacuum, approximately  $3.00 \times 10^8 \text{ m s}^{-1}$
- $\lambda$  is the wavelength of the photon (in meters).

Relative humidity  $70 \pm 10\%$ , and temperature of  $24 \pm 1^\circ\text{C}$ . Water spray irrigation was one min every two hours. The nutrient solution for the plants was made with (meq/L) 10.5 of N, 11.6 of P, 9 of K, 6.1 of Ca, 1.1 of Mg, and 0.5 of S.

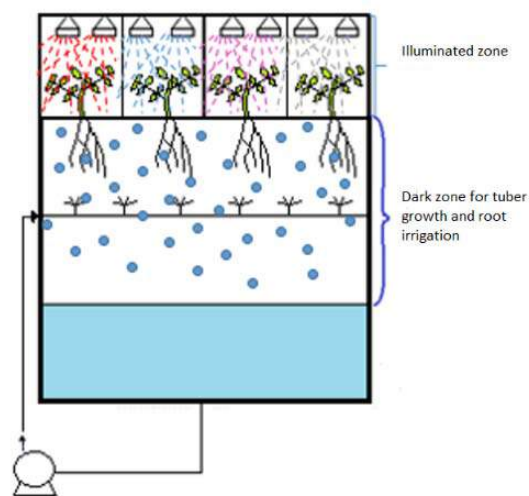


Figure 1: Aeroponic chamber scheme for potatoes growth.

The experiment was performed twice for a randomized design of growth conditions.

## 2.2 Net photosynthetic rate determination

To determine the net photosynthesis,  $P_n$ , as the  $\text{CO}_2$  measurements were made at 100 minutes of illumination, the  $P_{n_{100}}$ , not the total  $P_n$  for the plant. The equation proposed by Long & Hallgreen (1993) for closed systems was used:

$$P_n = \frac{C_1 - C_2}{(t_1 - t_2)} * \frac{V}{22.4} * \frac{P}{101.325} * \frac{273.15}{T * S} \quad (7)$$

Where:

- $P_n$  = Net photosynthetic rate ( $\mu\text{mol m}^{-2} \text{ s}^{-1}$ )
- C = molar fraction  $\text{CO}_2$  inside the cabin ( $\mu\text{mol mol}^{-1}$ )
- V = cabin's volume (L)
- P = atmospheric pressure
- S = leaf surface ( $\text{m}^2$ )
- t = time (h)

The Light Compensation Point, LCP, is obtained from the graph Pn vs irradiance for each treatment, more specifically, it is the point cross with the x axis. The slope in this point is called apparent quantum yield,  $\phi$ , and it indicates, the amount of CO<sub>2</sub> fixed by mol of photons incident not absorbed (Long & Hällgren, 1993).

### 2.3 Phenological monitoring

At the end of the treatment, each plant was measured in order to determine shoot length, shoot diameter, number leaves, and root length. Plant tissue samples were dried in a drying oven for 48 h at 90°C before weighing.

### 2.4 Statistical Analyses

Statistical analyses were carried out using InfoStat Professional v.1.1 program. All the parameters were subjected to analysis of variance (ANOVA) and t-test. Differences were accepted as statistically significant when  $P < 0.05$ . Tuckey's test was carried out to identify significance among the samples.

## III. RESULTS AND DISCUSSION

### 3.1 CO<sub>2</sub> Absorption at Different Light Intensities

In Figure 5, the CO<sub>2</sub> absorption by the plants in the cabinet is presented. The first 100 minutes correspond to measurements in the dark. The dotted lines indicate the boundary between dark and illuminated conditions. Overall, the results show that the higher the illumination, the higher the CO<sub>2</sub> absorption.

Under 10 PPFD, all treatments maintained a slope similar to that of mitochondrial respiration. We can see that even when photosynthesis has started, this light intensity is not enough for disrupt the balance between the CO<sub>2</sub> absorbed via the photosynthesis and the CO<sub>2</sub> produced by mitochondrial respiration. When exposed to 50 PPFD, only plants under blue light reached the equilibrium point (CO<sub>2</sub> absorbed = CO<sub>2</sub> produced). Plants treated with white and mixed lights reached this equilibrium at approximately 100 PPFD. Meanwhile, plants under red light required 100 PPFD to generate a positive CO<sub>2</sub> balance. Blue light proved the most efficient in facilitating CO<sub>2</sub> absorption, while red light was the least efficient.

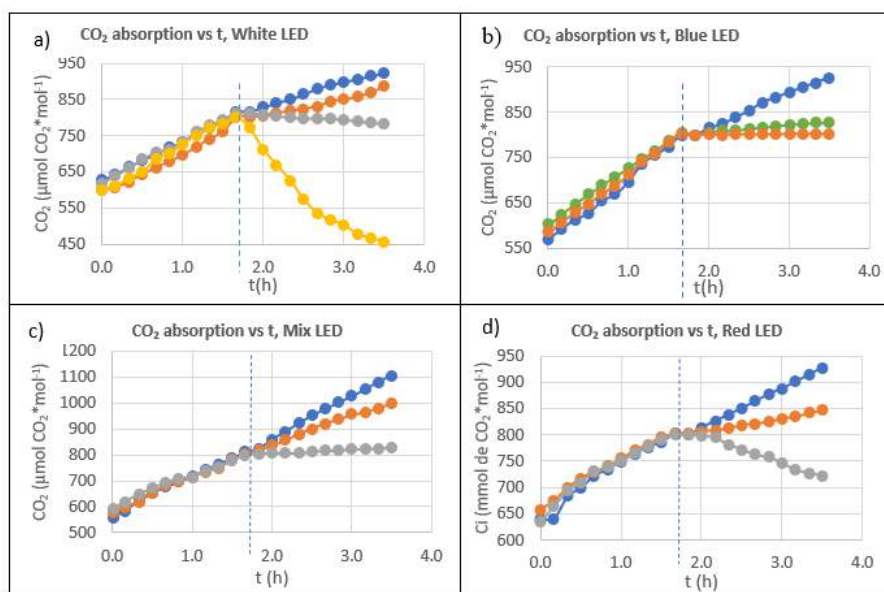


Figure 5: CO<sub>2</sub> absorption at different light intensity. CO<sub>2</sub> absorption vs time measured under a) white LED, b) blue LED, c) mix LED, d) red LED. Blue line shows the CO<sub>2</sub> absorption by the plant to 10 PPFF,

green line to 30 PPFF to range line is the answer of the plant to illumination of 50 PPFF, gray line is the answer to 100 PPFF and yellow line to 200 PPFF. The vertical dotted lines indicate the beginning of the illumination.

### 3.2 Light Compensation Point and Quantum Yield

Figure 6 illustrates the relationship between net photosynthesis (Pn<sub>100</sub>) and light intensity for each treatment. Table 4 summarizes the light compensation point (LCP) and quantum yield ( $\phi$ ) calculated from these data. Blue light treatments achieved the lowest LCP ( $\sim 50 \mu\text{mol m}^2 \text{s}^{-1}$ ), significantly lower than white ( $84.6 \mu\text{mol m}^2 \text{s}^{-1}$ ), mixed ( $108.5 \mu\text{mol m}^2 \text{s}^{-1}$ ), and red light ( $67 \mu\text{mol m}^2 \text{s}^{-1}$ ) treatments. This suggests that blue light promotes higher photosynthetic efficiency under low light conditions, consistent with findings that highlight the role of blue spectra in enhancing CO<sub>2</sub> uptake and stomatal conductance (Li et al., 2017; Paradiso et al., 2019). The quantum yield was highest for white light ( $\phi = 0.1543$ ), followed by mixed, red, and blue lights. However, plants under blue light operated near their LCP, which limited the calculation of a reliable quantum yield equation.

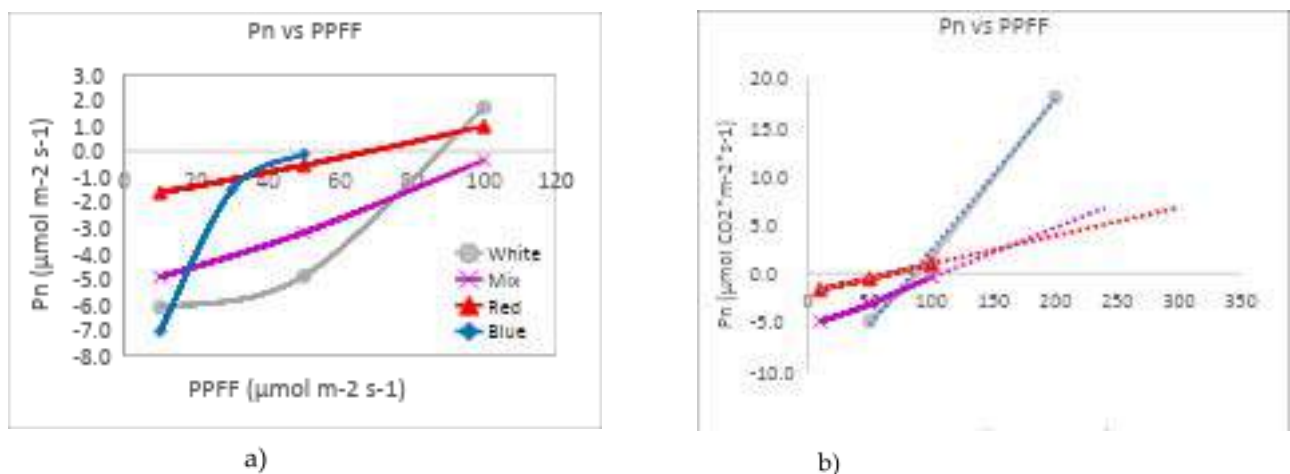


Figure 6: a) Pn<sub>100</sub> vs irradiance until 100 μmol m<sup>-2</sup> s<sup>-1</sup>, b) Pn<sub>100</sub> vs irradiance until 200 μmol m<sup>-2</sup> s<sup>-1</sup>, showing the extrapolation lines and their equations at the LCP.

Light compensation point is mainly related with respiration and light source for the treatments (Azcón & Osmond, 1983). Due the characteristics of this experiment, where dark respiration was established at 800 ppm, there is not other differences between the experiments and calculated LCP was affected only by light source. Plants were grown in a light level very close to the compensation point for blue and red light. This helped these plant to grow and pass through the vegetative stage. It is necessary to increase the PPFD of light upper the compensation point with these treatments in order to understand the light effect on biomass and tuberisation process.

The light compensation point (LCP), (Table 4, Figure 5) was achieved at the lowest photon flux density with blue light ( $50 \mu\text{mol m}^2 \text{s}^{-1}$ ), making it the most efficient for photosynthesis. Red, mix, and white light required higher fluxes ( $67$ ,  $84.6$ , and  $108.5 \mu\text{mol m}^2 \text{s}^{-1}$ , respectively). The high efficiency of blue light correlates with its ability to increase stomatal conductance and net photosynthesis (Li et al., 2017). The quantum yield ( $\Phi$ ) of white light, however, was the highest among treatments, indicating its potential for efficient energy use once plants grow above the LCP. These results support further testing of white light under optimized conditions.

*Table 4:* Equations of the graphs Pn vs PPFF on light compensation

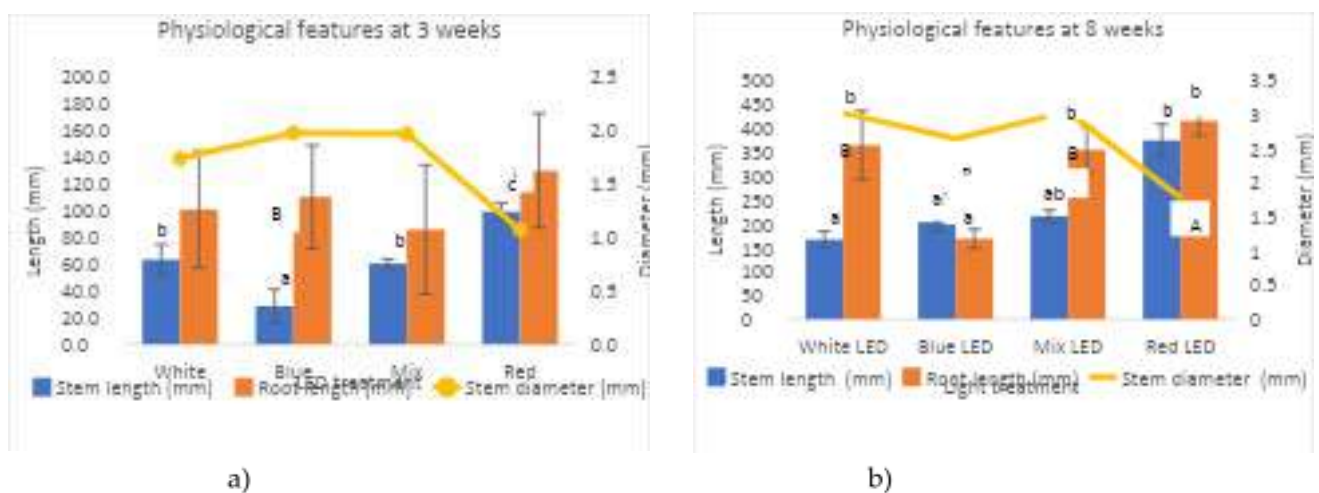
LED light treatment	Eq. Pn vs PPFF	LCP ( $\mu\text{mol m}^{-2} \text{s}^{-1}$ )	$\Phi$ ( $\mu\text{mol CO}_2 / \mu\text{mol photon}$ )
White	$y = 0.1543x - 13.045$ $R^2 = 0.997$	84.6	0.1543
Blue	----	~50	---
Mix	$y = 0.051x - 5.54$ $R^2 = 0.9948$	108.5	0.051
Red	$y = 0.029x - 1.94$ $R^2 = 0.9986$	67	0.029

LCP, Light Compensation Point;  $\phi$ , Quantum yield

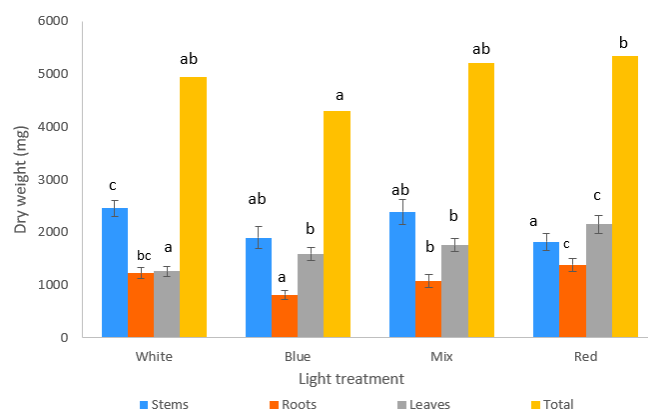
### 3.3 Dry Biomass

The results in Figure 7 show that at three weeks, all plants developed longer roots than stems. At this stage, plants grown under red light displayed the longest roots, whereas those under mixed light had the shortest roots. In some studies made with *in vitro* plantlets, finding with monochromatic red and far-red were stem elongation and leaf thinning, in those seedlings producing fragile plants (Chen et al, 2020; Miyashita et al, 1995). These researcher found that blue light caused leaf expansion and increased leaf thickness (Chen et al, 2020) as well as negative effect on the growth of potato seedlings, but the plants were more robust and had broader leaves (Miyashita et al, 1995).. In our experiments same tendencies of growth were observed in plants at three weeks of trasplanting, but at the end of eight weeks, these differences disappeared as observed in Figure 7b, main taining only the thickness in stem and fragility for plants under red light treatment, which also shown the thinniest leaves, but to compensate, plants increased the amount of leaves biomass, translating this in bigger number of leaves per plant.

Blue light produced the shortest roots at both time points, and they were significantly smaller compared to other treatments.



*Figure 7:* Physiological growth at week 4 (a) and eight weeks (b) after transplanting. Left axes is referred to length in mm for stems and root. Right axes is referred steam diameter. Data are presented as means  $\pm$  standard error (n = 4). Different letters indicate significant differences between values (p < 0.05).



**Figure 8:** Dry weight of three plants after 8 weeks of light treatment at  $10.65 \text{ J m}^{-2} \text{ s}^{-1}$ . Data are presented as means  $\pm$  standard error ( $n = 4$ ). Different letters indicate significant differences between values ( $p < 0.05$ ).

#### IV. CONCLUSIONS

*Solanum tuberosum L.* plants derived from *in vitro* culture successfully grew in our aeroponic chamber illuminated with LED lights in blue, red, mix (50:50 red and blue), and white spectrums over an eight-week period. The findings highlight significant differences in the physiological responses of plants to varying light qualities and intensities. Blue light was the most efficient in achieving  $\text{CO}_2$  equilibrium at the lowest light compensation point ( $\sim 50 \mu\text{mol m}^{-2} \text{ s}^{-1}$ ), promoting robust plant structures. Red light, although less efficient, produced the highest foliar biomass. The mix and white light treatments required higher irradiance levels to reach the compensation point, yet they balanced structural development and biomass production.

From these results, white and mixed light spectrums have been identified as promising candidates for further studies, particularly under conditions above their respective light compensation points. However, this study did not progress to the tuberization stage, a critical phase to validate the suitability of these light treatments for mini-tuber production. Future experiments should focus on evaluating these light spectrums under a range of intensities above the compensation point to determine the optimum light conditions for maximizing tuber yield and quality. Based on these findings, we recommend using white light for future experiments due to its high  $\Phi$  and mix light for its balance between energy use and biomass output. Further research should explore the impact of light intensities above the LCP to optimize growth and tuberization.

#### REFERENCES

1. Azcón-Bieto, J., Osmond, C.B. (1983). Relationship between Photosynthesis and Respiration: The Effect of Carbohydrate Status on the Rate of  $\text{CO}_2$  Production by Respiration in Darkened and Illuminated Wheat Leaves. *Plant Physiol.* 71, 574–581. <https://doi.org/10.1104/pp.71.3.574>.
2. Chen, L., Zhang, K., Gong, X., Wang, H., Gao, Y., Wang, X., Zeng, Z., & Hu, Y. (2020). Effects of different LEDs light spectrum on the growth, leaf anatomy, and chloroplast ultrastructure of potato plantlets *in vitro* and minituber production after transplanting in the greenhouse. *Journal of Integrative Agriculture*, 19(1), 108–119. [https://doi.org/10.1016/S2095-3119\(19\)62633-X](https://doi.org/10.1016/S2095-3119(19)62633-X).
3. Domurath, N., Schroeder, F.-G., & Glatzel, S. (2012). LIGHT RESPONSE CURVES OF SELECTED PLANTS UNDER DIFFERENT LIGHT CONDITIONS. *Acta Horticulturae*, 956, 291–298. <https://doi.org/10.17660/ActaHortic.2012.956.33>.

4. Janda, M., Navrátil, O., Haisel, D., Jindřichová, B., Fousek, J., Burketová, L., Čerovská, N., & Moravec, T. (2015). Growth and stress response in *Arabidopsis thaliana*, *Nicotiana benthamiana*, *Glycine max*, *Solanum tuberosum* and *Brassica napus* cultivated under polychromatic LEDs. *Plant Methods*, 11(1), 31. <https://doi.org/10.1186/s13007-015-0076-4>.
5. Lakhari, I. A., Gao, J., Syed, T. N., Chandio, F. A., & Buttar, N. A. (2018). Modern plant cultivation technologies in agriculture under controlled environment: A review on aeroponics. *Journal of Plant Interactions*, 13(1), 338–352. <https://doi.org/10.1080/17429145.2018.1472308>
6. Li, C.-X., Chang, S.-X., Khalil-Ur-Rehman, M., Xu, Z.-G., & Tao, J.-M. (2017). Effect of irradiating the leaf abaxial surface with supplemental light-emitting diode lights on grape photosynthesis: Supplemental light and grape photosynthesis. *Australian Journal of Grape and Wine Research*, 23(1), 58–65. <https://doi.org/10.1111/ajgw.12267>
7. Long, S. P., & Hällgren, J.-E. (1993a). Measurement of CO<sub>2</sub> assimilation by plants in the field and the laboratory. In D. O. Hall, J. M. O. Scurlock, H. R. Bolhàr-Nordenkampf, R. C. Leegood, & S. P. Long (Eds.), *Photosynthesis and Production in a Changing Environment* (pp. 129–167). Springer Netherlands. [https://doi.org/10.1007/978-94-011-1566-7\\_9](https://doi.org/10.1007/978-94-011-1566-7_9)
8. Miyashita, Y., Kitaya, Y., Kozai, T., & Kimura, T. (1995). EFFECTS OF RED AND FAR-RED LIGHT ON THE GROWTH AND MORPHOLOGY OF POTATO PLANTLETS IN VITRO: USING LIGHT EMITTING DIODE AS A LIGHT SOURCE FOR MICROPROPAGATION. *Acta Horticulturae*, 393, 189–194. <https://doi.org/10.17660/ActaHortic.1995.393.22>
9. Morrow, R. C., & Tibbitts, T. W. (1988). Evidence for Involvement of Phytochrome in Tumor Development on Plants. *Plant Physiology*, 88(4), 1110–1114. <https://doi.org/10.1104/pp.88.4.1110>
10. Nobel, P. S. (2009). Leaves and Fluxes. In *Physicochemical and Environmental Plant Physiology* (pp. 364–437). Elsevier. <https://doi.org/10.1016/B978-0-12-374143-1.00008-9>.
11. NORMA Oficial Mexicana NOM-041-FITO-2002, Requisitos y especificaciones fitosanitarios para la producción de material propagativo asexual de papa. (2003). Diario Oficial de la Federación. [http://dof.gob.mx/nota\\_detalle.php?codigo=698254&fecha=04/03/2003](http://dof.gob.mx/nota_detalle.php?codigo=698254&fecha=04/03/2003)
12. Paradiso, R., Arena, C., Roupheal, Y., d'Aquino, L., Makris, K., Vitaglione, P., & De Pascale, S. (2019). Growth, photosynthetic activity and tuber quality of two potato cultivars in controlled environment as affected by light source. *Plant Biosystems - An International Journal Dealing with All Aspects of Plant Biology*, 153(5), 725–735. <https://doi.org/10.1080/11263504.2018.1549603>
13. Rabara, R. C., Behrman, G., Timbol, T., & Rushton, P. J. (2017). Effect of Spectral Quality of Monochromatic LED Lights on the Growth of Artichoke Seedlings. *Frontiers in Plant Science*, 8. <https://doi.org/10.3389/fpls.2017.00190>
14. Taiz, L., & Zeiger, E. (2002). *Plant physiology* (3rd ed). Sinauer Associates.
15. Tapia y Figueroa, M. de L., Lorenzo, J. C., Escalona, M., & Mosqueda, O. (n.d.). Obtención de microtubérculos y minitubérculos como semilla pre-básica en tres cultivares peruanos de papa. 2017, 17 (3), 153–159.
16. Tunio, M. H., Gao, J., Shaikh, S. A., Lakhari, I. A., Qureshi, W. A., Solangi, K. A., & Chandio, F. A. (2020). Potato production in aeroponics: An emerging food growing system in sustainable agriculture for food security. *Chilean Journal of Agricultural Research*, 80(1), 118–132. <https://doi.org/10.4067/S0718-58392020000100118>
17. Xu, Y. (2019). Nature and Source of Light for Plant Factory. In *Plant Factory Using Artificial Light* (pp. 47–69). Elsevier. <https://doi.org/10.1016/B978-0-12-813973-8.00002-6>.



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# A New Method of Solving Chess Problems based on a Purely Mathematical Solution

*Mustapha Bakani*

## ABSTRACT

The practice of the game of chess leads to the development of skills related to memory, logic, concentration, rigor, strategy and the capacity for abstraction. In addition to the benefits observed on learning citizenship, by respecting the rules and others. Solving chess problems is an interesting variation for realizing intellectual development. The common way is to present problems on the chessboard or through diagrams. Here, we present a new method of solving chess problems based on a purely mathematical solution. Concretely, it is a question of solving a chess problem thanks to the solution of equations and the mathematical analysis. Thus with a basic knowledge of mathematics, generally of the secondary level, we can proceed to the resolution with a minimum of knowledge of chess, given that the resolution is done from the algebraic notation of the said problem. Here we advance definitions, properties and theorems. Also we present here an example of a chess problem solved by the method.

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# A New Method of Solving Chess Problems based on a Purely Mathematical Solution

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## ABSTRACT

*The practice of the game of chess leads to the development of skills related to memory, logic, concentration, rigor, strategy and the capacity for abstraction. In addition to the benefits observed on learning citizenship, by respecting the rules and others. Solving chess problems is an interesting variation for realizing intellectual development. The common way is to present problems on the chessboard or through diagrams. Here, we present a new method of solving chess problems based on a purely mathematical solution. Concretely, it is a question of solving a chess problem thanks to the solution of equations and the mathematical analysis. Thus with a basic knowledge of mathematics, generally of the secondary level, we can proceed to the resolution with a minimum of knowledge of chess, given that the resolution is done from the algebraic notation of the said problem. Here we advance definitions, properties and theorems. Also we present here an example of a chess problem solved by the method.*

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## I. INTRODUCTION

The resolution method that we have developed essentially concerns the didactics of mathematics. This is a new multidisciplinary practical field for teachers and students to test their abilities in mathematical problem solving, manipulation of data displayed on tables programmed by these same students in a simplistic way (Excel for example) and of course the patience and rigor specific to both disciplines. However, we believe that it would be wise to expose - briefly – some other aspects related to algorithms within the framework of game theory. The problem becomes difficult in the case of an  $n \times n$  board.

*For advocates of Chess in Education (CIE), chess offers a powerful tool to build a conceptual understanding of math in children. While the Internet is awash in clever programs that gamify the teaching of early math, chess provides an immediate, direct, and tactile offline tool for teachers. [1]*

The game of chess belongs to a set of games sharing common properties, called combinatorial games, of which here are the two main properties: "There is no random intervention. No dice are rolled and no cards are drawn. "These are full-information games. To choose his move, each player has all the information about the game to make his decision. This excludes for example the game of naval battle, where the board of the adversary is hidden". These two properties make it possible to eliminate any luck factor, so that in the presence of two players playing perfectly, the result of the game is known in advance. In education, the use of traditional games and especially chess is recommended. Indeed, on the cognitive level, the game chess "promotes the learning of logic and the development of the spirit of analysis and synthesis, or memory", essential skills for the student to solve a problem. [2]

Problems and modeling: A chess problem, also called a chess composition, is a puzzle set by the composer using chess pieces on a chessboard, which presents the solver with a particular task. For

instance, a position may be given with the instruction that White is to move first, and checkmate Black in  $n$  moves against any possible defence. A chess problem fundamentally differs from chess play in that the latter involves a struggle between Black and White, whereas the former involves a competition between the composer and the solver. Most positions which occur in a chess problem are 'unrealistic' in the sense that they are very unlikely to occur in chess play. There is a good deal of specialized jargon used in connection with chess problems. There are various different types of chess problems: Direct mates, Help mates, Self mates, Reflex mates, Series mates, Studies, Retrograde analysis problems, Shortest proof games, Construction tasks. There are other types of chess problem which do not fall into any of the above categories. Some of these are really coded mathematical problems, expressed using the geometry and pieces of the chessboard. Famous such problems are the Knight's tour and The Queen puzzle.

**Knight's tour problem:** The object of the puzzle is to find a sequence of moves that allow the knight to visit every square on the board exactly once. It is a direct mathematical problem, related to the Hamiltonian path problem in graph theory. It appeared for the first time in arabic manuscripts in the 9th century and was very popular among mathematicians from the 18th century due to all possible different solutions. Euler presented a very famous solution in the Berlin Academy of Science in 1759 based on the premise "divide and conquer." [3]

**8 queens problem:** The problem initially posed by K.F. Gauss in 1842, proposed by Max Bezzel in Germany in 1848 is as follows: is it possible to place 8 queens<sup>i</sup> on a chessboard without any queen threatening another? Gauss himself found 72 solutions to this problem, but there are 92, if you ignore the natural symmetries of the chessboard. The correct solution was found in 1972 with the help of computers and backtracking; 92 solutions were found in total where 12 of them are linear independent (Ramirez, 2004). It is rather the generalization to an  $n \times n$  chessboard that poses a problem when  $n$  is very large. [4]

These problems are usually solved by a classic technique called backtracking in computer science. Backtracking is a resolution process that is particularly suitable for this type of problem. We can describe the course of a game by a graph whose vertices are the different states of the game and the arcs represent the transitions from one state to another. In the special case where it is impossible to return to a past state in the game timeline, the graph has no cycles and is therefore a tree. The maximum number of alternatives to pass from one state to another of the game fixes the arity of the tree and the beginning of the game is logically the root of the tree. Such a tree is called a decision tree. Backtracking designates a way of traversing such a tree and this traversal is often implicit. [5]

**Chess solving software:** Many programs have also sprung up to verify the correctness of a chess problem. This type of program is very specific, because contrary to a game program, it must analyze all the possible moves, since a problem which would have other solutions than those wanted by the author would be demolished.

*What is meant by "solving the game of chess"? Every chess player has one day faced with a problem of the type "White plays and checkmates in  $n$  moves". For such a problem, regardless of Black's responses, White manages to checkmate in  $n$  moves or less. The problem is correctly analyzed once all Black answers are taken into account. As the human player progresses, he can study positions of more and more complex, but the length of analyzes required means that the problems rarely exceed 5 or 6 moves. The computer then comes to support the human player in its analysis, and for endgame positions, it can then be determined whether, assuming a perfect game, one of the two players is in a position to win, or if the game will end in a draw.[6]*

Chess problems take many different forms. The most common form is given by the specification of a position on the chessboard, the specification of the state of play and a statement of the solution condition. Note that this is an extremely general form and can cause many different kinds of problems.

In this article we wish to present a new method of solving chess problems different from current methods at the theoretical level and complementary to them on the level of general interest.

At the theoretical level, it seems that the subject "solving chess problems in a purely mathematical way: using the solution of equations and mathematical analysis" is not well present in the literature proper to these kinds of topics. In contrast, solving chess problems in a computer context is based on tree algorithms, which are good for the machine. Our method is based on new mathematical functions defining the movements of the pieces and their properties. Thus, we have avoided the theory of graphs present in the other methods, which makes it possible to solve compound chess problems by hand, taking into account the theoretical development carried out, thanks to the resolution of equations and the analysis through a dashboard that can be programmed using simple tools.

This way makes it possible to exploit the properties of chess in the learning of mathematics. A path that might be interesting to explore.

*Perspective:* Theoretical development reveals hidden links with physics. Indeed, within the complexity of chess lies a network of algorithms, models and structures that can be mathematically explored.

The interaction between moves, positions and strategies in chess can be interpreted as a dynamical system, similar to the behavior of particles in physical systems. This connection has opened new avenues for understanding chess problems and designing new methods for solving them. [7]

Here we present the theoretical development specific to our method of resolution.

## II. A MATHEMATICAL CHESS PROBLEM

Note: we denote: K=King, Q=Queen, R=Rook, N=Knight, B=Bishop, P=Pawn.

Without a chess board or diagram available, solve the following chess problem: "Find mate in two moves from the following position: (White: Ke1; Rh1; Ng3; Nf5; e2; h3) and (Black: Kg2; f3).

This problem is simple, and established composers or solvers may find it easy to solve (blindly). But for a beginner who finds it difficult to solve a problem laid out on a chess board or in the form of a diagram, mentally analyzing the initial position, it is not easy to think of all the eventualities from the initial position, provided in algebraic notation. However, a mathematical formalization of some rules of the chess game allows the transfer from the chessboard (3D) or diagram (2D) to a digital scoreboard, supporting mathematical analysis. So, with a modest knowledge of chess, we can try to solve chess problems using basic math (usually secondary school).

To do this, we introduce functions for moving chess pieces and controlling chessboard squares. Programming in Excel (or other) allows us to perform repetitive calculations and draw up control tables. Thanks to a mathematical analysis including the resolution of algebraic equations, one manages to solve certain chess problems composed for this purpose, using the usual mathematical tools; such as classical logic or solving equations and systems of equations, for example.

### III. DEFINITIONS AND NOTATIONS

#### 3.1 Geometric representation of the position

We project the chessboard on a suitable finite plane  $(\wp)$  provided with an orthonormal coordinate system  $(O, \vec{i}, \vec{j})$  of unit one and of origin  $O(o; o)$ . We thus define any square  $M$  of the chessboard by its strictly positive integer Cartesian coordinates  $(x_M; y_M)$  in the Plane  $(\wp)(O, \vec{i}, \vec{j})$

The pieces (or figures) are represented by capital letters. These same letters can also represent the squares on which the corresponding pieces are located, if there is no cause for any confusion. Each letter actually represents a type of piece (unique to Orthodox chess) that acts as test functions when it comes to checking the value of a given move, for example.

We will denote  $A$  a white piece and  $A'$  a black piece of the same nature. We denote by  $A$  a square occupied by a piece  $A$ .

#### a) Algebraic writing of the position of the diagram

The following example illustrates a position given in algebraic notation and how to relate it to a mathematical plane.

Whites: Kf8; R1e3; Bd5; Ne6; R2c7; g6. Blacks: Kd8; Ne4; Bf5; Rg7; c6.

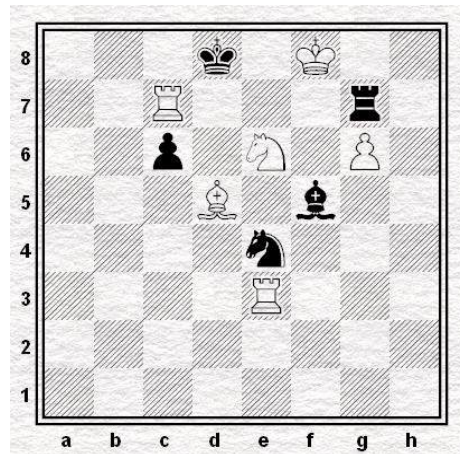


Fig. 1: Position diagram

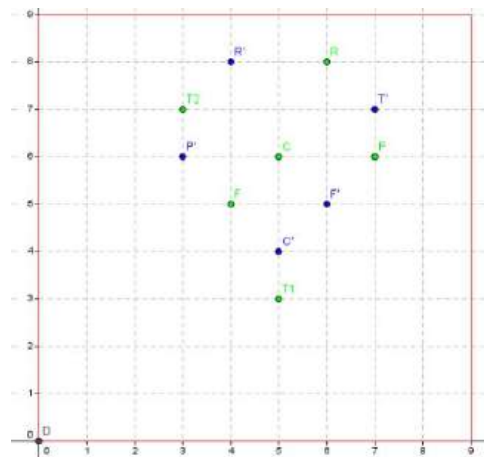


Fig. 2: Representation in Plan  $(\wp)$

b) Writing in Cartesian coordinates

Consider that the letters from a to h correspond respectively to integers from 1 to 8.

a	b	c	d	e	f	g	h
1	2	3	4	5	6	7	8

Fig. 3: From letters to numbers

We replace the letters corresponding to the columns by the numbers to have the x-axis. The y-axis are the line numbers.

We thus obtain, for each piece and square, a representation in Cartesian coordinates.

WHITE :  $K(6_8); R_1(5_3); B(4_5); N(5_6); R_2(3_7); P(7_6)$

BLACK :  $K'(4_8); N'(5_4); B'(6_5); N'(7_7); P'(3_6)$ .

Note: The transfer of the pieces to our plane is done from the algebraic notation of the position. The diagram is therefore not necessary to deal with the problem!

3.2 Lines and circles

For some, the movement of the pieces was posed in an arbitrary way by the "inventor" of Chess. Perhaps this proposal is not entirely sound!

In our benchmark, the pieces seem to follow a certain geometric logic.

Explanations:

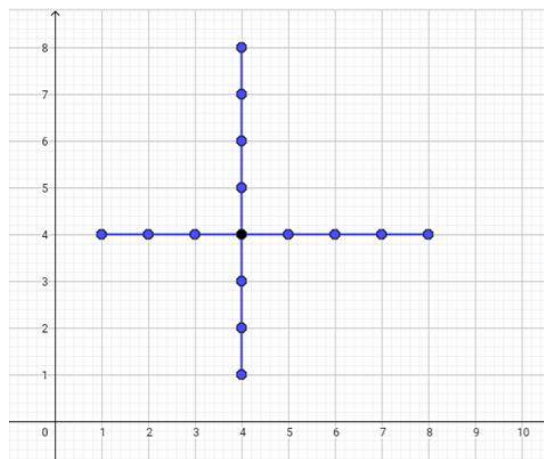
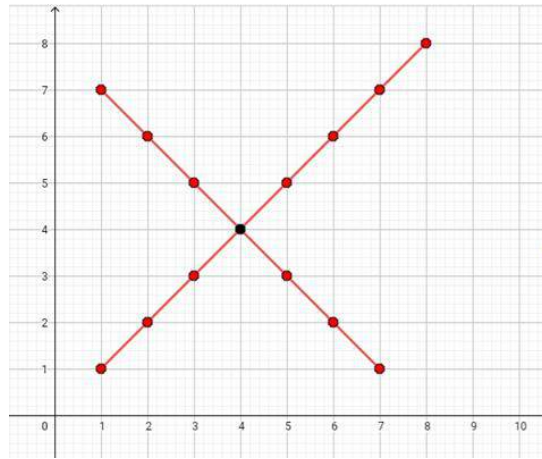
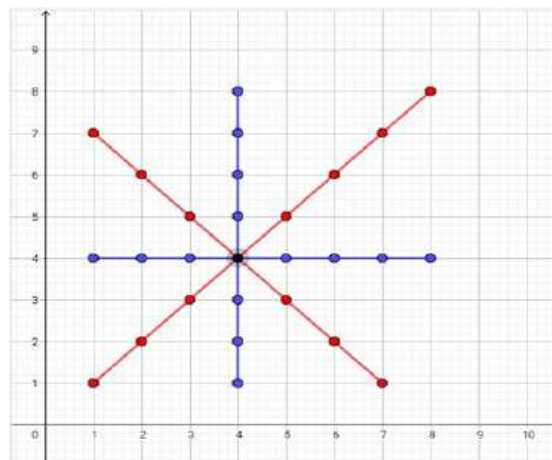


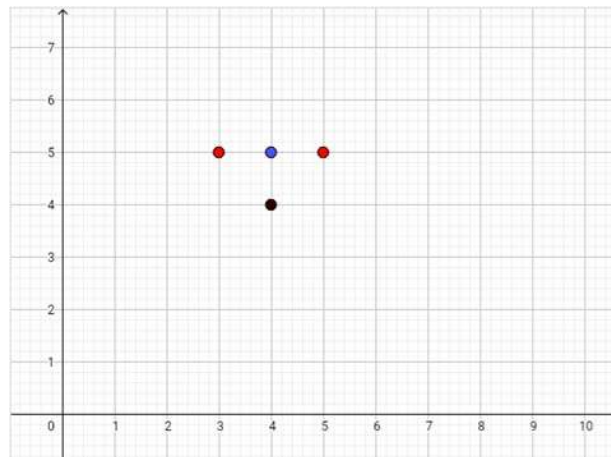
Fig. 4: The Rook moves on the horizontal and vertical squares



*Fig. 5:* The Bishop moves on the diagonal squares.



*Fig. 6:* The Queen moves on the horizontal, vertical and diagonal squares. It possesses both the qualities of the Rook and those of the Bishop.



*Fig. 7:* The pawn plays on the adjacent vertical square and takes on the 2 adjacent diagonal squares above for a white pawn and below for a black pawn.

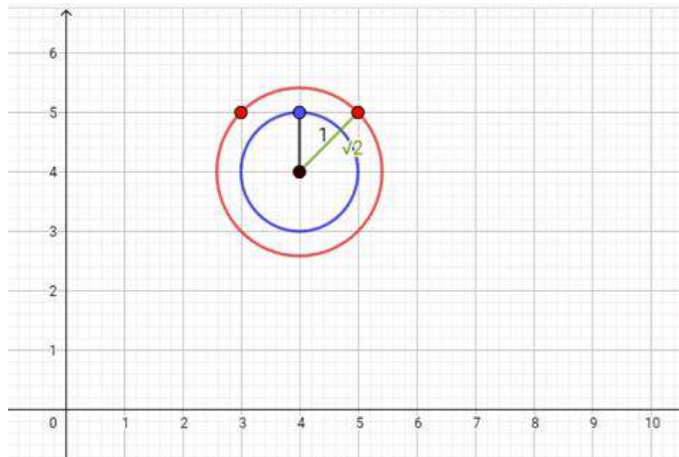


Fig. 8: The arrival points of the pawn belong respectively to the circles of origin the starting point and of radii 1 and  $\sqrt{2}$ .

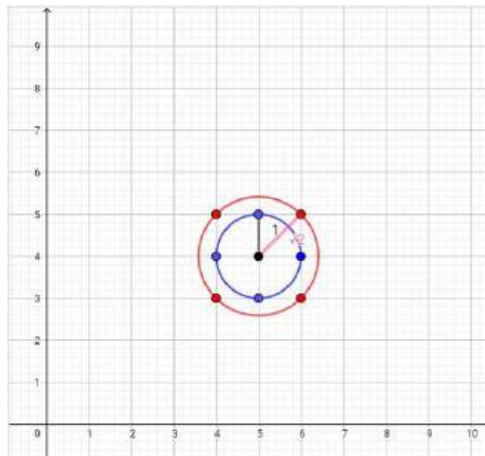


Fig. 9: The King moves on adjacent horizontal, vertical and diagonal squares. The arrival points of the King belong respectively to the circles of origin the starting point and of radii 1 and  $\sqrt{2}$ .

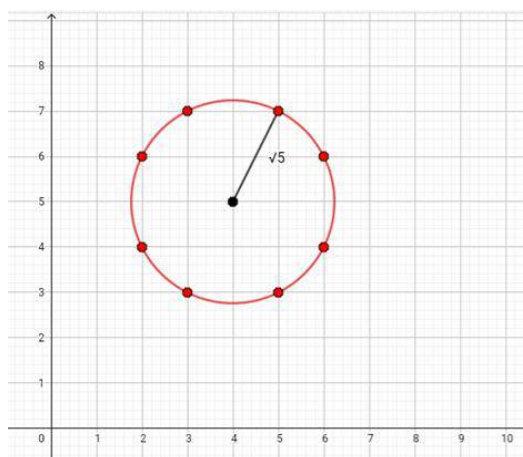


Fig. 10: The end points of the Knight belong to the circle of origin the starting point and of radius  $\sqrt{5}$ .

Here we distinguish two families: that of straight lines (Queen, Bishop and Rook) and that of circles (King, Knight and Pawn). In this order the chess pieces seem to represent a good part of the reality of the old wars.

### 3.3 Squares and Neighborhoods

Let us denote by  $\xi$  the set of usual pieces of orthodox chess<sup>ii</sup>

$$\xi = \{K; P; N; B; R; Q\} \cup \{K'; P'; N'; B'; R'; Q'\}$$

Let  $P = \{K; P; N; B; R; Q\}$  and  $P' = \{K'; P'; N'; B'; R'; Q'\}$

With:

K = King; P = Pawn; K = Knight; B = Bishop; R = Rook and Q = Queen, white pieces and K', N', B', R' and Q', black pieces.

Denote by  $\Omega$  the set of squares on a chessboard. L and M  $\in \Omega$ ,

We denote by  $\forall(M)$  the set of adjacent squares called here neighboring to square M.  $\forall(M) = \{N; NW; W; SW; S; SE; E; NE\} \subset \Omega$ .

$\forall(M)$  is said Immediate neighborhood of M.

NW	N	NE
W	M	E
SW	S	SE

Fig. 11: Neighborhood of M

### 3.4 Pieces and Control Functions

#### a. Movement Control

Definition: a piece A controls an M square if A can reach M after the next move.

For each type of piece of  $\xi$  and any square M of  $\Omega$ , we define P, N, B, R, Q and K as applications of  $\Omega$  in  $\mathbb{Z}$ , indicative of the possibility or not of the existence of a given piece on a given square after play of the next move.

These notations can also represent two-variable functions of  $\mathbb{Z}^2$  in  $\mathbb{Z}$  defined by :

- Knight :  $N(M) = f_c(x; y) = x^2 + y^2 - 5$
- Bishop :  $B(M) = f_b(x; y) = x^2 - y^2$
- Rook :  $R(M) = f_r(x; y) = xy$
- Queen :  $Q(M) = f_d(x; y) = xy(x^2 - y^2)$
- King :  $K(M) = f_k(x; y) = \prod (x^2 + y^2 - m)^{2m=1}$
- Pawn :  $P(M) = f_p(x; y) = (y - 4) \prod (x^2 + y^2 - m)^{2m=1}$

Such as  $m = 1$  (Vertical / horizontal movement) or  $m = 2$  (diagonal movement), And  $y > 0$  (if white pawn) or  $y < 0$  (if black pawn) ;

With, for each square M of  $\Omega$  and each piece A of  $\xi$  :

$$x = x_M - x_A \text{ and } y = y_M - y_A \text{ Where } M(x_M; y_M) \text{ and } A(x_A; y_A).$$

*b. Neighborhood control*

The restrictions of the control applications on  $V(M)$  allow to test the controllability of the M square and its immediate neighborhood (its adjacent L squares) by a piece of  $\xi$ . They are

9 defined by:

- Knight :  $N(L) = (X + \alpha_L)^2 + (Y + \beta_L)^2 - 5$
- Bishop :  $B(L) = (X + \alpha_L)^2 - (Y + \beta_L)^2$
- Rook :  $R(L) = (X + \alpha_L)(Y + \beta_L)$
- Queen :  $Q(L) = (X + \alpha_L)(Y + \beta_L) [(X + \alpha_L)^2 - (Y + \beta_L)^2]$
- King :  $K(L) = \prod ((X + \alpha_L)^2 + (Y + \beta_L)^2 - m) \quad m=1$
- Pawn :  $P(L) = (X + \alpha_L)^2 + (Y + \beta_L)^2 - 2$ , such as :

$Y + \beta_L > 0$  (if white pawn) or  $Y + \beta_L < 0$  (if black pawn).

With, for each square M of  $\Omega$  and each piece A of  $\xi$  :

- $X = x_M - x_A$  et  $Y = y_M - y_A$  where  $M(x_M; y_M)$  and  $A(x_A; y_A)$ ;

$\alpha_L$  et  $\beta_L$  parameters of the set  $\{0; \pm 1\}$  linked to the neighboring cells of M and are defined in the following table :

L	M	N	NW	W	SW	S	SE	E	NE
$\alpha_L$	0	0	-1	-1	-1	0	1	1	1
$\beta_L$	0	1	1	0	-1	-1	-1	0	1

Fig. 12: Neighboring squares of M parameters

#### IV. PROPERTIES

Suppose the trait is white.

Either  $A \in P$  and generally  $A_i (i = 1, 2, 3, \dots)$  white pieces of  $\xi$ .

$A' \in P', A'_i (i \in \mathbb{N})$  black pieces of  $\xi$ .

In what follows we will deal with the concepts within the strict framework of orthodox chess and we will assume that:  $A(x_A; y_A) \in \xi : (x_A; y_A) \in \mathbb{N}^2 / 1 \leq x_A \leq 8$  et  $1 \leq y_A \leq 8$ .

##### 1. Check the King

In a given position of a chess problem, the Black King  $K'$  is put on failure by one (or more) piece A ( $A_i, i \in \mathbb{N}$ ) if and only if  $A(K') = 0$  ( $A_i(K') = 0$ ).

##### 2. Control of the royal neighborhood

In a given position of a chess problem, a square L adjacent to that of the Black King is controlled by one (or more) piece A ( $A_i, i \in \mathbb{N}$ ) if and only if  $A(L) = 0$  ( $A_i(L) = 0$ ).

##### 3. Self-locking

###### a. Definition

Let  $A' \in P'$  and  $L \in V(K')$ . Let's ask :  $\mathcal{B}_{A'}(L) = (x_L - x_{A'})^2 + (y_L - y_{A'})^2$  Where  $A'(x_{A'}; y_{A'})$  and  $L(x_L; y_L)$ .

In a given position of a chess problem, Black King  $K'$  is blocked by  $A'$  if and only if it exists  $L \in V(K')$  such that  $\mathcal{B}_{A'}(L) = 0$ .

*b. Lemma*

In a given position of a chess problem, Black King  $K'$  is blocked by  $A'$  if and only if  $K'(A') = 0$ .

*4. Small castling*

For castling to be possible, it is necessary and sufficient that all of the following conditions be verified:

- a) The King and the Rook must be on their original primitive (game) spaces. We must therefore necessarily have  $K(5; 1)$  and  $R(8; 1)$  for whites and  $K'(5; 8)$  and  $R'(8; 8)$  when it comes to castling blacks.
- b) The spaces  $B(6; 1)$  and  $N(7; 1)$  for white castling and  $B'(6; 8)$  and  $N'(7; 8)$  in the case of black castling must not be controlled or blocked.
- c) The King is not in check and must not have moved before.
- d) The execution of castling produces the following condition:  $K(7; 1)$  and  $R(6; 1)$  for whites and  $K'(7; 8)$  and  $R'(6; 8)$  for blacks.

*5. Large castling*

For Large castling to be possible, it is necessary and sufficient that all of the following conditions be verified:

- a. The King and the Rook must be on their original primitive (game) squares. We must therefore necessarily have  $K(5; 1)$  and  $R(1; 1)$  for whites and  $K'(5; 8)$  and  $R'(1; 8)$  when it comes to castling blacks.
- b. The spaces  $Q(4; 1)$ ,  $B(3; 1)$  and  $N(2; 1)$  for the white castling and  $Q'(4; 8)$ ,  $B'(3; 8)$ ,  $N'(2; 8)$  in the case of the black castling, must not be controlled or blocked.
- c. The King is not in a state of check and must not have moved before. d. Large castling produces the following condition:  $K(3; 1)$  and  $R(4; 1)$  for whites and  $K'(3; 8)$  and  $R'(4; 8)$  for blacks.

### III. INITIALIZATION AND CONTROL TABLES

The results needed to deal with a problem are gathered in tables (non-exhaustive list) that help us determine the squares checked and the pieces involved, carry out tests and build a checkmate plan.

These tables expose the following data:

Table (A): Initialization / pieces coordinates

Table (B): The movement indicators applied to the cells of the opposing Kings Table (C): Control of the Black King's neighborhood by white pieces

Table (D): Possible blockages of the Black King

Table (E): Coordinates of neighboring cells and satisfaction of the rules Note: We use the symbol  $\emptyset$  to denote a correct result at the level of the application but which contradicts the rules of the game. For example, a pawn cannot retreat; a Rook cannot ensure its own defense against the opposing King on the ground one of its neighboring squares... etc.

## VI. APPLICATION (PROBLEM)

Solve the following chess problem, using its algebraic notation only:

Position A: White: Ke1 ; Rh1 ; Ng3 ; Nf5 ; e2 ; h3 and Blacks: Kg2 ; f3. White mate in 2 moves.

### 4.1 Reformulation

Initialization

- a) Redefine the pieces as moving objects, located by their Cartesian coordinates, and the squares as points in a suitable discrete plane;
- b) Draw up the control tables using a calculation program (Excel for example); c) Check whether one or the other of the Kings is in a state of failure; d) Define a resolution plan;

To analyze

- e) Study the possible movements of blacks;
- f) Deduce a first move (the most satisfactory) for White;
- g) Play\* the White move (the key) and mathematically define the new position;
- h) After analyzing the possibilities of Black, play the black move and define the new position;
- i) Find the checkmate from this position.

\* play = in the control table, replace the old coordinates of the square on which the piece concerned is located with the new ones.

### 4.2 Solution

Note: To avoid repetitive manual work in the sense of formulas, you can use a simplistic program such as Excel to perform the calculations, knowing that the tables are reusable for each new position (new move). This should lighten the work, but also serve as a guide for mathematical analysis.

#### a. Initial position

In what follows we consider the following notations:

$$R = \text{king (Roi)}, D = \text{queen (Dame)}, T = \text{rook (Tour)}, C = \text{knight (Cavalier)}, \\ P = \text{pawn (Pion)}$$

Whites: R(5; 1); T(8; 1); C<sub>1</sub>(6; 5); C<sub>2</sub>(7; 3); P<sub>1</sub>(5; 2); P<sub>2</sub>(8;3),

Blacks: R'(7; 2); P'(6; 3).

#### b. Satisfaction with preliminary rules

According to (B),  $\forall B \in \mathcal{P} : B(R') \neq 0$ , so the black king is not in a state of failure and like  $P'(R) \neq 0$ , the white king is not either, in the state of the initial position of the problem.

#### c. Resolution strategy

To solve a chess problem such as a straight checkmate in 2 moves, one starts by studying the possibilities of Black; the King's escape spaces and the possible movements of the pieces. However, the escape squares are the squares adjacent to the Black King. We therefore calculate the movement and control indicators for each white piece applied to the squares adjacent to the Black King. If the indicative specific to a piece is zero for a square then the latter is controlled by said piece.

BLANCS						
<i>B</i>	<i>R</i>	<i>T</i>	<i>C1</i>	<i>C2</i>	<i>P1</i>	<i>P2</i>
<i>x</i>	5	8	6	7	5	8
<i>y</i>	1	1	5	3	2	3
<i>x</i>	2	-1	1	0	2	-1
<i>Y</i>	1	1	-3	-1	0	-1
<i>B(R)</i>	12	-1	5	-4	2	-2

NOIRS		
<i>R'</i>	<i>P'</i>	<i>B'</i>
7	6	<i>x</i>
2	3	<i>y</i>
-2	-1	<i>x'</i>
-1	-2	<i>Y'</i>
-1	<i>B'(R)</i>	

Fig. 13: Tables A and B

<i>L</i>	<i>R(L)</i>	<i>T(L)</i>	<i>C1(L)</i>	<i>C2(L)</i>	<i>P1(L)</i>	<i>P2(L)</i>	$\alpha_L$	$\beta_L$	NC	PC
<i>N</i>	42	-2	0	-5	3	-1	0	1	1	C1
<i>NW</i>	12	-4	-1	-4	0	2	-1	1	1	P1
<i>W</i>	0	-2	4	-3	-1	3	-1	0	1	R
<i>SW</i>	0	0	11	0	0	6	-1	-1	3	R, T, C2
<i>S</i>	6	0	12	-1	3	3	0	-1	1	T
<i>SE</i>	56	0	15	0	8	2	1	-1	1	C2
<i>E</i>	72	0	8	-3	7	-1	1	0	1	T
<i>NE</i>	132	0	3	-4	8	-2	1	1	1	T

Fig. 14: Table C

<i>L</i>	$x_L$	$y_L$	$Y+\beta_L P1$
<i>N</i>	7	3	1
<i>NW</i>	6	3	1
<i>W</i>	6	2	0
<i>SW</i>	6	1	-1
<i>S</i>	7	1	-1
<i>SE</i>	8	1	-1
<i>E</i>	8	2	0
<i>NE</i>	8	3	1

Fig. 15: Table E

Notes :

- NC = Number of Controls on square L and PC = Pieces which control L
- $P_1(SW) = \emptyset$  Because  $Y + \beta_{SW} < 0$  and  $T(SE) = \emptyset$  since  $T \equiv SE$ , according to the Tables (A) and (E).

To analyze:

In our example, all of the Black King's escape squares are controlled according to the Table (C) ; since  $\forall L \in V(R'), \exists A \in \mathcal{P}$  tel que  $A(L) = 0$ . The King therefore cannot move from the initial position.

Let's study the movement of the black Pawn. Let M be an end square that we want to determine its position. So we have  $P'(M) = 0$ .

Which implies  $P'(M) = x^2 + y^2 - m = 0$ , with  $m = 1$  ou  $m = 2$  s depending on the nature of the movement.

If  $m = 1$ , we operate with the vertical displacement indicator; so  $x^2 + y^2 = 1$ . As  $x$  et  $y$  are relative integers, this equation has for solutions  $(0; \pm 1)$  and  $(\pm 1; 0)$ . Now,  $y < 0$  since this is a black pawn. There is only one solution which therefore holds:  $(0; -1)$ .

As a result,  $x = x_M - x_{P'} = 0$  et  $y = y_M - y_{P'} = -1$ . Thus,  $x_M = x_{P'}$  and  $y_M = y_{P'} - 1$ . The square that will be occupied by the black pawn after the next move is therefore  $M(6; 2)$ .

Let's take a look at the Black Pawn's control indicator on the White King's throne, once it has been moved. For this, we calculate  $P'(R) = x^2 + y^2 - 2$  since it is a control test.

We have  $x = x_R - x_{P'} = 5 - 6 = -1$  et  $y = y_R - y_{P'} = 1 - 2 = -1$ ,  
So  $P'(R) = (-1)^2 + (-1)^2 - 2 = 0$ .

The indicator being zero, the King will therefore be put in a state of failure by the black pawn. This fatal blow would jeopardize any white plan to do Mate in 2 moves. We must therefore remember to avoid it. Let us continue our analysis with which implies a diagonal displacement. Determine the arrival square M and check if it is indeed occupied by an opponent's piece so that the movement is possible.

$P'(M) = x^2 + y^2 - 2 = 0 \Rightarrow x^2 + y^2 = 2$ . The solutions are  $(\pm 1; \pm 1)$ . Like  $y < 0$ , we hold back  $(\pm 1; -1)$ . As a result,  $x = x_M - x_{P'} = \pm 1$  and  $y = y_M - y_{P'} = -1$ .

Thus,  $x_M = x_{P'} \pm 1$  and  $y_M = y_{P'} - 1$ .

The square that can be occupied by the black pawn after the next move, if possible, is  $M(7; 2)$  or  $M(5; 2)$ .

According to Table (A), the squares  $(7; 2)$  and  $(5; 2)$  are respectively occupied by the black king and a white pawn, so it is possible to move to  $(5; 2)$ . However, these are 2 opposing pawns; which means that the black pawn can also be taken by the white pawn. It will therefore be interesting to verify this white move.

In Table (A), we replace the coordinates of the white Pawn P1 by those of the black Pawn P' which we eliminate by granting it the origin  $O(0; 0)$  which is off the board.

BLANCS							NOIRS		
<i>B</i>	<i>R</i>	<i>T</i>	<i>C1</i>	<i>C2</i>	<i>P1</i>	<i>P2</i>	<i>R'</i>	<i>P'</i>	<i>B'</i>
<i>x</i>	5	8	6	7	6	8	7	0	<i>x</i>
<i>y</i>	1	1	5	3	3	3	2	0	<i>y</i>
<i>x</i>	2	-1	1	0	1	-1	-2	5	<i>x'</i>
<i>Y</i>	1	1	-3	-1	-1	-1	-1	1	<i>Y'</i>
<b><i>B(R)</i></b>	12	-1	5	-4	∅	-2	<b>#### <i>B'(R)</i></b>		

FIG.16: Tables A1 and B1

<i>L</i>	<i>R(L)</i>	<i>T(L)</i>	<i>C1(L)</i>	<i>C2(L)</i>	<i>P1(L)</i>	<i>P2(L)</i>	$\beta\_L$	NC	PC
<i>N</i>	42	-2	<b>0</b>	-5	-1	-1	1	1	C1
<i>NW</i>	12	-4	-1	-4	-2	2	1	<b>0</b>	<b>inc</b>
<i>W</i>	<b>0</b>	-2	4	-3	-1	3	0	1	R
<i>SW</i>	<b>0</b>	<b>0</b>	11	<b>0</b>	$\emptyset$	6	-1	3	R,T,C2
<i>S</i>	6	<b>0</b>	12	-1	3	3	-1	1	T
<i>SE</i>	56	$\emptyset$	15	<b>0</b>	6	2	-1	1	C2
<i>E</i>	72	<b>0</b>	8	-3	3	-1	0	1	T
<i>NE</i>	132	<b>0</b>	3	-4	2	-2	1	1	T

Fig.17: Table C1 inc = square not checked

According to Table (C1), the NW space is not controlled by any white piece and is occupied by the white Pawn P1 (Tables (E) and (A1)). The Black King therefore has only one escape space on which there is an opponent's piece. King R' is forced to take the P1 pawn since it is the only black piece on the board.

In the new position the black king gains access to the square (6 ; 3) while the white pawn disappears from the board. In Table (A1), we therefore replace the coordinates of King R' by (6 ; 3) and we assign to P1 those of the origin.

The following results are obtained:

<i>L</i>	<i>R(L)</i>	<i>T(L)</i>	<i>C1(L)</i>	<i>C2(L)</i>	<i>P2(L)</i>	$\alpha\_L$	$\beta\_L$	NC	PC
<i>N</i>	72	-6	-4	-3	3	0	1	<b>0</b>	<b>inc</b>
<i>NW</i>	56	-9	-3	<b>0</b>	8	-1	1	1	C2
<i>W</i>	6	-6	<b>0</b>	-1	7	-1	0	1	C1
<i>SW</i>	<b>0</b>	-3	5	<b>0</b>	8	-1	-1	2	R, C2
<i>S</i>	<b>0</b>	-2	4	-3	3	0	-1	1	R
<i>SE</i>	12	-1	5	-4	$\emptyset$	1	-1	<b>0</b>	<b>inc</b>
<i>E</i>	42	-2	<b>0</b>	-5	-1	1	0	1	C1
<i>NE</i>	132	-3	-3	-4	<b>0</b>	1	1	1	P2

Fig. 18: Table C2

In this new position, the Black King has in front of him 2 escape spaces: N and SE. Knowing that it is now about to checkmate, the white move must cover the 2 spaces N and SE in addition to the one where R' sits. This is too much for a single piece like the Knight or the Pawn, but not for the Rook by the nature of its movement.

So let's play this move of the Rook.

To do this, let's first determine which squares our Rook must land on in order to defeat the Black King.

Let M be one of these squares. So we have on the one hand  $T(M) = \mathbf{0}$  and on the other hand  $T(R') = \mathbf{0}$  when  $T \equiv M$ . In other words, we have to solve the following integer system:

$$\begin{cases} (x_M - x_T)(y_M - y_T) = \mathbf{0} \\ (x_{R'} - x_M)(y_{R'} - y_M) = \mathbf{0} \end{cases}$$

We therefore have the following 4 possibilities:

$$(x_M = x_T \text{ or } y_M = y_T) \text{ and } (x_M = x_{R'} \text{ or } y_M = y_{R'}).$$

- # If  $x_M = x_T$  and  $x_M = x_{R'}$ , then  $x_T = x_{R'}$ , that is  $8=6$ . Which is absurd.
- # If  $x_M = x_T$  and  $y_M = y_{R'}$ , then  $M \begin{pmatrix} x_T \\ y_{R'} \end{pmatrix}$ , from where  $M \begin{pmatrix} 8 \\ 3 \end{pmatrix}$ . However, according to Table (A) :  $P_2(8; 3)$ . Which means that  $M \equiv P_2$ . This possibility is excluded since the white Rook cannot access a square occupied by a white pawn.
- # If  $y_M = y_T$  and  $y_M = y_{R'}$ , then  $y_T = y_{R'}$  and  $1 = 3$ . Which is absurd.
- ✓ The last case gives us  $y_M = y_T$  and  $x_M = x_{R'}$ . So  $M(6; 1)$ .

We replace in (A2) the coordinates of the Rook by (6 ; 1) and we obtain the following tables:

BLANCS						NOIRS		
<i>B</i>	<i>R</i>	<i>T</i>	<i>C1</i>	<i>C2</i>	<i>P2</i>	<i>R'</i>	<i>P'</i>	<i>B'</i>
<i>x</i>	5	6	6	7	8	6	0	<i>x</i>
<i>y</i>	1	1	5	3	3	3	0	<i>y</i>
<i>x</i>	1	0	0	-1	-2	-1	5	<i>x'</i>
<i>Y</i>	2	2	-2	0	0	-2	1	<i>Y'</i>
<i>B(R)</i>	12	0	-1	-4	####		#####	<i>B'(R)</i>

Fig. 19: Tables A3 and B3

<i>L</i>	<i>R(L)</i>	<i>T(L)</i>	<i>C1(L)</i>	<i>C2(L)</i>	<i>P2(L)</i>	$\beta\_L$	NC	PC
<i>N</i>	72	0	-4	-3	3	1	1	T
<i>NW</i>	56	-3	-3	0	8	1	1	C2
<i>W</i>	6	-2	0	-1	7	0	1	C1
<i>SW</i>	0	-1	5	0	8	-1	2	R, C2
<i>S</i>	0	0	4	-3	3	-1	2	R, T
<i>SE</i>	12	1	5	-4	$\emptyset$	-1	0	inc
<i>E</i>	42	2	0	-5	-1	0	1	C1
<i>NE</i>	132	3	-3	-4	0	1	1	P2

Fig. 20: Table C3

Note: Like  $\beta_{SE} < 0$ , we have  $P_2(SE) = \emptyset$  From (B3),  $T(R') = 0$ . The Black King is therefore in a state of failure by the Rook, but he still has an escape space (the SE space according to (C3)). The Rook move is therefore not the right one. Note, however, that  $R(5; 1)$  and  $T(8; 1)$  one of the conditions for white castling is fulfilled and the Rook after having played his last move ( $T(6; 1)$ ) does indeed occupy the finish square after performing a small white castling. What motivates us to play the small castling rather than the Rook.

In Table (A3), we change the coordinates of the White King to have  $R(7; 1)$ .

<i>B</i>	<i>R</i>	<i>T</i>	<i>C1</i>	<i>C2</i>	<i>P1</i>	<i>P2</i>	<i>R'</i>	<i>P'</i>	<i>B'</i>
<i>x</i>	7	6	6	7	0	8	6	0	<i>x</i>
<i>y</i>	1	1	5	3	0	3	3	0	<i>y</i>
<i>x</i>	-1	0	0	-1	6	-2	1	7	<i>x'</i>
<i>Y</i>	2	2	-2	0	3	0	-2	1	<i>Y'</i>
<i>B(R)</i>	12	0	-1	-4	$\emptyset$	###		###	<i>B'(R)</i>

Fig. 21: Tables A4 and B4

<i>L</i>	<i>R(L)</i>	<i>T(L)</i>	<i>C1(L)</i>	<i>C2(L)</i>	<i>P2(L)</i>	$\alpha_L$	$\beta_L$	<b>NC</b>	<b>PC</b>
<i>N</i>	72	<b>0</b>	-4	-3	3	0	1	<b>1</b>	T
<i>NW</i>	132	-3	-3	<b>0</b>	8	-1	1	<b>1</b>	C2
<i>W</i>	42	-2	<b>0</b>	-1	7	-1	0	<b>1</b>	C1
<i>SW</i>	12	-1	5	<b>0</b>	8	-1	-1	<b>1</b>	C2
<i>S</i>	<b>0</b>	<b>0</b>	4	-3	3	0	-1	<b>2</b>	R, T
<i>SE</i>	<b>0</b>	1	5	-4	<b>0</b>	1	-1	<b>1</b>	R
<i>E</i>	6	2	<b>0</b>	-5	-1	1	0	<b>1</b>	C1
<i>NE</i>	56	3	-3	-4	<b>0</b>	1	1	<b>1</b>	P2

Fig. 22: Table C4

It can be seen from (C4) that all of the black king's escape cells are under white control, since:  
 $\forall L \in V(R') : NC(L) \neq 0$ .

Since the Black King is in a check state according to (B4), it is concluded that he is checkmate.

By following the development of the position of the pieces involved in the checkmate, we manage to present the result in its digital form and then the chess form:

0.  $P_1(5; 2)$  1.  $P_1(6; 3)!$   $R'(6; 3)$  2.  $R(7; 1), T(6; 1)$

**Solution : 1. e × f3! R' × f3 2. 0 – 0 #**

#2 Math Solution (6+2)

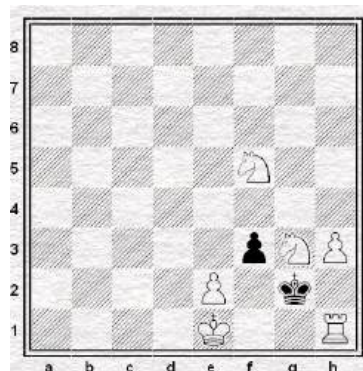


Fig. 23: mate in 2 moves to solve with math tools

The application example shows us that it is quite possible to solve chess problems with the help of equations and analysis using properties and theorems. This brings closer the two disciplines Chess and Mathematics and opens the way to new ideas around a composition of problems oriented towards education.

## VII. CONCLUSION

So there are other ways to value chess problems. The fact of exploring the other possibilities should increase the presence, in various forms, of Chess in particular the resolution of the problems. Certainly some fun and useful combinations come out of it.

What can we think about the diversification of published problem genres?

A mathematics teacher who seeks, in special sessions, to offer a playful challenge to his students may find no better than to give them chess problems to solve using mathematical tools. But first he will have to teach them a method, which should be done quickly and without hassle, then compose himself problems or seek out them from the composers who indicate under their compositions the terms: "math

solution". These will have to respect certain rules (which will have to be defined) and certainly the level of the target audience; A one-move problem being easier than a two-moves one. For the student it is a fun session solving puzzles and proving his ability to use mathematical analysis with the appropriate tools and for the teacher it is a question of testing at fair value certain technical and moral skills in its students.

The link between the two disciplines gives rise to connections whose result can be in favor of the promotion of the chess problem and the human cognitive values it must convey.

## REFERENCES

1. <https://chessineducation.org/conceptual-learning/>
2. Archambault, Jean-Pierre, A school of the mind, US Magazine, October 2000.
3. Hooper, David, Whyld, Kenneth (1996) [First pub. 1992]. The Oxford Companion to Chess (2nd ed.). Oxford University Press. ISBN 0-19-28004.
4. Hoffman E. J. and al., Construction for the Solutions of the Queens Problem, Mathematics Magazine, Vol. XX (1969).
5. [www.zanotti.univ-tln.fr/ALGO/I51/Reines.html](http://www.zanotti.univ-tln.fr/ALGO/I51/Reines.html)
6. Lemoine Julien, Viennot Simon, It is not impossible to solve the game of chess, 1024 – Bulletin de la Société Informatique de France, numéro 6, July 2015, pp. 15–40
7. Bakani M. A new method of solving chess problems: Unveiling a mathematical-physics approach. J Mod Appl Phy. 2024;7(1):1. <https://www.pulsus.com/scholarly-articles/a-new-method-of-solving-chess-problems-unveiling-a-mathematicalphysics-approach.pdf>

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# Multivariable Analysis and Phenotypic Diversity Studied for some Barley Genotypes under Heat Stress

*Samah A.Mariey, Karima R. Ahmed & Anas .H. Ahmed*

## ABSTRACT

Heat stress is one of most domineering abiotic stress influences that border barley production. Herein, three different field screening locations were carried out at Sakha, Mallawi and New-valley research stations, to identify the response of ten barley genotypes to different temperatures degrees using phenotypic diversity and, multivariable analysis during two consecutive seasons 2020/2021 and 2021/2022 under different temperatures degrees. Heat stress index (HI) activated a reduction in all traits ranged from lowest average reduction in plant height PH by (5.43 and 20.37%) to highest average reduction in no. of tillers /m<sup>2</sup>TM by (14.49 and 40.83 %) under Malawi (T2) and New Valley (T3) locations respectively as camper by Sakha, also high temperature enhancement all the genotypes to quicken flowering and days to maturity by average (7.24 and 8.35 %) under New Valley. Days to heading HD and to maturity MD exhibited a strong and significant negative relationship with all studied traits Loading principal component analysis PCA accounted 86.1% of the total variability, which PCA2 clarified 24.2 % of the total variability influenced by HD and MD which placed in the left side (negative). Scatter plot of PCA categorizing all the barley genotypes in four groups indicated that the Egyptian barley genotypes (Giza 137, Giza 138, line5, line 1 and line 3) were separate from the other genotypes and located in the right side with of PCA1 analysis cluster with a significant distance which could be considered as a heat tolerance genotypes.

*Keywords:* hordeum vulgar, phenotypic diversity - heat stress index hi, pca, heatmap analysis.

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## ABSTRACT

Heat stress is one of most domineering abiotic stress influences that border barley production. Herein, three different field screening locations were carried out at Sakha, Mallawi and New-valley research stations, to identify the response of ten barley genotypes to different temperatures degrees using phenotypic diversity and, multivariable analysis during two consecutive seasons 2020/2021 and 2021/2022 under different temperatures degrees. Heat stress index (HI) activated a reduction in all traits ranged from lowest average reduction in plant height PH by (5.43 and 20.37%) to highest average reduction in no. of tillers /m<sup>2</sup>TM by (14.49 and 40.83 %) under Malawi (T<sub>2</sub>) and New Valley (T<sub>3</sub>) locations respectively as camper by Sakha, also high temperature enhancement all the genotypes to quicken flowering and days to maturity by average (7.24 and 8.35 %) under New Valley. Days to heading HD and to maturity MD exhibited a strong and significant negative relationship with all studied traits Loading principal component analysis PCA accounted 86.1% of the total variability, which PCA<sub>2</sub> clarified 24.2 % of the total variability influenced by HD and MD which placed in the left side (negative). Scatter plot of PCA categorizing all the barley genotypes in four groups indicated that the Egyptian barley genotypes (Giza 137, Giza 138, line5, line 1 and line 3) were separate from the other genotypes and located in the right side with of PCA<sub>1</sub> analysis cluster with a significant distance which could be considered as a heat tolerance genotypes. A cluster heatmap according their resulted form all studied traits showing that the ten barley genotypes were clustered into two main clusters, Giza 137, Giza 138, line5, line 1 and line 3 were the most closed genotypes together due to their tolerance to heat stress while line 2 and line 8 were the most closed genotypes together due to their sensitive to heat stress. Thus, we could use them as a source for future barley breeding programs for heat stress such as important step to get a new genotype with high heat tolerance and high yield.

**Keywords:** hordeum vulgar, phenotypic diversity - heat stress index hi, pca, heatmap analysis.

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## I. INTRODUCTION

Heat stress is one of most vital climate change influences, which there was a universal will increase the average of temperature by 1.8–4 °C in the 21st century, the increasing will due to a significant yield losses with great dangers for the future global food safety around the worldwide (Mariey et al., 2023a, Horváth et al., 2024 and Habouh and Abo-Sapra, 2025). In Egypt temperature changes from low and worm in coastal zone to hot in the Upper area that the winter season is from December to February and the summer season from June to August, so these weather change had negative influence on Egypt agricultural strategy. (Elbasiouny et al., 2017 and Mahmoud et al., 2017 and Mariey et al., 2023a ).

Barley (*Hordeum vulgare* L.) is a chief cereal crop that has well improved to numerous abiotic stresses in dry areas, it was found to be moderately tolerant to drought stress, due to it is the restricted amount

of water that is available for irrigation (*Habib et al., 2021 and Mariey et al., 2022*). In Egypt, barley is a major winter crop cultivated in old and newly reclaimed lands that hurt from a dearth of irrigation, low soil fecundity, and salinity of both soil and water. However, there is a lack of consciousness of the nutritional role of barley for both humans and animals (*Mariey et al., 2023b and Horváth et al., 2024*) The important responsibilities for plant breeders is to increase yield per unit area by evolving high tolerant genotypes to be suitable for sowing in bad area which surfing from abiotic stresses, These tasks could be realized over using effective methods that help the breeders to screening and documentation the response of genotypes for stress. Agro-morphological parameters and yielding period were the most useful selection criteria to evaluation barley response to heat salinity stress under real field conditions. (*Mariey et al., 2021, 2023 a , Horváth et al ., 2024 ,Kim et al., 2024 and Habouh and Abo-Sapra, 2025*).

Multi-traits include several related traits that could be included in a multivariate analysis. Bi-plot, principal component and cluster analysis were the most influence mathematical methods that could provide a simultaneous analysis of multiple variables to improve the ranking accuracy of the genotypes for abiotic stress in barley (*Mansour et al., 2021, Mariey et al., 2022 & 2023b, Habouh , Abo-Sapra, 2025 and Kumar et al ., 2025* )

Consequently, understanding the phenotypic diversity among genotypes will help to ensure that the breeding program has the genetic diversity to improve biotic and abiotic stresses tolerance by crossing genetically-diverse parents having desirable characters, estimate of genetic diversity using phenotypic diversity is one of the primary and important steps in breeding programs for abiotic stresses tolerance (*Mariey et al., 2021 & 2023, Horváth et al., 2024 and Habouh and Abo-Sapra, 2025* ).

The present study aimed to investigate the phenotypic diversity of ten Egyptian barley genotypes using some relative importance of some agronomical traits and classify them using multivariable analysis in order to offer genetic evidence for the future breeding programs for heat to intensification the production of barley under heat stress

## II. MATERIAL AND METHODS

### 2.1. Barley plant materials

Ten barley genotypes were kindly provided by Barley Dep., , Field Crops Research Institute, ARC, Egypt, were used in this study their names and pedigree shown in (Table 1).

*Table 1:* Name, and pedigree of ten barley genotypes used in this study

No.	Name	Pedigree
1	Line 1	Giza 124/6/Alanda//Lignee527/Arar/5/Ager//Api/CM67/3/ Cel/WI2269//Ore/4/ Hamra,01
2	Line 2	BLLU/PETUNIA1//CABUYA/3/Alanda// Lignee527 / Arar
3	Line 3	Giza 118/3/Alanda/Hamra//Alanda,01
4	Line 4	Rihane03/7/Bda/5/Cr.115/Pro/Bc/3/Api/CM67/4/Giza120/6/Dd/4/Rihane,03
5	Line 5	Giza 2000/6/Alanda//Lignee527/Arar/5/Ager//Api/CM67/3/ Cel/WI2269//Ore/4/ Hamra,01
6	Line 6	Giza 119/3/Alanda/Hamra//Alanda,01
7	Line 7	Giza 117/6/Alanda//Lignee527/Arar/5/Ager//Api/CM67/3/ Cel/WI2269//Ore/4/ Hamra,01
8	Line 8	Giza 123/5/Furat 1/4/M,Att,73,337,1/3/Mari/Aths*2//Attiki
9	Giza 137	Giza 118 /4/Rhn-03/3/Mr25-//Att//Mari/Aths*3-02
10	Giza 138	Acsad1164/3/Mari/Aths*2//M-Att-73-337-1/5/Aths/ lignee686 /3/Deir Alla 106//Sv.Asa/ Attiki /4/Cen/Bglo."S")

2.2. Field investigational description

2.2.1. Field experimental Locations

Three field experimentations were achieved in three dissimilar heat stress sites were growing during two winter sowing seasons of 2020/ 2021 and 2021. /2022 to study the effect of heat stress on ten barley genotypes yield production as shown in Fig 1:

1. Sakha station, locating in the center of the Delta -Kafer EL-Sheik governorate, has an elevation of 8.30 above sea level, with Latitude: 31° 6' 22.75" N" and Longitude: 30° 56' 31.11" E" .
2. Malawi station, locating in Minya governorate with Latitude: 27° 43' 53.04" N Longitude: 30° 50' 29.94" E
3. EL-Dakhla, Oasis station, locating in New valley research governorate with Latitude: 25° 30' 59.99" N and Longitude: 29° 09' 60.00" E,.

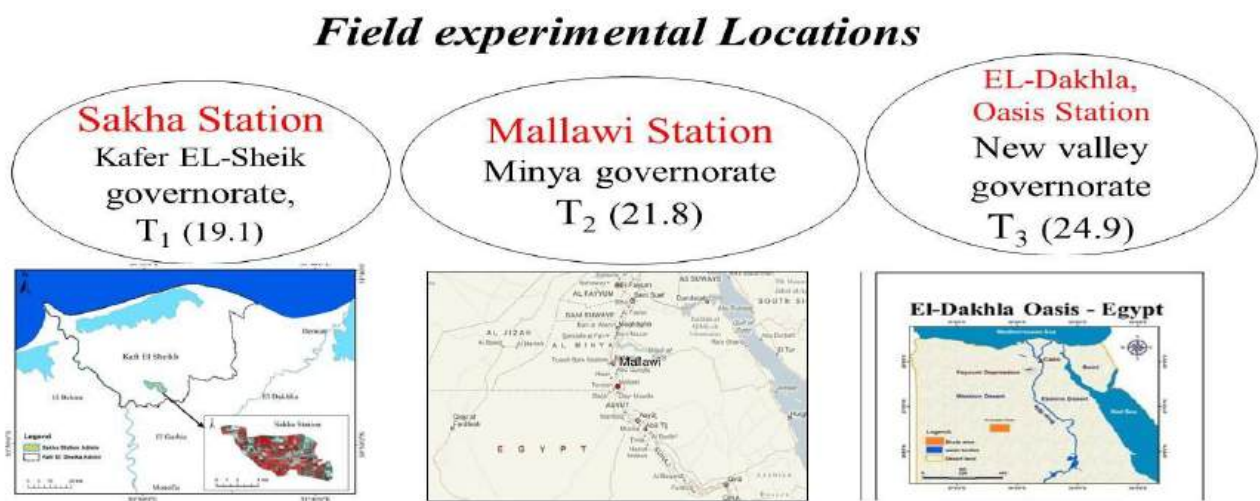


Fig. 1: The maps of Field experimental Locations

2.2.2. Field experimental design

The ten genotypes were growing in a randomized complete block design (RCBD) with three replicates using (plot area =3.6 m<sup>2</sup>) for each plot, to evaluate the related traits to grain yield and heat stress index

2.2.3. Field experimental Soil samples

Soil samples were taken before land preparation in two depths from the soil surface; i.e. 0-15 cm and 15-30 cm. The physical and chemical analysis of different experimental sites were presented (Table 2)

Table 2: The average of physical and chemical properties for soil samples from the field experiments sites during two growing seasons 2020/2021 and 2021/2022

Soil analysis	Sakha Station	Mallawi Station	New valley Station
A: Physical analysis			
Sand (%)	18.94	14.1	67.1
Silt (%)	28.15	43.1	9.0

Clay (%)	51.35	40.2	23.9
Texture	Clayey	Silty caly	Sandy clay loam
B: Chemical analysis			
EC(dSm <sup>-1</sup> )	2.76	1.62	5.78
PH	7.6	7.86	7.85
K <sup>+</sup> meq100 <sup>1</sup> g soil	0.1	0.57	0.58
CaCO <sub>3</sub> <sup>-</sup> meq100 <sup>1</sup> g soil	0	2.21	4.52
So <sub>4</sub> <sup>-</sup> meq100 <sup>1</sup> g soil	4.95	0.55	-

**2.2.4. The Agro- meteorological information**

The data of average month maximum and minimum temperatures (C°) and relative humidity (RH., %), were documented for weather station belonging to the Sakha (T<sub>1</sub>), Mallawi (T<sub>2</sub>) and New Valley (T<sub>3</sub>) Station, Egypt during two growing winter seasons 2020/2021 and 2021/2022 were shown in (Table 3).

*Table 3:* The Meteorological of the experimental area during the two-growing seasons of barely 2020/2021 and 2021/2022 under three different locations Sakha, Mallawi and New valley sites

Season	Month	Temperature, C°									Relative humidity, RH %		
		Sakha (T <sub>1</sub> ) Normal temperature			Mallawi (T <sub>2</sub> ) Medium temperature			New valley (T <sub>3</sub> ) High temperature			Sakha	Malawi	New valley
		Max.	Min	Mean	Max.	Min	Mean	Max	Min	Mean			
Season 2020/2021	Dec.	21.4	13.4	17.4	20.7	9.15	14.93	21.9	9.9	15.9	86.9	64.83	54.2
	Jan.	18.4	11.8	15.1	18.7	6.13	12.42	28.0	7.6	17.8	86.7	64.83	54.5
	Feb.	20.4	12.7	16.6	22.7	9.82	16.26	25.7	9.8	17.75	84.6	61.81	41.9
	Marc.	22.6	15.6	19.1	28.7	14.2	21.45	30.0	13.8	22.9	81.1	61.19	32.9
	Apr.	26.0	18.9	22.5	32.53	17.1	24.82	35.5	18.2	26.85	80.0	53.46	25.2
	seasonal	21.71	14.4	18.4	24.6	11.8	17.9	28.2	11.8	20.4	83.6	61.2	39.9
Season 2021/2022	Dec.	22.9	13.7	18.3	25.0	14.0	19.50	25.3	11.2	18.25	87.7	54.56	51.7
	Jan.	21.0	13.5	17.25	24.5	12.5	18.50	23.1	6.6	14.85	86.7	54.54	44.9
	Feb.	21.5	12.5	17.0	23.5	9.71	16.61	25.2	8.4	16.8	87.5	54.20	43.6
	Marc.	23.8	15.2	19.5	29.3	13.9	21.60	31.6	14.6	23.1	83.8	52.35	35.2
	Apr.	27.6	19.4	23.5	31.0	14.6	22.80	31.9	15.5	23.7	74.6	45.51	27.3
	seasonal	23.35	14.8	19.11	26.6	12.9	19.8	27.4	11.26	19.34	84.06	52.23	40.5

**2.2.5. Measured Characteristics:**

At the heading stage days to heading were recorded, at maturity stage days to maturity were recorded and at the harvest stage ten guarded plants were randomly taken from each plot to measure plant height cm, number of tillers m<sup>-2</sup>, number of grains spike<sup>-1</sup>, and grain yield was determined using the full plot area (3.6 m<sup>-2</sup>).

**2.2.6 Multivariable studied analysis**

1. *Heat stress index:* The relative change due to heat stress was computed for each trait according to (Bousslama and Schapaugh, 1984)

2. *Correlation coefficient*: person and matrix were used to study the relationship between each two studied traits were done using Minitab 18.1 statistical software (Minitab Inc., Coventry, UK) and
3. *Principal Component Analysis (PCA)*: Loading and scatted plot were performed to study the differences and interrelations between genotypes with respect to measured phenotypic traits using Minitab 18.1 statistical software (Minitab Inc., Coventry, UK) and
4. *Heatmaps cluster*: ClustVis: is a web tool for visualizing clustering of multivariate data, was used to constructed heatmaps (<https://biit.cs.ut.ee/clustvis/>) (Metsalu, et al., 2015)

## 2.7. Data analysis

All the data of the examined traits from the two seasons were homogeneity and statistically analyzed were exposed to ANOVA in a randomized complete block design (RCBD) to conclude the effects of genotypes, salinity levels and their interaction on the studied traits was performed using SAS software ver. 9.1 (SAS 2011). Duncan's test was used to compare mean values at 95% levels of probability (Duncan, 1955).

## III. RESULTS

### 3.1 Effect of different temperatures degrees on studied traits for barley genotypes

The ANOVA analysis of all phenotypic studied traits including days to heading (HD days), days to maturity (DM, days), plant height (PH, cm), number of tillers  $m^2$  (TM, tillers / $m^2$ ), number of grain spike<sup>-1</sup> (NGS<sup>-1</sup> grain /spike), thouded kernel weight (TKW, g) and grain yield (GY ard/fad) indicated a significant statistical effect ( $P < 0.01$ ) by different temperatures degrees under three locations Sakha ( $T_1$ ), Malawi ( $T_2$ ) and New Valley ( $T_3$ ), cultivars (C), and years (Y) as shown in (Table 4).

A significant two-way interaction between temperatures degrees and barley genotypes (G X T) were observed for all studied traits. While, the two-way interaction between years x temperatures degrees (Y X T) and years x genotypes (Y X G) were significant. across all traits expect the, TM were non-significant. Similarly, the combined ANOVA indicated significant effect for three-ways interaction (G X T X Y) across all traits, expect for TM, which were non-significant

The results indicate that high temperatures at Malawi and New Valley ( $T_2$  and  $T_3$ ) caused a significant decrease in all studied characters, while caused a significant increase in HD and MD as compared with normal temperature at Sakha station ( $T_1$ ). which induced all genotypes to flowering and maturity early in Malawi and New Valley ( $T_2$  and  $T_3$ ) more than Sakha station ( $T_1$ ). Correspondingly, the results showed diverse significantly which were found among all the Egyptian barley genotypes according their average of mean performances of all studied traits due to the differ temperature ( Table 4), which the results showed that Giza 138, Line 1 line 3, line 5 and Giza 137 high average values for all studied under the high temperatures degrees than other genotypes recognized as greater heat tolerance barley cultivars traits, while Line 2 and Line 8 had low average values which they were more affected by heat stress .For grain yield GY, the results indicated that the heat stress significant reduced GY at Malawi and New Valley ( $T_2$  and  $T_3$ ) more than Sakha station ( $T_1$ ) as showed in Fig 2. Giza138 gave the maximum values was (21.11, 19.03 and 17.47 ard/fad) at Sakha, Malawi and New Valley sites respectively with followed by Giza 137, Line 5, Line 3 and line 1, which all get high grain yield at Sakha, Malawi and New Valley sites respectively. However, line 2 had minimum GY values (15.11,13.33 and 9.47 ard/fad) at Sakha, Malawi and New Valley sites respectively followed by Line 8 get lower GY were (15.89, 13.47 and 10.14 ard/fad) at Sakha, Malawi and New Valley locations respectively as showed in Fig 2. These results were in gooh harmony with (Kaseva et al., 2023, Mariey et al 2023a Kim et al., 2024, Horváth et al., 2024 and Habouh and Abo-Sapra, 2025). They reported that heat stress significantly reduced most of the grain-yield related traits, which they confirm that interaction  $G \times E$  in

grain yield was strong when applied across the years for each barley genotypes under heat stress, and they conveyed that high temperatures also have indirect negative significances on yield by margarine the plant cycle or unsettling optimal development forms.

**Table 4:** ANOVA analysis of years, different temperature degrees (three location) and barley genotypes on agronomic, and their interactions during two growing seasons 2020/2021 and 2021/2022.

Studied traits	Days to heading days	Days to maturity days	Plant height cm	No. of tillers /m2	No. of grain spike-1	Thoued kernel weight (g)	Grain Yield ard/fad
Years							
2019/20	85.18	113.37	98.07	379.48	56.72	44.91	17.81
2020/21	84.77	113.26	99.33	378.36	55.73	44.36	17.50
Temperature Degrees							
( Sakha ) T1	85.67	118.02	109.05	488.83	66.77	53.24	19.78
(Malawi) T1	82.15	115.81	103.13	418.01	62.08	49.85	17.59
(New valley) T2	79.47	108.17	86.83	289.23	44.52	34.19	14.78
Barley cultivars							
line 1	80.67	114.00	101.11	396.25	56.78	48.79	19.08
line 2	84.44	113.72	83.67	256.19	52.11	43.21	13.53
line 3	80.17	115.22	104.17	427.67	56.00	48.99	19.89
line 4	83.83	113.67	96.89	359.89	61.00	44.92	18.61
line 5	81.11	112.67	105.61	407.89	57.33	45.66	19.70
line 6	84.46	113.82	92.70	373.01	55.43	45.56	17.42
line 7	83.06	112.39	95.89	373.44	53.39	44.00	18.09
line 8	85.78	113.44	93.56	259.17	49.44	42.78	14.86
Giza 137	80.72	112.06	102.61	438.78	55.00	41.98	18.96
Giza 138	80.22	112.72	105.22	453.44	62.33	43.18	19.54
ANOVA analysis							
Years (Y)	*	**	*	NS	**	**	**
Cultivars ( C )	**	**	**	**	**	**	**
Temperature (T)	**	**	**	**	**	**	**
LSD (0.05)							
Years (Y)	0.381	0.876	0.891	4.54	NS	3.45	1.166
Cultivars (C)	0.808	1.123	1.79	11.15	3.34	3.54	2.18
Temperature (T)	0.442	0.876	0.969	5.56	1.831	1.56	1.906
Interaction							
C X Y	**	**	**	NS	NS	NS	**
TX Y	**	**	**	NS	NS	NS	NS
C X T	**	**	**	**	**	**	**
CX T X Y	**	**	**	NS	NS	**	**

Which *Ns*, \* and \*\* non-significant and significant at the 0.05 and 0.01 levels of probability, respectively

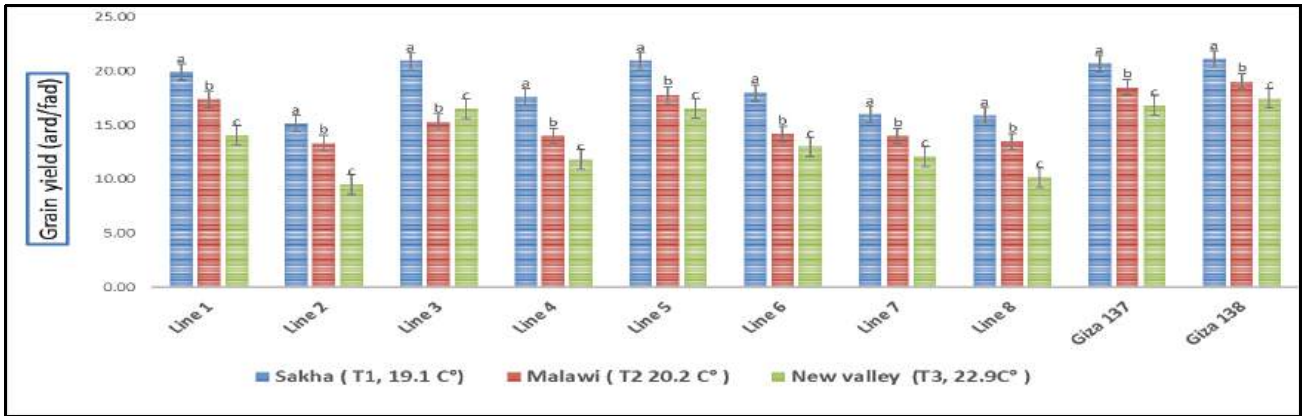
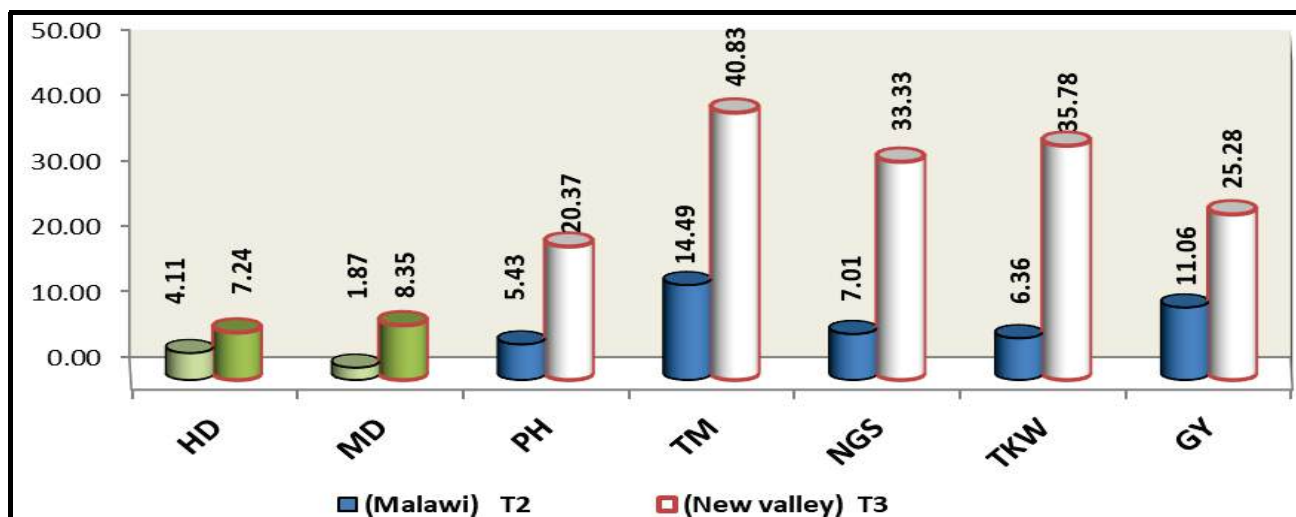


Figure 2: Effect of different temperatures degrees on grain yield among ten barley genotypes at Sakha, Malawi and New Valley locations

### 3.2 Multivariable analysis

#### 3.2.1 Heat stress index (HI)

The virtual changes reduction due heat stress (HI) on morphological studied traits, were presented in (Figure 3), the results showed that heat stress activated a reduction in all traits ranged from lowest average reduction in PH by (5.43 and 20.37%) to highest average reduction in TM by (14.49 and 40.83 %) under Malawi (T2) and New Valley (T3) locations respectively as camper by Sakha T<sub>1</sub>. About the heat index due heat stress on grain yield the results showed that there a reduction was happened due heat stress by average values. Nevertheless, heat stress induced all genotypes to flowered and maturity earlier by an average (4.11 and 1.87 %) respectively under Malawi (T2) and by an average (7.24 and 8.35 %) under New Valley (T3) location respectively as camper by Sakha T<sub>1</sub> as shown in (Figure 3). On behalf of the relative changes due heat stress on grain yield the results showed that there a reduction was happened due heat stress by average values (8.06 and 29.0 %) under Malawi and New Valley location respectively. However, heat stress induced all cultivars to flowered earlier by an average (3.93 and 11.39 %) respectively as shown in (Figure 3). This results were agree with (Devi et al 2021, Bhagat et al., 2023, Mariey et al 2023a, Kimet al 2024, Habouh and Abo-Sapra, 2025 and Kumar et al., 2025) whom, found that barley heat tolerant genotypes were significantly less affected by stress factors than heat sensitive genotypes which heat stress index is an inductor for detect the barley heat tolerant genotypes depends on it grain yielded values.



**Figure 3:** Heat stress index of studied traits under Malawi T2 and New valley T3 as compared by T1 at Sakha station which days to heading (HD), days to maturity (MD), plant height (PH), number of tillers m<sup>2</sup> (TM), number of grain spike-1 (NGS-1), thousands kernel weight (TKW) and grain yield (GY), which the green refer to inducing flowering and maturity days.

### 3.3.2 Correlation coefficient

Both Pearson and matrix correlation coefficient was done to recognize the relationships among all studied characters across the three different temperature degrees (three locations) (Figure 4 & 5). Results designated clearly that the correlation coefficients among grain yields GY and PH, TM, TKW and NGS traits were highly positive and significantly correlated. Days to heading HD exhibited a strong and negative relationship with all studied traits grain yield, and days to maturity MD showed negative relationship with all studied traits except NGS. These results were in agreement with (Mariey et al 2023a, Kim et al., 2024 and Horváth et al., 2024) whom reported that there was a significant correlation between the heat stress-induced changes in grain-yield related traits.

	HD	MD	PH	TM	TKW	NGS
MD	0.055					
PH	-0.842	-0.148				
TM	-0.878	-0.174	0.872			
TKW	-0.564	-0.039	0.586	0.692		
NGS	-0.395	0.734	0.325	0.301	0.223	
GY	-0.821	-0.068	0.919	0.942	0.712	0.461

**Figure 4:** Pearson correlation coefficient heatmap among grain yield (GY) and days to heading (HD), days to maturity (MD), plant height (PH), number of tillers m<sup>2</sup> (TM), number of grain spike-1 (NGS-1), thousands kernel weight (TKW) across the three heat stress locations. Correlation key and the scale reads, red box indicated strong negative correlation, green box indicated strong positive correlation, white yellow box mean medium positive correlation, orange box mean medium negative correlation

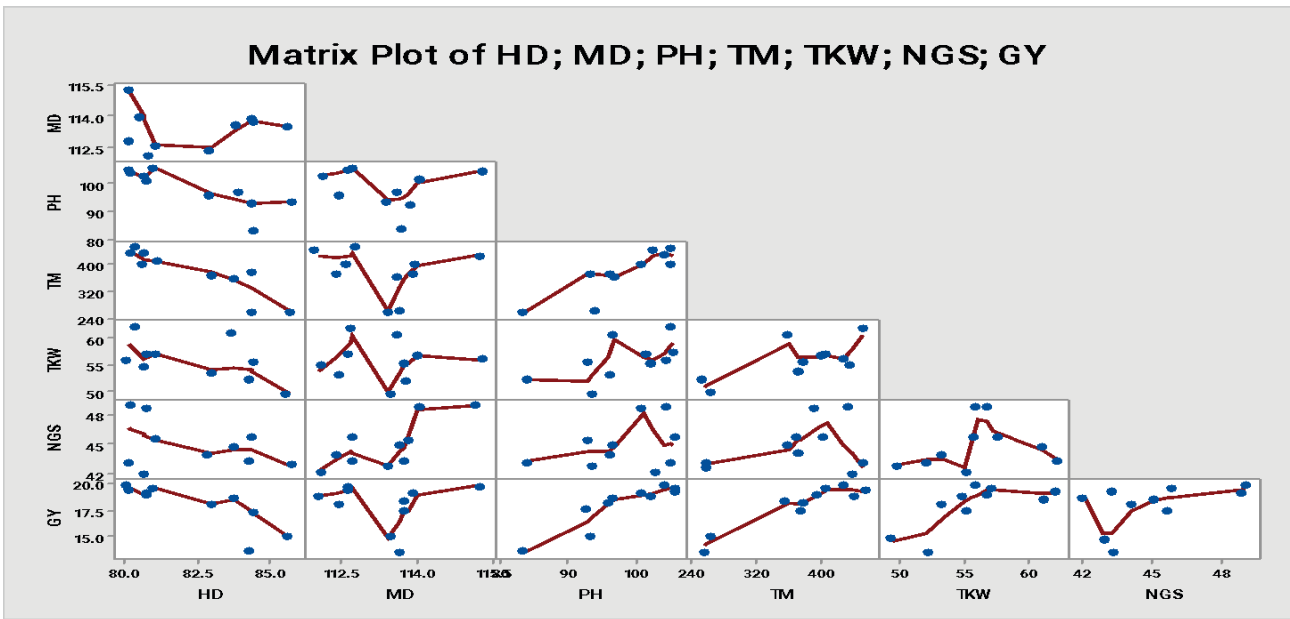
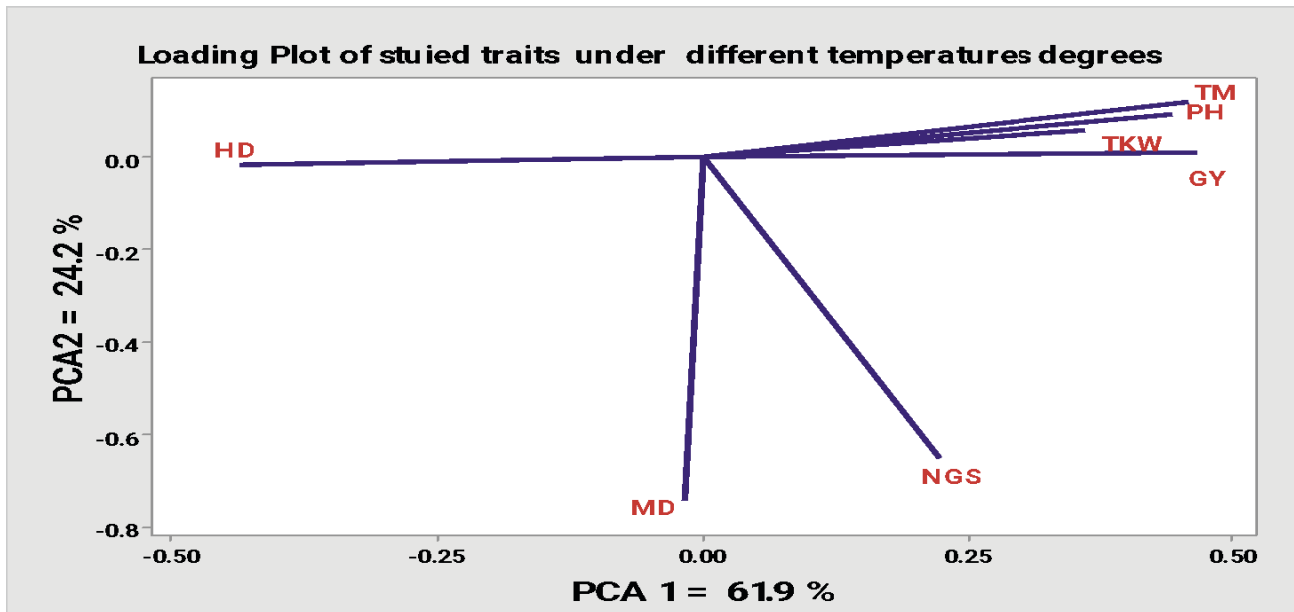


Figure 5: matrix plot correlation coefficient heatmap among grain yield (GY) and days to heading (HD), days to maturity (MD), plant height ( PH ), number of tillers m2 ( TM ), number of grain spike-1 (NGS-1), thousands kernel weight( TKW ) across the three heat stress locations.

### 3.2.3. Principal component analysis (PCA)

#### 3.2.3.1. Loading plot PCA

Loading plot was realized using distance matrix available in the horizontal axis chosen the direction of association among all studied characters was showing in (Figure 6). The results presented that principal component analysis PCA accounted 86.1% of the total variability. PCA1 lit 61.9 % of total variation partial by PH, MT, NGS, TKW and GY characters were positioned in positive direction (right side) of the horizontal axis according to their positive significant correlations with other characters under study. The second PCA2 clarified 24.2 % of the total variability influenced by HD and MD which placed in the left side (negative) of the horizontal axis conferring to its negative significant correlations with other characters under this study.



*Figure 5:* Loading plot graph, showing the first two principal components (PCA) of the correlation matrix among the studied characters which leaf area index (LAI), chlorophyll fluorescence (Fv/Fm), Total chlorophyll content SPAD, days of heading (HD), plant height (PH) numbers of tiller m<sup>2</sup> (TM), number of grains spike, 1 (NGS,1), thousand kernel weight (TKW) and grain yield (GY)

### 3.2.3.2. PCA scatter plot

The scatter plot of PCA analysis based on all studied traits categorizing all the barley genotypes in four groups as shown in (Figure 6), which PCA analysis indicated that the Egyptian barley genotypes (Giza 137, Giza 138, line no5 ) and ( line 1 and line 3) were separate from the other genotypes and located in the right side with of PCA1 analysis cluster with a significant distance, which they had achievement high average of all studied traits under study that could be documented them as heat tolerance genotypes. line 4, line 6 and line 7 which were distributed distance from one other in the scatter plot of PCA analysis cluster based in their medium average value of studied traits could be documented them as moderated heat tolerance genotypes. Whereas both of line 2 and 8 genotypes were scattered distance far from the two other groups which located on left side, as selected by cluster analysis of PCA2 affording to their lowest values of all studied traits with high reduction that could recognized as heat sensitive genotypes. The results agree with (Mariey et al., 2023 and Kumar et al., 2025) they confirmed that principal component analysis PCA was the most impact mathematical devices that could provide a concurrent analysis of multiple variables to improve the position correctness of the genotypes for abiotic stress in barley.

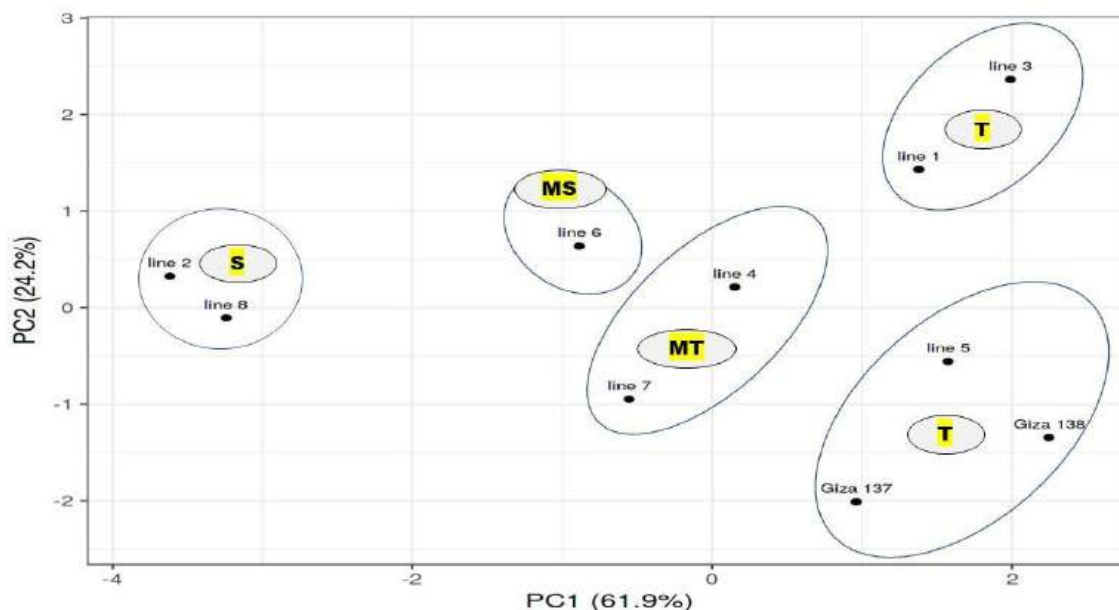
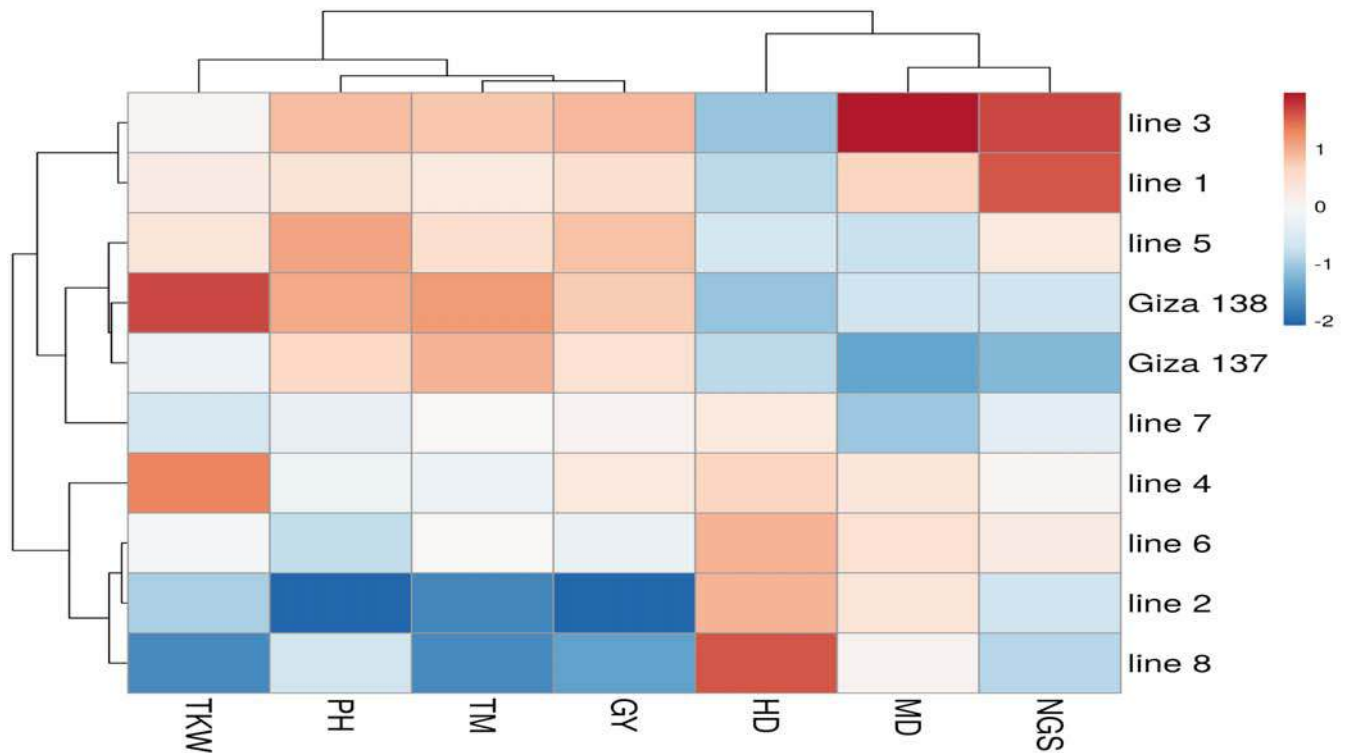


Figure 6: PCA scatter plot of all the ten barley genotypes based on studied traits

#### 2.2.4. Heatmap Cluster Analysis

The heatmap cluster analysis were constructed to investigated the effect of different temperatures on morphological traits of ten barley genotypes which use Euclidean distance and average linkage by R software (Figure 7), which confidential all the genotypes and the traits in two chief dendrograms. Column dendrograms drawing all the morpho studied traits. Row dendrogram design the ten barley genotypes which the analysis clustered them into two main clusters, first cluster include the tolerance and moderated heat divided to sub cluster, first sub includes the heat tolerance barley genotypes (Giza 137, Giza 138, line 5, line 1 and line 3), second sub cluster consisted of moderated heat tolerant genotypes line 7. While Second cluster include the sensitive and moderated sensitive heat first cluster divided to sub cluster, first sub includes the heat sensitive barley genotypes (line 1 and line 8) and second sub cluster consisted of moderated heat sensitive genotypes line 4 and line 6. Our results were in good harmony with (Mohamed, et al 2021, Mariey et al 2022 and Mariey et al 2023 a&b) which They reported that heatmap cluster analysis had used positively in sympathetic the information of phenotypic evaluations of the barley genotypes as a significant factor using to helps the breeders to have good plan for their programs for specific environments using targeted traits



*Figure 7:* Multivariate heatmap illustrating the phenotypic diversity of ten barley genotypes, based on morpho traits using the module of a heatmap of ClustVis, days of heading (HD), days to maturity (MD), plant height (PH), number of tillers m<sup>2</sup> (TM), number of grains spike<sup>-1</sup> (NGS), thousand kernel weight (TKW) and grain yield (GY).

#### IV. CONCLUSIONS

Ten barley genotypes were grown in three different field screening locations carried out at Sakha, Malawi and New-valley research stations, which studied their response to three different temperatures degrees by using phenotypic diversity and, multivariable analysis, which the results indicated that Giza 138, Giza 137, Line 1, Line 3 and Line 5 we could consider them as heat tolerance genotypes and both of Line 2 and Line 8 were heat sensitive genotypes, these differences enable the breeders to use the tolerant cultivars as a good parent in heat stress breeding programs in Egypt due to increase the farmer's income.

#### REFERENCES

1. Bartlett MS. (1937). Properties of sufficiency and statistical tests. Proceedings of the Royal Statistical Society Series A 160: 268
2. Bates LS., Waldern RP., Teave ID. (1973). Rapid determination of free proline for water stress studied. Plant and Soil. 39, 205-207.
3. Bouslama, M and Schapaugh, W. T. (1984). Stress tolerance in soybean. Part 1: Evaluation of three screening techniques for heat and drought tolerance. Crop Sci., 24, 933-937.
4. Bradford MM. (1976). A rapid and sensitive method for quantitation of microgram quantities of protein utilizing the principle of protein-dye binding. Annals of Biochem. 72:248-254.
5. Devi S, Y Kumar and S Shehrawat. (2021). Identification of heat tolerant barley genotypes based on heat susceptibility index. Journal of Cereal Research 13(2): 197-204
6. Elbasiouny H., Abowaly M., Gad A., Abu\_Alkheir A., Elbehiry F. (2017). Restoration and

- sequestration of carbon and nitrogen in the degraded northern coastal area in Nile Delta, Egypt for climate change mitigation. *J. Coast. Conserv.* 21, 105–114.
7. Habib, Md., Mannan, Md., Karim, Md., Miah, Md and Singh, H. (2021). Water deficit stress tolerance assessment in barley cultivars using drought tolerance indices.
  8. Habouh, M., and Abo-Sapra, H. (2025). Impact of Late Sowing on Performance of Barley Genotypes Under Aswan Conditions. *Assiut Journal of Agricultural Sciences*, 56(1), 33-47.
  9. Horváth Ádám, Zita Berki, Krisztina Balla, Judit Bányai, Marianna Mayer, András Cseh, Tibor Kiss, Ildikó Karsai (2024), Field versus controlled environmental experiments to evaluate the heat stress response of barley (*Hordeum vulgare* L.), *Environmental and Experimental Botany* 228,
  10. Kaseva, J., Hakala, K., Högnäsbacka, M., Jauhiainen, L., Himanen, S. J., Rötter, R. P., and Kahiluoto, H. (2023). Assessing climate resilience of barley cultivars in northern conditions during 1980–2020. *Field Crops Research*, 293, 108856.
  11. Kim, J., Savin, R., & Slafer, G. A. (2024). Quantifying pre-and post-anthesis heat waves on grain number and grain weight of contrasting wheat cultivars. *Field Crops Research*, 307, 109264.
  12. Kumar ogender, Suman Devi, Divya Phougat and Harsh Chaurasia ( 2025) Estimation of stress tolerance indices for identification of heat tolerant genotypes in barley . *Electronic Journal of Plant Breeding*, 15 4, 890-902.
  13. Mariey A. Samah , Khatab, I. A., Shahein, A. M., Abo-Marzoka, E. S. A., Noreldin, T., Badawy, A. F., and El-Naggar, A. A. (2022). Comprehensive Selection Criteria for Some Barley Genotypes under Different Water Stress Treatments. *International Journal of Plant & Soil Science*, 34(23), 1777-1791.
  14. Mariey A. Samah, Mohamed N Eman, Ghareeb E. Zeinab, Abo Zaher S. Engy (2021). Genetic Diversity of Egyptian Barley using Agro–physiological Traits, grain quality and molecular markers. *Current Science International*, 10 :58-71
  15. Mariey A. Samah,, El-Bialy MA, Khedr RA, Mohamed EN, Meleha AMI, Khatab IA. (2023a). Comprehensive evaluation and economic analysis in some barley genotypes under soil salinity. *Asian J Agric* 7: 20-33
  16. Mariey, A. Samah Omnia SM Hashem, Anas H. Ahmed, Karima R. Ahmed, and Hayam IA Elsayy. "Phenotypic and genotypic diversity analysis of some Egyptian barley cultivars (*Hordeum vulgare* L.) under different heat stress conditions. (2023b) . *Egyptian Journal of Agricultural Research* 101, no. 2 412-423.
  17. Metsalu T., Vilo J., ClustVis (2015). A web tool for visualizing clustering of multivariate data using Principal Component Analysis and heatmap. *Nucleic Acids Res.* 43: 566.
  18. Mohamed, A.H.; Omar, A.A.; Attya, A.M.; Elashtokhy, M.M.A.; Zayed, E.M.; Rizk, R.M. (2021). Morphological and Molecular Characterization of Some Egyptian Six-Rowed Barley (*Hordeum vulgare* L.). *Plants*, 10: 2527
  19. Pour-Aboughadareh, A., Barati, A., Koohkan, S. A., Jabari, M., Marzoghian, A., Gholipoor, A., and Kheirgo, M. (2022). Dissection of genotype-by-environment interaction and yield stability analysis in barley using AMMI model and stability statistics. *Bulletin of the National Research Centre*, 46(1), 19.

#### الملخص العربي

دراسة التحليل متعدد المتغيرات والتنوع المظهري لبعض التركيب الوراثية من الشعير تحت الإجهاد الحراري  
 سماح عبدالله مرعي 1 كريمة رشاد أحمد 1 وأنس حسين أحمد 1

1- قسم بحوث الشعير ، معهد بحوث المحاصيل الحقلية، مركز بحوث زراعية، الجيزة 12619، مصر  
 الإجهاد الحراري هو واحد من أكثر الإجهاد البيئية الحيوية التي تؤثر على إنتاج الشعير. وهنا تم إجراء ثلاثة تجارب في ثلاث مواقع بحثية مختلفة في محطات أمحة بحوث سخا وملوي والوادي الجديد ، للتعرف على استجابة عشرة تراكيب وراثية للشعير لدرجات الحرارة مختلفة باستخدام التنوع الظاهري، والتحليل متعدد المتغيرات خلال موسمين متتاليين 2020/2021 و 2021/2022 تحت درجات حرارة مختلفة. أدى

مؤشر الإجهاد الحراري (HI) إلى انخفاض جميع الصفات المدروسة حيث تراوح بين أدنى متوسط انخفاض في طول النبات بنسبة (5.43 و 20.37%) إلى أعلى متوسط انخفاض في عدد الفروع متر مربع بنسبة (14.49 و 40.83%) تحت محطة بحوث ملاوي و محطة بحوث الوادي الجديد على التوالي كمقارنة بمحطة بحوث سخا ، وكذلك درجات الحرارة العالية شجعت جميع التركيب الوراثية الى التبرير وتسريع عدد أيام النضج بمتوسط (7.24 و 8.35%) تحت محطة بحوث الوادي الجديد. كما أظهرت كلا من عدد أيام التزهير والنضج علاقة ارتباط معنوية قوية وسلبية مع جميع السمات المدروسة . تحليل المكونات الرئيسية شكلت 86.1% PCA من إجمالي التباين ، والتي أوضحت المكون الثانى من التحليل 24.2% PCA من إجمالي التباين المتأثرة بعدد أيام التزهير والنضج حيث كان موقعها في الجانب الأيسر (سلبية). وأشارت Scatter plot ل PCA الى تصنيف جميع التركيب الوراثية للشعير إلى أربع مجموعات حيث وجد ان التركيب الوراثية للشعير المصرية (الجيزة 137 والجيزة 138 وسلالة 5 وسلالة 1 وسلالة 3) كانت منفصلة عن التركيب الوراثية الأخرى وتقع في الجانب الأيمن مع مجموعة تحليل المكون الرئيسى الاساسى PCA1 بمسافة كبيرة , لذا يمكن اعتبارها هذه التركيب الوراثية متحملة للحرارة. خريطة النسب الوراثية بناء على كل الصفات المدروسة اوضحت أن التركيب الوراثية العشرة للشعير تم تجميعها في مجموعتين رئيسيتين ، الجيزة 137 ، الجيزة 138 ، سلالة 5 ، سلالة 1 وسلالة 3 كانت أكثر الأنماط الجينية قريبة معا بسبب تحملها للإجهاد الحراري بينما كان سلالة 2 وسلالة 8 أكثر التركيب الوراثية قريبة معا بسبب حساسيتهما للإجهاد الحراري . وبالتالي ، فإننا نستخدمها كمصدر لبرامج تربية الشعير المستقبلية للإجهاد الحراري كخطوة مهمة للحصول على تركيب وراثية جديدة ذات تحمل عالي للحرارة وإنتاجية عالية.



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# Experimental Determination of Suction Pressure using the filter Paper Method for Unsaturated Clay Soils in the Moscow Region

*M.A. Novgorodova & D.N. Gorobtsov*

*University for Geological Prospecting*

## ABSTRACT

**Introduction:** In the world, practice of soil research there is ASTM Standard Test Method for Measurement of Soil Potential (Suction) Using Filter Paper D 5298-16. However, unfortunately, in Russia there are no defining documents related to negative pore pressure, or matrix suction, or SWCC, yet.

**Materials and methods.** In this study, as a practical example of the effect of suction pressure on slope stability, the authors considered an object located in Zelenograd. Because of the experiment, SWCC was obtained and a geomechanical model was created.

**Results.** Thus, for cover loams, the value of the initial suction pressure is 17 kPa. The value of matrix suction is 199 kPa. For fluvioglacial loams, the value of the initial suction pressure is 14 kPa. The value of matrix suction according to the graph is 207 kPa.

**Discussion and conclusion:** Subsequently, calculations were performed in the Plaxis software without taking into account the suction pressure and with taking into account the suction pressure, while all other model parameters remained unchanged.

**Keywords:** matric suction; suction pressure; unsaturated soils; filter paper method; SWCC.

**Classification:** LCC Code: TA710, TA703

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## I. FOR CITATION

In Russian engineering practice, the classical concept of soil mechanics assumes solving problems involving either fully saturated or fully dry soil, which are actually two limiting states of soil. In most cases, soils used as foundations for buildings and structures are in an unsaturated state and their degree of water saturation can vary from 0% to 100%. In this case, designers use the Van Genuchten-Mualem equation in their models, which is intended for unsaturated soils. Unsaturated soils are commonly found in many parts of the world, at a shallow depth (from 2-3 m to 10-15 m) from the surface, as well as in arid regions, where the natural groundwater level is usually at a greater depth. Common to all these soils is their negative pore water pressure, which plays an important role in assessing mechanical properties and also complicates their studies in the laboratory. The presence of air and water in the pore spaces between soil particles causes capillary action, which creates suction.

The relationship between soil suction pressure and water content is determined by SWCC and is an important tool for predicting and interpreting the behavior of unsaturated soils, including under load. SWCC is the relationship between matric suction (chemical potential) and water content (gravimetric or volumetric) or degree of saturation. As the soil passes from a saturated state to an unsaturated state, the distribution of mineral, water and air phases changes as the stress state changes. The relationships between these phases take different forms and affect the engineering properties of unsaturated soils.

In the world practice of soil research, there is ASTM Standard Test Method for Measurement of Soil Potential (Suction) Using Filter Paper D 5298-16. But, unfortunately, in Russia there are no defining documents related to negative pore pressure, or matrix suction, or SWCC.

The advantages of the filter paper method are the ability to measure total suction, which is the sum of osmotic and structural suction, as well as technical simplicity, low cost and sufficient accuracy.

Based on ASTM D 5298, we conducted experimental studies to determine the characteristic curve. The tests were carried out using the filter paper method with the Whatman No. 42.

As a practical example of the effect of suction pressure on slope stability, we will consider an object located in Zelenograd.

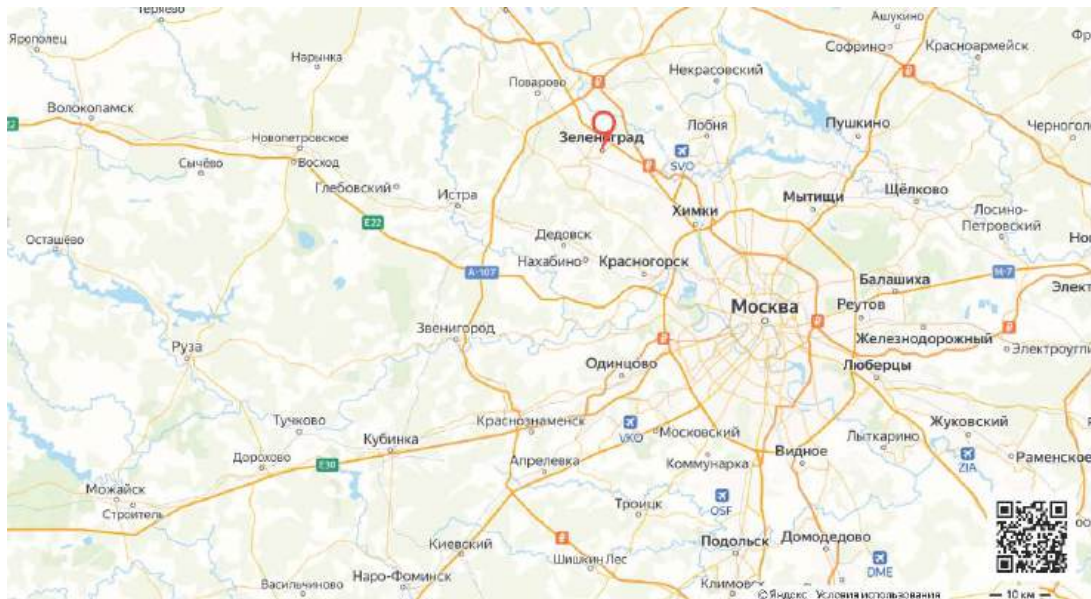


Figure 1: The city of Zelenograd (marked with a red dot).

The engineering-geological section is represented by:

- Modern technogenic (fill) accumulations (tQIV), represented by refractory loams; fine sands of medium density;
- Upper Quaternary cover deposits (prQIII) represented by gray-brown, heavy, semi-hard loams with ferrugination interlayers;
- Middle Quaternary fluvio-glacial water-glacial and lacustrine-glacial deposits of the Moscow horizon (f,lgQIIms), represented by soft- and refractory loams, rarely peaty.

As a result of the experiment, the characteristic soil-water curves presented in Figures 2 and 3 were obtained. Thus, for the mantle loams, the value of the initial suction pressure, or air entry value (AEV), is 17 kPa. Before this point, there is a boundary effect zone. The value of matrix suction according to the graph is 199 kPa, and after it, there is a residual zone. Between the points of air entry and matrix suction values, there is a transition zone. For fluvio-glacial deposits, the value of the initial suction pressure, or air entry value (AEV), is 14 kPa. Before this point, there is a boundary effect zone. The value of matrix suction according to the graph is 207 kPa, and after it, there is a residual zone.

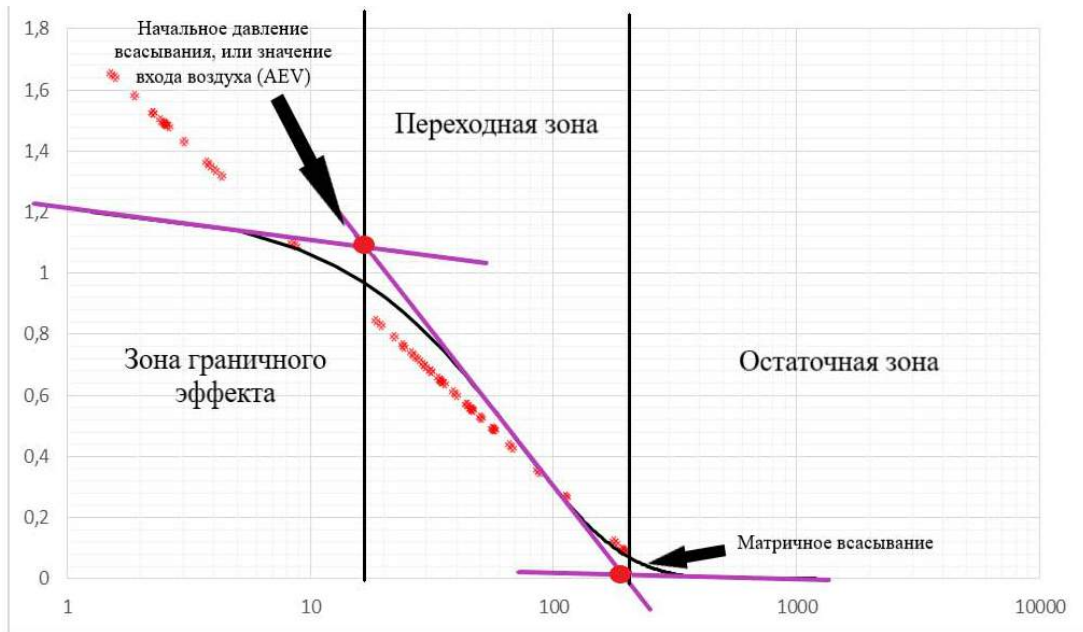


Figure 2: Results of constructing SWCC for cover loams

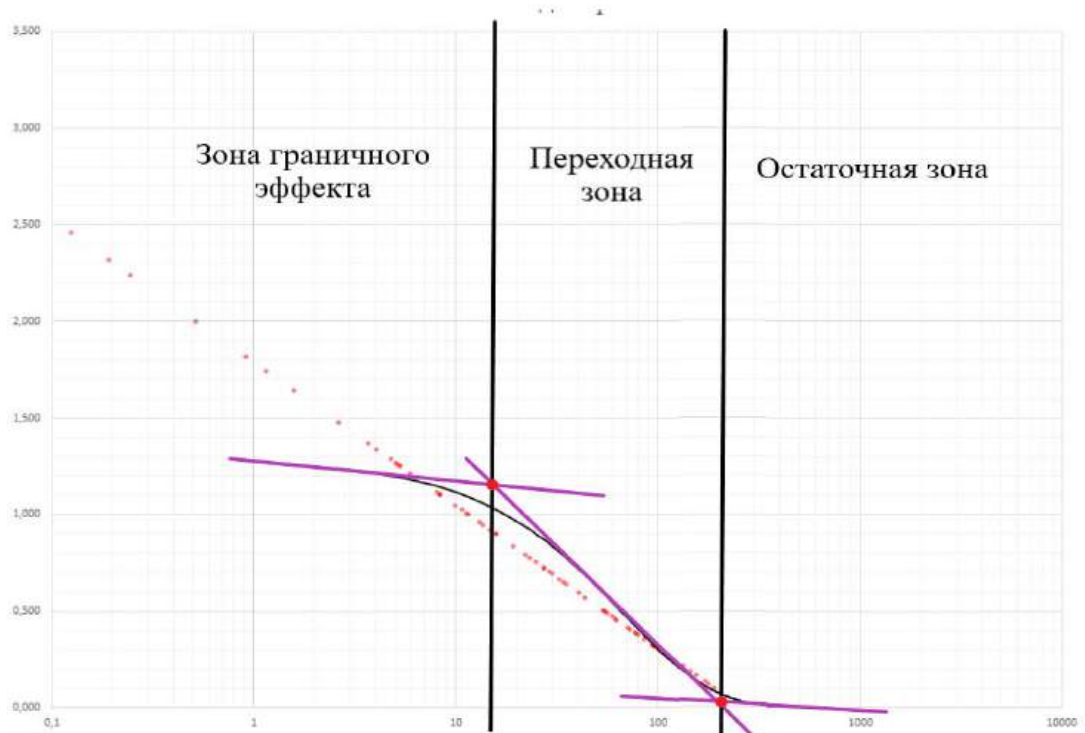


Figure 3: Results of SWCC construction for fluvioglacial loams

In addition, X-ray diffraction quantitative analysis was performed for qualitative assessment. The results of mineral analysis are presented in Table 2:

Sample	smectite *	illite	chlorite	kaolinite	palygorskite	quartz	calcite	Actinolite	Aragonite	dolomite	potassium feldspar (микронит)	plagioclase (альбит)	Gypsum	Pyrite	Anatase
Sheet	8,7	23,8	10,7	4,0	-	17,2	2,2	-	-	20,1	8,7	4,6	-	-	-
Glacial	13,6	5,1	2,1	3,3	2,0	45,2	11,6	-	-	7,5	7,6	2,0	-	-	-

\*smectite and mixed-layer illite-smectite minerals

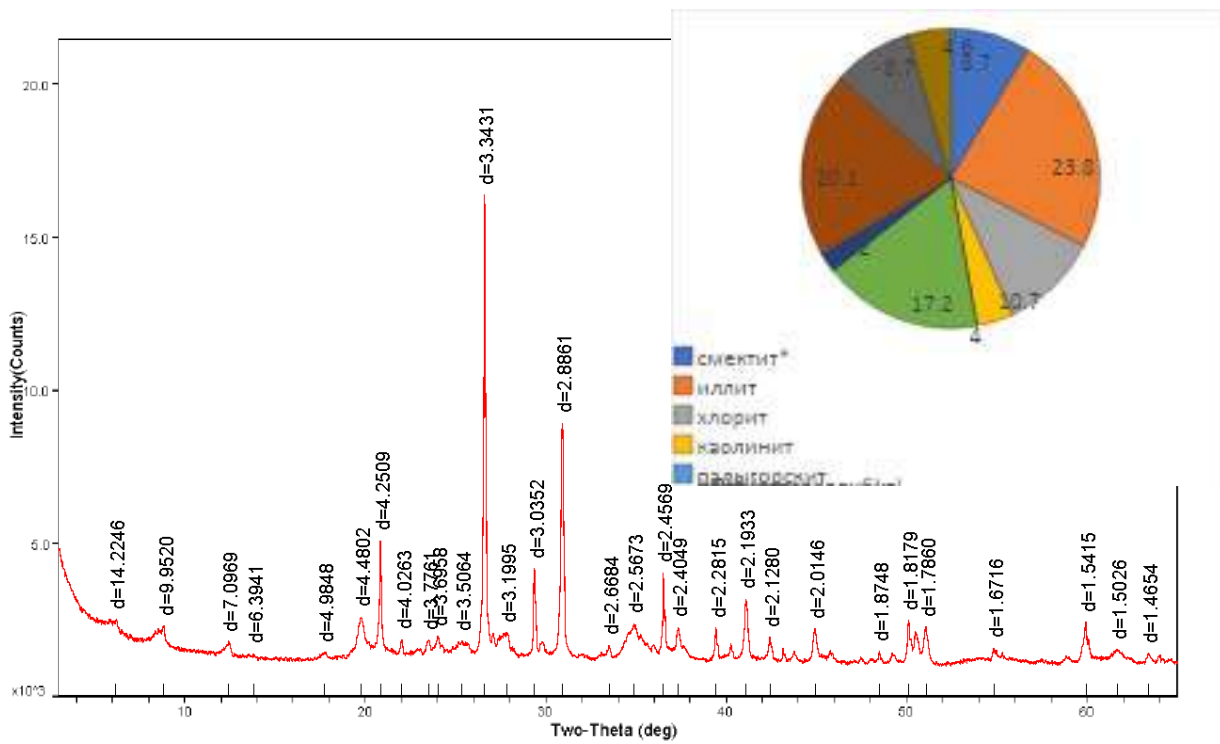


Figure 4: Results of mineral analysis for cover loams

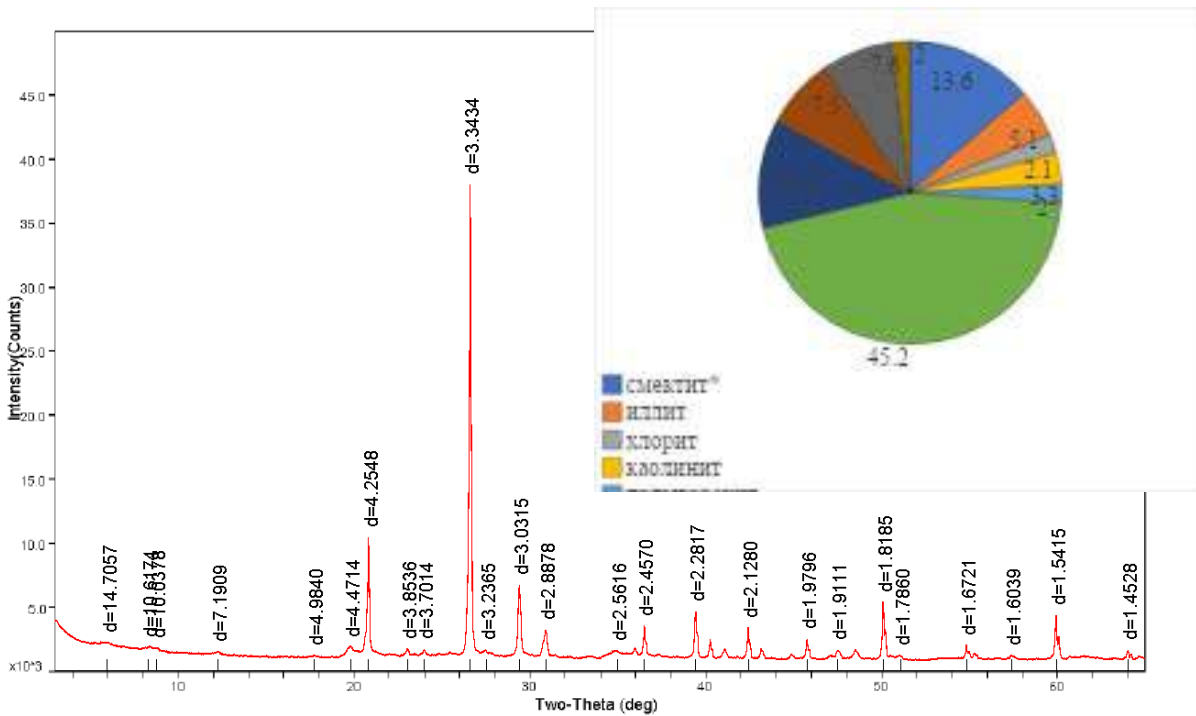


Figure 5: Results of mineral analysis for fluvioglacial loams

As mentioned above, suction pressure and capillarity can affect the physical and mechanical properties of soils, which in turn affect the value of the slope stability coefficient.

The solution of the Mualem-Van Genuchten model is presented in the Plaxis software for geotechnical calculations, but it is necessary to use the SWCC soil-water characteristic curve, constructed on the basis of laboratory tests or adopted according to recommendations for different types of soils.

Subsequently, calculations were performed in the Plaxis software without taking into account the suction pressure and with it, while all other parameters of the model remained unchanged.

The modeling results are presented in Figures 6-10.

The first calculation was performed for a preliminary assessment of the effect of suction pressure. For this purpose, a stability calculation was performed for a slope composed of one type of soil. The calculation results are shown in Figures 6 and 7. The modeling analysis showed that even if the slope is formed by one type of soil, there is a difference between the values of the stability coefficient without taking into account suction and with it. Without suction and capillarity, the stability coefficient was 1.21. With suction and capillarity, the stability coefficient was 1.38.

The analysis of the figures showed that the slope stability coefficient without taking into account suction and capillarity of the soil is 1.46 (Figure 8). With suction and capillarity, the stability coefficient increases to 1.67 (Figure 9). If only the suction pressure is taken into account in the modeling process without taking capillarity into account (Figure 10), the stability coefficient is 1.66.

In addition, the pore pressure was calculated, including the negative one. The maximum value was 39.94 kPa, and the negative value was -50 kPa according to the boundary condition. The maximum suction pressure was also 39.94 kPa.

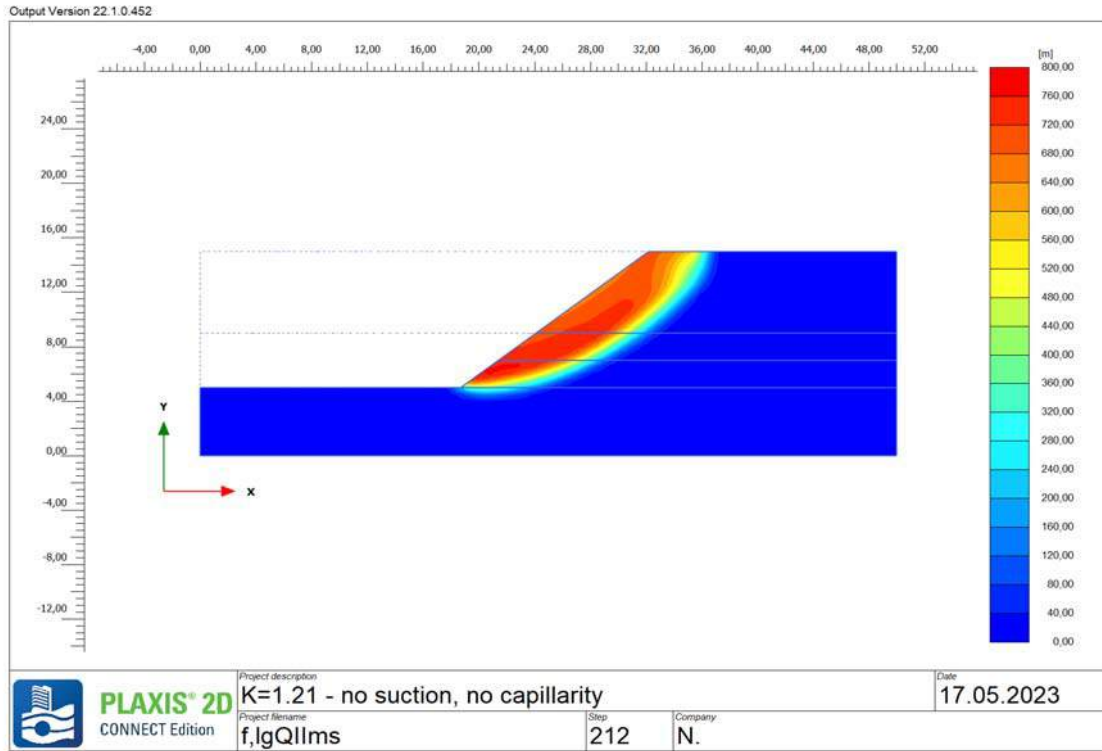


Figure 6

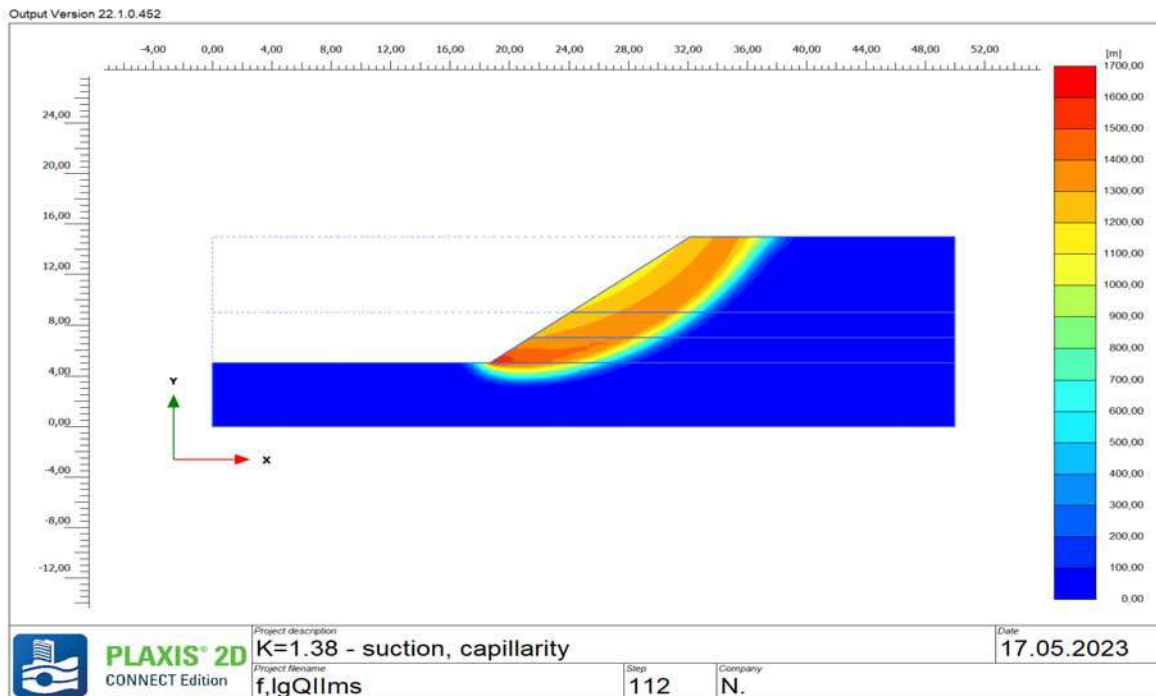


Figure 7

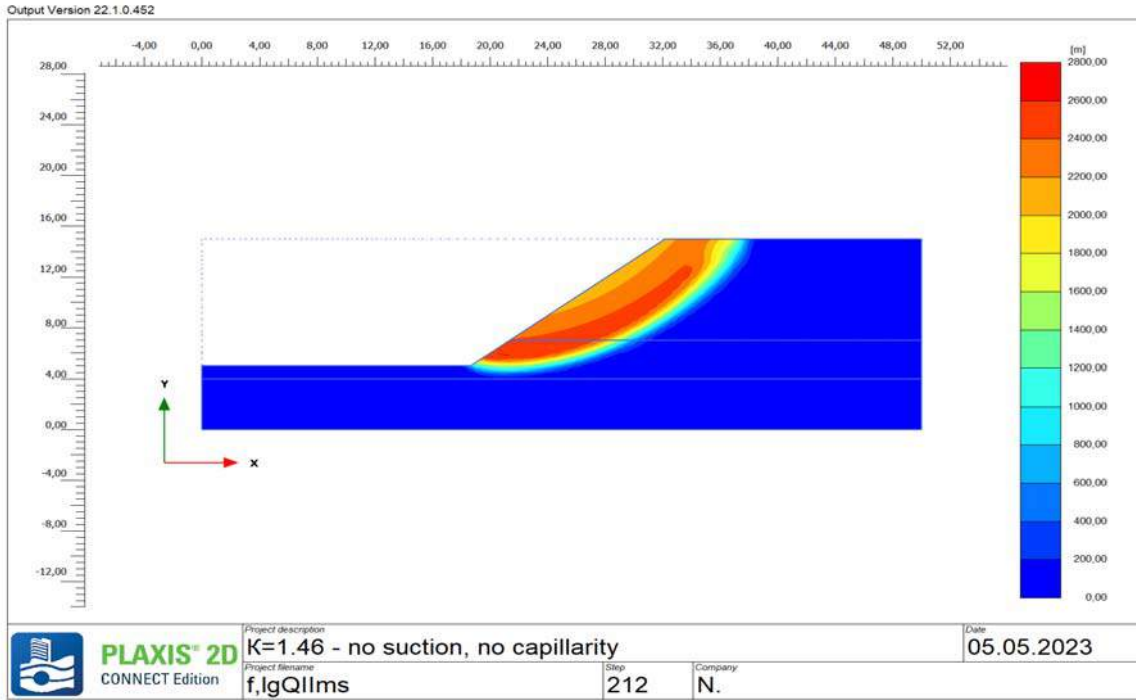


Figure 8

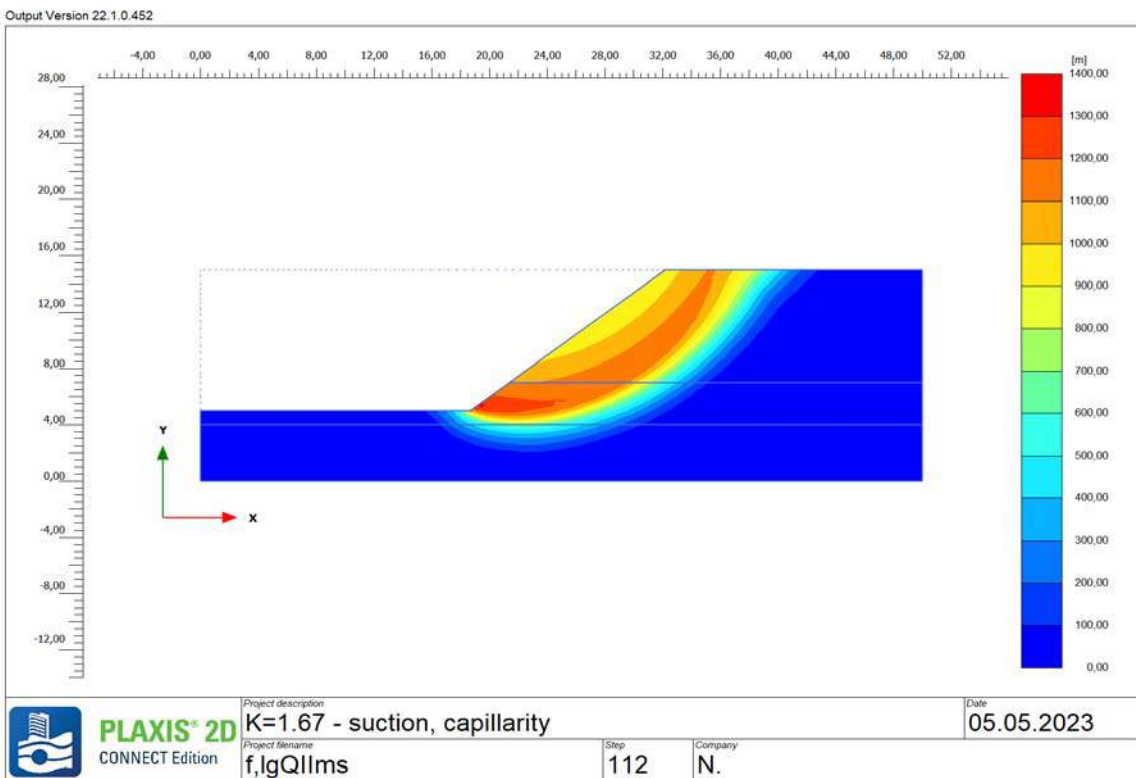


Figure 9

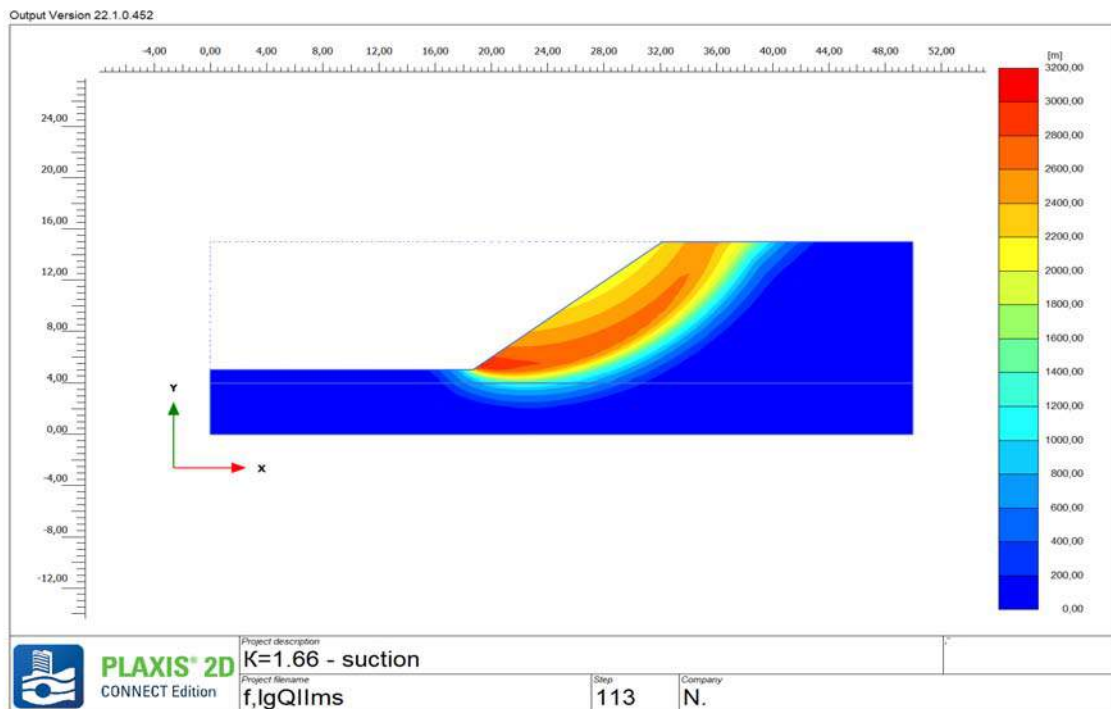


Figure 10

## REFERENCES

1. Bishop, A. W. (1959). "The principle of effective stress." *Teknisk Ukeblad I Samarbeide Med Teknikk*, 106(39), 859–863.
2. Fredlund, D. G., and Rahardjo, H. (1993). *Soil mechanics for unsaturated soils*, Wiley, New York.
3. Fredlund, D.G., *Soil Suction Monitoring for Roads and Air Fields*, 1989, In *Proceedings of Symposium on the State of the Art of Pavement Response Monitoring Systems for Roads and Airfields*, pp.113-121.
4. Greco, Roberto. *Rainfall-induced landslides in shallow granular soil covers: hydrological processes controlling slope response to seasonal climatic forcing*. iRALL School, October 20, 2021, Chengdu University of Technology. Microsoft PowerPoint Presentation.
5. Houlsby, G. T. (1979). "The work input to a granular material." *Geotechnique*, 29(3), 354–358.
6. Houlsby, G. T. (1997). "The work input to an unsaturated granular material." *Geotechnique*, 47(1), 193–196.
7. Khalili, N., and Khabbaz, M. H. (1998). "A unique relationship for  $X$  for the determination of shear strength of unsaturated soils." *Geotechnique*, 48(5), 681–688.
8. Lu, N., and Likos, W. J. (2006). "Suction stress characteristic curve for unsaturated soil." *J. Geotech. Geoenviron. Eng.*, 132(2), 131–142.
9. *Methodical recommendations for the course "Fundamentals of Hydrogeology"* Authors: K.V. Belov, A.B. Lisenkov. MGRI. Moscow, 2019, 47 p.
10. Mitchell, J. K., and Soga, K. (2006). *Fundamentals of soil behavior*, Wiley, New York.
11. Muraleetharan, K. K., and Wei, C. (1999). "Dynamic behavior of unsaturated porous media: Governing equations using the theory of mixtures with interfaces (TMI)." *Int. J. Numer. Analyt. Meth. Geomech.*, 23, 1579–1608.
12. Murray, E. J. (2002). "An equation of state for unsaturated soils." *Can. Geotech. J.*, 39, 125–140.
13. Ning Lu, F.ASCE. *Is Matric Suction a Stress Variable?* *Journal of Geotechnical and Geoenvironmental Engineering* Volume 134, Issue 7 [https://doi.org/10.1061/\(ASCE\)1090-0241\(2008\)134:7\(899\)](https://doi.org/10.1061/(ASCE)1090-0241(2008)134:7(899))

14. Novgorodova M.A., Gorobtsov D.N., Ushakov A.S. Effect of suction pressure of clayey partially saturated soils on slope stability. *Proceedings of Higher Educational Establishments: Geology and Exploration*. 2024;66(2):69-79. <https://doi.org/10.32454/0016-7762-2024-66-2-69-79>.
15. Skorobogatko K.V. 2022. Soil mechanics of saturated and unsaturated soil regions // Specialized calculation complexes MIDAS URL: <https://midasoft.ru/blog/mekhanika-gruntov-nasyshchennoy-i-nevodonasyshchennoy-oblasti-grunta/> (date of access: 02.03.2024).
16. Skorobogatko K.V. Modeling of soil properties in the unsaturated zone above the groundwater level // Specialized calculation complexes MIDAS. URL: <https://midasoft.ru/blog/modelirovanie-svoystv-grunta-v-nevodonasyshchennoy-oblasti-vyshe-urovnya-gruntovykh-vod/> (date of access: 08.10.2024).